Exploring Stable Population Concepts from the Perspective of Cohort Change Ratios: Estimating the Time to Stability and Intrinsic r from Initial Information and Components of Change*

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Abstract. Cohort Change Ratios (CCRs) have a long history of use in demography. Except in their restrictive form as survival rates, CCRs, appear, however, to have been overlooked in regard to a major canon of formal demography, stable population theory. We believe that it is worthwhile to move beyond this restrictive form and examine full CCRs because they contain information about both migration and mortality. As a means of exploiting this information, CCRs are explored as a tool for examining the transient dynamics of a population as it moves toward the stable equivalent that is captured in most formal demographic models based on asymptotic population dynamics. We employ simulation and a regression-based approach to model trajectories toward this stability. This examination is done in conjunction with the Leslie Matrix and data for 62 countries selected from the US Census Bureau's International Data Base. We use an Index of Stability (***S***), which defines stability as the point when ***S*** is equal zero (operationalized as ***S*** = 0.000000). The Index also is used to define initial stability for a given population and four subsequent "quasi-stable" points on the temporal path to stability (S=.01, S=.05, S=.001, and S=.0005). The Stability Index can be readily calculated and provides an easy-to-interpret measure of the distance to stability. We use ergodicity as a guide in that it directly states that initial conditions are "forgotten" when a population reaches stability and only vital rates play a role. Given this, we not only explore the effect of the initial Stability Index and vital rates, but also the effect of migration on the temporal path to stability The regression-based analysis reveals that the initial conditions as defined by the initial Stability Index along with fertility and migration play a role in determining time to stability up until the quasi-stable point of ***S*** =.0005 is reached. After this point, the initial conditions are no longer a factor and mortality joins the fertility and migration components in determining the remaining time to stability. These findings are consistent with ergodicity. Overall all, we find that fertility and mortality have an inverse relationship with time to stability while migration has a positive relationship. The initial Stability Index has an inverse relationship with time to quasi-stability at ***S*** = .01, ***S*** = .005, ***S*** = .001, and ***S*** = .0005. Continuing the use of regression analysis, we also find that a regression model works very well in estimating the intrinsic rate of increase from the initial rate of increase, but this model can be improved by adding the components of change. We also compare time to stability and intrinsic r as estimated using the CCR Leslie Matrix approach to, respectively, estimates of time to stability and intrinsic r found using analytic methods and find that the former are consistent with the latter. We discuss our findings and provide suggestions for future work.

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12. 1 Introduction

Cohort Change Ratios (CCRs) have a long history of use in demography, dating back to at least Hardy and Wyatt (1911). Under the rubric of "Census Survival Ratios," they have been used to estimate adult mortality (Swanson and Tedrow 2012, United Nations 2002) and under the rubric of the "Hamilton-Perry" method, they are used to make population projections (Hamilton and Perry 1962, Smith Tayman and Swanson 2013, Swanson and Tayman 2013, Swanson and Tayman 2014, Swanson and Tedrow 2013, Swanson Schlottmann and Schmidt 2010). However, CCRs appear to have been largely overlooked in regard to examining the concept of a stable population (Caswell 2001, Coale 1972, Dublin and Lotka 1925, Lotka 1907, Preston et al. 2001, Schoen 2006, United Nations 1968).

We believe that it is worthwhile to examine CCRs in regard to the concept of a stable population because they contain information about both migration and mortality. As a means of exploiting this information, we explore CCRs as a tool for examining the transient dynamics of a population as it moves toward the stable equivalent that is captured in most formal demographic models based on asymptotic population dynamics. We employ a Leslie Matrix framework with an invariant set of CCRs and a regression-based approach to model trajectories toward stability.

The usual approach to generating a stable population is the use of a constant set of fertility and mortality rates applied to an arbitrarily chosen age distribution (Caswell 2001, Coale 1972, Dublin and Lotka 1925, Lotka 1907, Preston et al. 2001). When a given population is subjected to constant fertility and mortality rates, it will eventually reach stability and have a constant rate of growth (Caswell 2001: 79-92). This constant rate of change is known by several names, but in this chapter we use the term “intrinsic r,” which is denoted in this study by “***r***.” It also is the case that ergodicity stipulates that the initial age distribution is “forgotten” by the time the population in question reaches stability (Caswell 2001:79-92, 386-397). Because CCRs are invariant and always positive, the Leslie Matrix framework we use represents a process will lead to a stable population that is ergodic.

Preston et al. (2001), among others, observe that a stable population also will result if a constant set of migration rates is included with sets of constant fertility and mortality rates. A number of papers have extended stable population models to include migration, and have examined its impact on long-term population stability. Espenshade (1986: 249), for example, states: "When migration is recognized, it is often to note that migration rates can be incorporated into survival rates so that no substantial modifications of the stable model are required.” Sivamurthy (1982) also considers net-migration within the standard stable population model in the same manner. We find that this approach is both similar and dissimilar to the Hamilton-Perry model we employ in this chapter. It is similar in that net-migration schedules are components of the CCRs we use in the sub-diagonal of the Leslie matrix; it is dissimilar in that the approach developed by Sivamurthy (1982) and applied by Espenshade (1986) allows only for net in-migration, while ours allows for both net in-migration and net-outmigration.

It is important to acknowledge that there are analytic solutions to some of the questions we are asking. For example, the eigenvalues and eigenvectors of the CCR Leslie matrix can be computed and analyzed to yield "damping ratios," which provide a basis for estimating the time to stability (Caswell 2001: 95-97). Similarly, Caswell (2001: 74-75) provides an analytic basis for estimating ***r*** that can be used with CCRs. In Section 12.8, we use these analytic approaches to develop estimates of time to stability and ***r***, which are compared to those found using the CCR Leslie Matrix approach. We also acknowledge that while a CCR approach has not before been used, it has been proven that any population subject to a constant set of positive rates (such as CCRs) will converge to stability (Alho 2008, Cohen 1979a, Espenshade 1986, Mitra and Cerone 1986). This is consistent with the Perron-Frebonius and ergodicity theorems (Caswell 2001: 79-87, 369-370). While we know that analytic solutions exist that should work with the CCR approach and expect a population subject to a constant set of CCRs to converge to stability, in point of fact, however, the CCR approach has not yet been examined Moreover, neither the Perron-Frebonius nor ergodicity theorems actually produce a stable population. Hence, we believe that the CCR approach is worth exploring.

In addition to using CCRs as a new way to examine the concept of population stability, we also use a measure that, like CCRs, has been employed by demographers (and others) for a long time, but appears to have been overlooked in regard to population stability. This is the Index of Dissimilarity (Hobbs 2004:

157-158), which we use as a measure of both stability itself and the distance to stability. In our employment of the Index of Dissimilarity, we refer to it as the "Stability Index" (***S***). Its application is useful here because it is a bounded measure that is easy to interpret and it has characteristics not found in existing measures of the distance to stability.

We use a CCR Leslie Matrix framework in conjunction with a series of regression models to estimate the number of years to stability at selected levels of ***S***, which represent not only stability itself, but "quasi-stability" points on the way to stability. As we discuss later, we find that these regression models work reasonably well in providing an estimate of the time both to these selected points of quasi-stability and stability itself for a given population within the CCR Leslie Matrix framework. Importantly, these models provide information on the role of the components of change in determining the time to stability, as well as the initial conditions (as measured by the initial Stability Index). Continuing the use of regression analysis, we also find that a regression model works reasonably well in estimating the intrinsic rate of increase (***r***) from the initial rate of increase (IRI)

Including this section (12.1), this chapter is composed of nine sections. In the next section (12.2), we discuss the CCR method while in Section 12.3 we briefly discuss stable population concepts. The Stability Index and the CCR approach are discussed in Section 12.4. Section12.5 discusses the data we employ along with the Leslie Matrix we use to implement this approach. Section 12.6 describes the regression models used in the estimation of time to selected points of quasi-stability and stability while Section 12.7 describes the estimation of ***r*** from the initial rate of increase (IRI). Section 12.8 provides a comparison of the estimates of time to stability and ***r*** found using the CCR approach with results found using analytical methods for the same populations. Section 12.9 concludes this chapter with a discussion.

12.2 Cohort Change Ratios

Because we use a constant set of CCRs to project a population to stability, we discuss them in conjunction with the Hamilton-Perry method. The Hamilton-Perry Method is a variant of the cohort-component method that has far less intensive input data requirements. Instead of mortality, fertility, migration, and total population data, which are required by the cohort-component method, the Hamilton-Perry method requires data only from two census counts (or estimates) that provide population data by age (Hamilton and Perry 1962, Smith Tayman and Swanson 2013, Swanson and Tayman 2013, Swanson Schlottmann and Schmidt 2010).

The Hamilton-Perry method moves a population by age (and sex) from time t to time t+k using CCRs typically computed from data in the two most recent censuses.[1] It consists of two steps. The first uses existing data to develop CCRs and the second applies the CCRs to the cohorts of the launch year population to move them into the future. The second step can be repeated infinitely, with the projected population serving as the launch population for the next projection cycle. The formula for the first step, the development of a CCR is:

$$ {}_{n}CCR_{x,i} = {}_{n}P_{x,i,t} \,/\, {}_{n}P_{x-k,i,t-k} \qquad [1] $$

where

${}_nP_{x,i,t}$ is the population aged x to x+n in area i at the most recent of the two points in time for which the data are available(t),

${}_nP_{x-k,i,t-k}$ is the population aged x-k to x-k+n in area i at the earlier of the two points in time for which the data are available (t-k),

k is the number of years between the two points in time for which the population data are available in area i and it needs to be consistent with the age groups (${}_nP_x$) used for the population in question and not greater than 10.

The basic formula for the second step, moving the cohorts of a population into the future is:

$$ {}_nP_{x+k,i,t+k} = ({}_nCCR_{x,i})*({}_nP_{x,i,t}) \qquad [2] $$

where

${}_nP_{x+k,i,t+k}$ is the population aged x+k to x+k+n in area i at time t+k

$$ {}_nCCR_{x,i} = {}_nP_{x,i,t} / {}_nP_{x-k,i,t-k} $$

${}_nP_{x,i,t}$ is the population aged x to x+n in area i at the most recent point in time for which the data are available (t),

k is the number of years between the two points in time for which the population data used to construct the CCRs were available. This time interval becomes the length of the forecast cycle and must be consistent with the age groups (${}_nP_x$) used for the population in question and should not be greater than 10.

The CCRs reflect differential net undercount error and both the effect of mortality and migration. CCRs can be less than one (1.00) or greater than one (1.00). In the absence of differential net undercount error, the following observations hold: (1) in any age group where a CCR is greater than one, net in-migration is occurring; (2) in young ages (i.e., 20-24, 25-29, and 30-34) where mortality rates are low, CCRs less than one generally indicate net out-migration; and (3) at older age groups where mortality is high, CCRs less than one generally provide a picture of cohort survival rates. Thus, in the absence of differential net undercount error, CCRs and the combined effects of mortality and migration change with age, with mortality becoming a dominant component of a CCR at older ages (e.g., 60-64, 65-69, and 75+).[2]

Given the nature of the CCRs, 5-9 is the youngest age group for CCRs can be calculated if there are 5 years between the points in time that the data are assembled. If there are ten years between the data points, then 10-14 is the youngest age group for which CCRs can be calculated. To project the population aged 0-4 (and 5-9) one can use the Child Woman Ratio (CWR), or more generally a "Child Adult Ratio" (CAR). It does not require any data beyond what is available in the decennial census. There are different ways to develop a CAR (Hamilton and Perry 1962, Smith Tayman and Swanson 2013: 176-180, Swanson and Tayman 2013, Swanson Schlottmann and Schmidt 2010). As we discuss in Section 12.5, we do not use "CARs" because we use employ age-specific fertility rates to generate the number in the youngest age group, which given our five-year data structure is 0-4.

CCRs for the oldest open-ended age group differ slightly from the CCRs for the age groups up to the oldest open-ended age group and for which a CAR is not required. If, for example, there are five years between the points in time for which the data are assembled (k=5) and the final closed age group is 70-74, with 75+ as the terminal open-ended age group, then calculations for the $_{\infty}CCR_{75,i,t}$ require the summation of the appropriate age groups to get the population age 70+ at time t-k, which is then used as the denominator in finding the CCR for those aged 75+:

$$_{\infty}CCR_{75+,i,t} = {}_{\infty}P_{75,i,t} / {}_{\infty}P_{70+,i,t-k} \qquad [3]$$

The formula for projecting the population 75+ of area i for the year t+k is:

$$_{\infty}P_{75,imt+k} = ({}_{\infty}CCR_{75,i,t})^{*} ({}_{\infty}P_{70,i,t}) \qquad [4]$$

Table 1 provides an illustrative example of the Hamilton-Perry Method for Austria, which uses data from the US Census Bureau’s International Data Base for 2000 and 2005 to generate a 2010 population projection of the population by age for both sexes combined.

(Table 1 About Here)

Table 1 shows that launching from a total population of 8,184.691 in 2005, the Hamilton-Perry Method generates a 2010 total population of 8,268,696 for Austria using the 2000-2005 CCRs and a midpoint (2002.5) set of age-specific fertility rates. The increase largely reflects Austria’s net in-migration among young adults and their children (all of the CCRs from age 5-9 to age 35-39 exceed 1.000).

12.3 A Stable Population: A Brief Overview of the Traditional Approach

As noted earlier, a stable population has an invariable relative age structure and a constant rate of growth. That is, the proportion of people in each age group remains constant over time and the population as a whole has a constant rate of increase (Coale 1972, Dublin and Lotka 1925, Lotka 1907, Preston et al. 2001).

As mentioned earlier, an important feature of the stable population model is ergodicity, whereby over time a population "forgets" its initial age distribution as it convergences on stability (Coale 1972, Cohen 1979a, Preston et al. 2001). There is both a strong and weak form of the ergodicity theorem (Caswell 2001: 79-92 & 386-387, Cohen 1979a). We use ergodicity as a general guide for part of our analysis.

Alfred J. Lotka is generally credited with formulating the idea of a stable population and exploring many of its important features, including the finding that in the absence of migration, a population subject to constant fertility and mortality rates would eventually have a constant rate of natural increase (Dublin and Lotka 1925, Lotka 1907). Continuing the analytical tradition established by Lotka, many researchers have examined the idea of a stable population and refined its underlying theory and extended its applications (Alho and Spencer 2005, Arthur 1981, Arthur and Vaupel 1984, Bacaër 2011, Bennett and Horuchi 1984, Caswell 2001, Coale 1972, Cohen 1979a, Kim and Sykes 1976, Le Bras 2008, Pollard et al. 1974, Popoff and Judson 2004, Preston et al. 2001, Preston and Coale 1982, Rogers 1985, Schoen 1988, Schoen 2006, United Nations 1968). Much of this research has, however, been confined to examining a population not affected by migration. Preston et al. (2001) and others (Espenshade 1986, Sivamurthy 1982) have suggested that this is an un-necessarily restrictive assumption. Nonetheless, other than a few exceptions, such as Espenshade et al. (1982), Rogers (1985, 1995), and (Rogers et al. 2010), this restriction appears to remain a governing force in the examination of stable population ideas. It is useful to note that even the approach for dealing with migration developed by Sivamurthy (1982) and used by Espenshade (1986) is limited in its application because it requires that a population have only net in-migration at all ages, a condition not always found in human populations.

Another restrictive assumption that has governed much of the work on stable populations is defined by the so-called “two-sex” problem (Pollak 1986, Preston and Coale 1982). In this problem (which evidently stems from Lotka’s 1907 formulation of a stable population), only one sex (virtually always women) was examined in the context of a stable population because of problems reconciling the numbers of births resulting from including both sexes. However, as Preston et al. (2001) show a “female-dominant” approach to fertility offers a convenient way around this problem, one that has been employed in different ways by others (Barclay, 1958: 216-222; Keyfitz and Flieger, 1968). Yet another somewhat restrictive idea associated with the traditional approach is that if one is using a discrete approach, such as found in this chapter, a discrete version of Lotka’s equation is required. Caswell (2001: 197), however, observes that the best way to implement a discrete version of Lotka’s continuous equation is to use a discrete-time model rather than attempt to write discrete versions of Lotka’s equation. This is the approach we follow, as discussed in the next section.

12.4 A Stable Population: The CCR Approach and the Index of Stability

The CCR approach simply takes the cohort change ratios found at a given point in time and holds them constant until the population reaches stability. In terms of our implementation of this approach within the Leslie Matrix framework, this also means we hold the initial ASFRs constant as well.

To determine when a population has reached stability, the well-known "Index of Dissimilarity" is employed as an "Index of Stability" (***S***).[3] The index is defined as:

$$\boldsymbol{S} = \{0.5 * \sum \left| ({}_np_x / \sum {}_np_x)_{t+y} - ({}_np_x / \sum {}_np_x)_t \right| \}. \qquad [11]$$

where

p = population
y = number of years between census counts/projection cycles
x = age
n = width of the age group (in years)
t = year

S compares the relative age distribution at one point in time (t+y) with the relative age distribution at the preceding point in time (t) within the forecast cycle (the forecast cycle we employ is five years) and measures the proportion of one population would have to be re-allocated to match the relative age distribution of the other. ***S*** ranges from 0 to one (1); a score of zero means that there is no difference between the two relative age distributions and no re-allocation is needed, which is the minimum re-allocation that can take place. A score of one (1) means that maximum re-allocation is required for the two relative age distributions to match. A score of one (1) can be interpreted in several ways, but a common interpretation is all of the numbers by age in one population would have to be re-allocated in order to match the distribution of the numbers by age in the comparison population. Since we are dealing with the same population at viewed at two successive points in time, this leads to viewing a score of one (1) as an indication that all of the numbers by age at time t would have to be reallocated to match the numbers by age of the same population at the preceding point in time in terms of the forecast cycle.

S exploits the idea that when a population is stable, the sum of the differences between the relative size of corresponding age groups at time t+y and time t is zero (which we have operationalized as ***S*** = 0.000000). Thus, at a point when the sum of the differences across all of the corresponding age groups is zero between the time point at the end of a five-year forecast cycle and the preceding time point of the five-year forecast cycle, the population has reached stability. The advantage of using the Index of Dissimilarity as ***S*** is that it provides a bounded measure (between zero and 1) and has a clear interpretation. As mentioned earlier, this index can be used both to define stability and provide a measure of the distance to stability and we use it here in both regards. It could, of course, be used in conjunction with the traditional approach, but this appears not be the case in that our search of the literature found nothing in regard to the use of the Dissimilarity Index to either define stability (see e.g., Caswell 2001, Preston et al. 2001, Schoen 2006) or measure the distance to stability (see, e.g., Caswell 2001, Schoen 2006, Schoen and Kim 1991, Tuljapurkar 1982). We believe that ***S*** possesses several desirable characteristics not found in other measures. First, in regard to stability itself, it is a summary measure of population age structure. Second, as a measure of the distance to stability, it only requires information on two successive current age structures, unlike, say, the Kullback Index, which requires information on the current age structure and the age structure at stability (Schoen 2006, Schoen and Kim 1991, Tuljapurkar 1982). In addition, the Dissimilarity Index can also be calculated between an initial age structure of a population (or the population's age structure at any point on the path to stability) and its age structure at stability, which makes it conceptually similar to the Kullback Index. This use of the Dissimilarity Index is described by Keyfitz (1968: 47). Thus, we find that there are four useful features of the Index of Dissimilarity in regard to the concept of population stability. First, it provides a summary measure of relative age structure at origin, an important aspect of initial conditions. Second, it provides a measure of stability itself in that when $\boldsymbol{S}$ = zero (in the context of the using the CCR or other approach within a Leslie Matrix framework) a population has converged to stability. Third, by looking at $\boldsymbol{S}$ at any given point on the path to stability, we get an idea of the distance to stability in that we can see how far it is from zero. Fourth, by computing an Index of Dissimilarity between the age structure at origin (or any other point on the path to stability) and the age structure at stability, we can see how much of the initial age structure must be "re-allocated" in order to match the age structure at stability. These four features are found neither in any other single measure of stability nor any other single measure of the distance to stability.

Using ***S*** in conjunction with the CCR approach is a natural fit because the latter is implemented using a chain of fixed forecast cycles, which in the case of moving to stability ends when successive age distributions are proportionately equal and ***S*** = zero. We generate a chain of fixed forecast cycles by using a Leslie Matrix (Caswell 2001: 8-34), which we illustrate here using our example, Austria. The CCRs and the ASFRs for Austria shown in Table 1 are held constant from the launch year (2005) to a year where ***S*** = zero (relative to the preceding year in the five-year projection cycle). This occurs at the year 2485.

Figure 1 provides the change in ***S*** from 2005 to 2485 for Austria as it proceeds to stability. As it shows, the path to stability is nearly monotonic and definitely not linear. It initially declines rapidly to the point where S is approximately equal to .005, but the change in ***S*** slows substantially around the year 2185, which 180 years from the launch year. From there to 2485, ***S*** moves incrementally to zero as can be seen in Figure 1.

(Figure 1 About Here)

With some variations, the path to stability shown for Austria in Figure 1 is generally found for all of the other 61 countries we use in this analysis. For many, the path is fully monotonic, others, nearly monotonic, but all are non-linear. This finding is consistent with findings elsewhere (Nair and Nair 2010, Schoen and Kim 1991). There is an initial and rapid decline in ***S***, the Index of Stability, which at some point slows. From the point at which it slows, it moves very slowly until stability is reached. As such, taking into account the slightly non-monotonic nature of the initial part in which ***S*** declines rapidly, these paths generally fit the form of “long-tailed” negative exponential distributions, where those showing monotonic decline would be better fits than those showing decline that is not precisely monotonic.

12.5 Data and Methods

As illustrated in the example for Austria, applying a constant set of CCRs and ASFRs to a given population will yield a stable population. We pursue this idea by applying this approach to 62 countries taken from the US Census Bureau’s International Data Base. These countries were selected using two major criteria: First, the United Nations (2008) identifies them as having “reasonably reliable data;” and, second, their launch year populations are greater than 500,000. The data in the International Data Base are provided on an annual basis, so clearly they represent estimates informed by census and register information. These data allowed us to select a five-year forecast cycle, which is consistent with the five-year age groups we use (0-4, 5-9, 10-14,…, 70-74, 75+).

Exhibit 1 provides a list of these 62 countries along with their initial population counts and the region of the world in which they are found.

(Exhibit 1 About Here)

For these countries we used data from the early part of the 21st century to develop the input data needed to perform the projections. For those countries which have census counts in years ending in one and six (e.g., Australia, Canada, Fiji, Ireland, the United Kingdom), we used data for 2001 and 2006; for countries which have census counts in years ending in zero or with excellent population registers, we used data for 2000 and 2005 (e.g., Austria, Cuba, Finland, Poland, the United States). We organized the population data into 16 age groups, 0-4, 5-9, 10-14,…, 70-74, and 75+. For these countries, we also obtained ASFRs for the same years for which we obtained the population data. In both cases, we selected data from these past points in time in order to ensure that the input data were, in fact, "reasonably reliable" in that these data could have been informed by subsequent census counts and administrative data (e.g., 2006 and 2011 for countries such as Australia, Canada, Fiji, Ireland, and the United Kingdom; or 2010 for countries such as Austria, Cuba, Finland, Poland, and the United States).

The population data were used to generate the (constant) set of CCRs that was applied to the most launch year (2005 or 2006) to take the country in question to stability. The ASFRs were averaged between the two years. Because they related only to the female population in each age group, they were "deflated" so that they applied to the total population (both males and females) and then multiplied by five to match the five year cycle used in the projection sequence.

The 16 age groups yielded a 16 x16 Leslie Matrix. The CCRs are found in the major diagonal and the ASFRs in the first row, elements 5 through 10 (which correspond to age groups 20-24, 25-29,…, 45-49). Exhibit 2 shows the layout of this matrix for Austria.[4]

(Exhibit 2 About Here)

The population data used to calculate CCRs were also used to calculate the initial Stability Index for each of the 62 countries. We also calculated a measure of net migration from the CCRs. This measure is the mean of the CCRs for age groups 20-24, 25-29, and 30-34. We selected these ages because they are closely associated with the ages at which adult migration is most likely to occur at the national level. The CCRs for these age groups also include mortality, but the mortality effects at these age groups are minimal.

In addition to the fertility data and the population data needed to develop CCRs, we acquired life expectancy data from the Census Bureau's International Data Base for the 62 countries used in this study. We did this to have a complete set of indicators for all three of the components of population change.

Exhibit 3 provides summary statistics for these measures as well as the average times to stability (when ***S*** = zero, which recall we have operationalized as ***S*** = 0.000000) and the selected points of quasi-stability used in the study, ***S*** =.01, ***S*** = .005, ***S*** = .001, and ***S*** = .0005. The selection of these points has no substantive significance beyond the fact that our visual inspections of the graphs showing the paths to stability for all 62 countries suggested that they generally encompass portions of the path to stability that in terms of time are rapid (.01), somewhat less rapid (005), slow, (.001) and very slow (.0005). An example of this can be seen in Figure 1.

(Exhibit 3 About Here)

12.6 Time to Stability

As the title suggests, in this section we are primarily interested in examining the time it takes for a population to reach stability, given its initial conditions and its components of change. However, we also are interested in an issue raised by ergodicity; namely, that at some point on the path to stability, the initial conditions are “forgotten” (Caswell 2001). Hence, in this section, we also explore the point(s) on the path to stability at which this occurs.

Given our earlier work (Swanson and Tedrow 2013) in exploring CCRs and the time to stability, we elected to again use regression as the major tool of inquiry. It is well suited to our task for several reasons: (1) regression models can be specified both in accordance with ergodicity and with our assumption that as a component of population change, migration should be examined along with births and deaths; (2) regression models use empirical data; and (3) characteristics of the models (e.g., the coefficient of variation, statistical tests of inference, and standardized regression coefficients) support analysis in regard to ergodicity. That is, we construct and examine regression models using a combination of variables representing initial conditions and the components of change as predictors of the time to selected quasi-stable points on the path to stability as well as to stability. In this regard, one would expect, for example, that the initial Stability Index would affect the time to a point on the path to stability, but after that point it would no longer have an effect – it would be “forgotten” before stability was reached. Swanson and Tedrow (2013), in fact, found support for this in that the initial Stability Index served reasonably well as a predictor of the time to $\boldsymbol{S}$ = .01, but not to $\boldsymbol{S}$ = zero. A related question is the role played by the components of change in determining time to stability. Clearly, they play a role, but what is the relative importance of fertility vs. mortality, vs. migration? This question has only been partially answered, and generally only in the context of fertility variation (Kim and Schoen, 1993a, Coale 1972, Liaw 1980). Again, we use regression analysis to explore this issue. We explored a number of regression models in terms of the time to stability using the NCSS statistical analysis system. What resulted is a model in which fertility, migration, and mortality all play a role, but the initial conditions (in the form of the initial Stability Index) do not. The model is provided below as equation [12], along with its characteristics:

Estimated N of Years to ***S*** = zero =

$$-824.79 + (6.73*e_0) + (927.84*MEAN\_CCR\_20\_34) - (69.82*TFR) \quad [12]$$

p=.007 *p=.0242* *p =.0001* *p=.0004*

n =62

R^2 =.597

Adj R^2 = .576

where

e_0 = life expectancy at birth (an index of mortality)

MEAN_CCR_20-34 = Mean of the CCRs, Age 20-24, 25-29 & 30-34 (an index of migration that is positively related to net in-migration: as it increases, so does net in-migration)

TFR = Total Fertility Rate (an Index of Fertility)

and the p values (α =.05) are found below the intercept term and each of the three regression coefficients.

In the model shown as equation [12], life expectancy is positively related to the time to stability, as is net in-migration, while fertility is inversely related to the time to stability. Since life expectancy is inversely related to mortality, we can see that: (1) as fertility and mortality increase, the time to stability declines; and (2) as net in-migration increases, the time increases.[5] In advance of generating this equation, we had no expectations in terms of the signs and magnitudes of the regression coefficients, and, therefore, we had no expectations regarding the effects of these variables beyond the idea that the initial Stability Index would not be likely to play a role and that the components of change would play roles. We return to this point in our discussion of Exhibit 4 below and again in the last section.

Because of the different scales at which the predictor variables are measured, we examine standardized regression coefficients to get an idea of the relative importance of these three components of change. The standardized versions of the coefficients found in equation [12] are, respectively, for the measures of life expectancy, migration, and fertility, .2376, .4286, and -.3327. These values suggest that in terms of the time to stability, the level of net in-migration plays the largest role, fertility the second largest, and life expectancy, the least. They also suggest that the time to stability is longer for a population with low mortality, low fertility and high net in-migration than it is for a population with high mortality, high fertility, and low net in-migration. While we do not show the full results, the former description fits Singapore very well (e_0 = 81.7, MeanCCR20-34 = 1.287, and TFR = 0.908) which takes 890 years to reach stability; and the latter description fits El Salvador (e_0 = 71.8, MeanCCR20-34 = 0.869, and TFR = 2.73), which takes only 225 years to reach stability.

To examine the question in regard to the effect of initial conditions on the path to stability, we examined regression models using the components of change in conjunction with the initial Stability Index as predictors of the time to ***S*** = .01,

S = .005, ***S*** = .001, ***S*** = .0005, and ***S*** = zero. We summarize the results of this investigation in the form of Exhibit 4.

(Exhibit 4 About Here)

As can be seen in Exhibit 4, the initial conditions (in the form of the initial Stability Index) have an effect all the way to the time when ***S*** = .001 and play the largest role in terms of the times to ***S*** = .01 and ***S*** = .005, respectively. As we move from ***S*** = .01, to ***S*** = .005, to ***S*** = .001, we reach the last point where this predictor variable is statistically significant and we can see that it declines steadily to this point (from .6175 at ***S*** = .01 to .4787 at ***S*** = .005, to .2297 at ***S*** = .001). By the time we reach the point of quasi-stability where ***S*** = .0005, the initial value of ***S*** is no longer statistically significant and it remains so to the point of stability when ***S*** = zero. Both migration (in the form of Mean CCR for age groups 20-24, 25-29 & 30-34) and fertility (in the form of the Total Fertility Rate) have an effect throughout the entire path, with migration having less of an effect initially (at ***S*** = .01) then having a larger effect than fertility from ***S*** = .005 all the way to when ***S*** = zero. Mortality, in the form of e_0, is not statistically significant on the path to stability when fertility and migration are present until the point where ***S*** = zero, at which time it has the smallest effect of the three predictor variables (.2376). As was the case with the discussion of our expectations regarding equation [12], we had no firm expectations regarding the regression coefficients found in Exhibit 4 other than the following: (1) the initial Stability Index would have an effect up to some point of quasi-stability but not to stability, per the concept of ergodicity; and

(2) it seemed likely to us that all else being equal, fertility would have an inverse relationship with the time to stability, as would mortality, all else being equal.

12.7 Estimating Intrinsic r

A number of methods exists for estimating intrinsic r, which, recall is denoted as ***r*** by us (Barclay 1958: 216-222, Coale 1957, Coale 1972, Dublin and Lotka 1925, Keyfitz and Flieger 1968; Lotka 1907, McCann 1973, Pressat 2009: 318-328, Preston et al. (2001:138-170, United Nations 1968), but we not aware of the direct use of regression analysis in which the initial rate of increase in a given population is used a predictor variable.[6] We note in regard to our use of regression analysis that analytic methods are preferable when relationships are understood. However, as Barclay (1958: 216) observes the determination of a non-stationary population is a complex task and the literature does not reveal a direct relationship between the initial rate of increase (which, recall we denote by IRI) in a given population to ***r*** (Barclay 1958, Coale 1957, Coale 1972, Dublin and Lotka 1925, Keyfitz and Flieger 1968, Lotka 1907, McCann 1973, Pressat 2009, Preston et al. 2001) As an initial exploration of this relationship, and given the results yielded from employing regression to estimate the time to stability for a given population, we, therefore, employ regression analysis.

In earlier work, Swanson and Tedrow (2013) used data for 67 countries found in Keyfitz and Flieger (1968) in a "proof of concept" test. These 67 cases represent are the most recent entries for national and ethnic populations in Keyfitz and Flieger (1968); they also were used by McCann (1973) in constructing a quadratic regression model to estimate mean generation length, which he then employed to estimate ***r*** in conjunction with the natural logarithm of the net reproduction rate. The independent variable is the natural rate of increase (denoted here by NRI), which Keyfitz and Flieger (1968) found by subtracting the crude death rate from the crude birth rate for these 67 populations. The dependent variable is the intrinsic rate of increase, ***r***, found by Keyfitz and Flieger (1968) for these same 67 populations. Swanson and Tedrow (2013) found that a simple bivariate regression equation worked very well in estimating ***r*** from NRI for these 67 countries:

Estimated Intrinsic rate of increase, ***r*** = -1.1719 + (1.0532*NRI) [13]

p= .0222 p <.0001

n=67

r^2 = .8992

The results strongly support the idea that ***r*** can be estimated from NRI using linear regression. The coefficient of determination is high (r^2= .8992) and the both the intercept and slope coefficient are statistically significant (given α=.05) at p =.0222 and p <.0001, respectively.

Given these results for the 67 countries taken from Keyfitz and Flieger (1968), we now turn our attention to the same 62 country data set used to generate the regression model used to estimate time to stability from the score of the initial Stability Index. Here we do not use the "Natural Rate of Increase (NRI), as found in the data provided by Keyfitz and Flieger (1968), but, instead the Initial Rate of Increase (IRI). As discussed earlier, the former takes into account only the difference between the crude birth rate and crude death rate while the latter takes into account all three of the components of change, births, deaths, and migration.

Estimated Intrinsic rate of increase, $\boldsymbol{r}$ = -0.0096 + (1.778*IRI) [14]

p <.0001 *p<.0001*

n =62

r^2 =.881

As was the case for using NRI as a predictor variable for the 67 country data set taken from Keyfitz and Flieger (1968), we find that a simple bivariate regression model works well for predicting $\boldsymbol{r}$ from NRI using our 62 country data set: the coefficient of determination is high (r^2= .881) and both the intercept and slope coefficient are statistically significant (given α=.05) at p <.0001 and p <.0001, respectively. Taking into account the differences between NRI and IRI, it appears that the regression approach to estimating intrinsic r from initial measures of population change is reasonably robust.

Given our results for estimating time to stability, it is a natural question to ask what role the components of change play in estimating $\boldsymbol{r}$. To answer this question, we constructed a multiple regression model using IRI and the components of change as predictor variables. The results are found below with the model shown as equation [15].

Estimated Intrinsic rate of increase, $\boldsymbol{r}$ =

-.1227 + (.2922*IRI) + (.0002*e_0) + (.0767*MEAN_CCR_20_34) + (.0128*TFR) [15]

p<.0001 *p=.0151* *p=0032* *p<.0001* *p=.0158*

n =62

R^2 =.952

Adj R^2 = .948

The model shown as equation [15] shows that the components of change play a role along with IRI in determining ***r***. Because the regression coefficients are all positive, we can see that each of the components has a positive relationship with ***r***. By looking at the standardized regression coefficients for the model shown in equation [15], we obtain an idea of their relative importance. In order of size, we find that fertility plays the largest role, with a standardized coefficient of 0.7345 for the variable TFR; migration has the second largest standardized coefficient, with .4805 for the variable, MeanCCR20-34; IRI has the third largest standardized coefficient at .2366, while the smallest effect is found for mortality, for which the standardized coefficient for e_0 is .1088.

12.8 Comparison of CCR-based estimates of time to stability and *r* with estimates found using the analytic approach.

Estimates of time to stability and the intrinsic growth rate found using the simulation and regression approaches utilized here should correspond directly with those arrived at by analytic solutions. To explore their similarity, we computed time to stability using the damping ratio within a matrix model, as described by Caswell (2001: 95-97). All calculations were conducted in the "**R"** software package (www.Rproject.org), using the PopBio package (Stubben and Milligan 2007). Within a matrix model framework, the dominant eigenvalue of a square projection matrix (such as what employ, one based on age-specific fertility rates and cohort change ratios) is the equivalent of the Euler-Lotka growth rate (Caswell 2001, Sykes 1969). This solution holds at stability, such that the ratio of the dominant eigenvalue to the absolute value of the second largest eigenvalue provides a measure of the percentage rate of convergence of the population on stability for each time-step in a population projection. In formulaic terms:

$$\rho = \frac{\lambda_1}{|\lambda_2|} \qquad [16]$$

The damping ratio may be used to approximate the time to stability in an asymptotic model (Caswell 2001) as:

$$t_x = \frac{\log(x)}{\log(\rho)} \qquad [17]$$

From this relationship, the number of years required for a population to reach convergence, which by convention is a point in time where x =10 (Caswell 2001) may be estimated as:

$$time\ to\ convergence = 5 * t_x \qquad [18]$$

This time should match to a high degree of precision the number of years required in each simulation.

This method provides an estimate of time to convergence that is widely used in population ecology (Caswell 2001, Rogers-Bennett and Leaf 2006), but differs from those traditionally presented in human demographic studies (Kim and Schoen 1993a, Kim and Schoen 1993b, Schoen 2006, Schoen and Kim 1991, Tuljapurkar 1982; Cohen 1979b). The damping ratio is specifically chosen here for comparison with the results of our simulation—as it provides a direct measure of the number of years required to achieve stability that can be compared to those estimated using projection. As such, it provides a straightforward basis of comparison in the correspondence between the results of an asymptotic model and those based on demographic projection reported here. The implications of the damping ratio for studies of population convergence should be similar to alternative approaches (Kim and Schoen 1993b) because all convergence measures (as well as patterns of fluctuation in age-structure or growth rate during the process of convergence) ultimately depend upon the relationship between the largest and second-largest eigenvalues of a projection matrix (Keyfitz, 1977).

These asymptotic estimates were compared to those arrived at via the damping ratio measure and the results of this analysis is presented in Exhibit 5. On average, small differences characterized discrepancies between analytic and simulated solutions for time to convergence. While the presence of some outlying values is clearly observable in the difference between mean and median and coefficient of variation measures for the CCR and analytic solutions in terms of years required to converge, on average these differences are five years or less in numeric terms. In percentage terms, the algebraic differences suggest that the CCR Leslie matrix-based approach provides a lower estimate of the time to convergence than does the analytic solution (mean = - 4.22 percent, median = - 2.92 percent). Absolute differences are less than seven percent on average between the two sets when the mean is used (mean = 6.99 percent) and under six percent when the median is considered (median = 5.91 percent). These relatively small differences likely stem from rounding issues associated with imprecision in floating point arithmetic in the Excel software package and from the arbitrary use of "10" as the converging scale in Equation 17 (Caswell 2001). Overall, these results suggest a strong correspondence between the estimates of time to convergence associated with both approaches. The correlation between the two is extremely high (r = 0.96).

(Exhibit 5 About Here)

To get an idea of the consistency between the CCR approach to estimating ***r*** and the analytic approach, the latter was estimated using a method suggested by Caswell (2001: 74-75) in which the natural logarithm of the ratio of each population age group at stability to its corresponding age group at the launch point is summed across all age groups using the proportion of each age group at origin as a weight in the summation process.

Moving on to the estimates of ***r***, Exhibit 6 provides a summary of the comparisons across the 62 countries found using the CCR approach and the analytic approach. As can be seen in Exhibit 6, there is close agreement between the two approaches. In terms of measures of centrality, the mean of the 62 values of ***r*** estimated using the CCR approach is -0.0050 while the mean for the 62 values of ***r*** estimated using the analytic approach is -0.0047, an algebraic difference of 0.00030 (subtracting the former from the latter). The mean algebraic percent difference is -4.05% and the mean absolute percent difference is 8.67%. The median of the 62 values of ***r*** estimated using the CCR approach is -0.0047 while the mean for the 62 values of ***r*** estimated using the analytic approach is -0.0045, an algebraic difference of 0.00020 (subtracting the former from the latter). The median algebraic percent difference is -3.20% and the median absolute percent difference is 4.49%.

In terms of dispersion, Exhibit 6 shows that the standard deviation of the estimates of r found using the CCR approach is 0.0106 while the standard deviation for the estimates found using the analytic approach is 0.0105. Given the close correspondence between the means found using the two approaches and their standard deviations, it is not surprising that the coefficients of variation are similar, -2.12 for the former and -2.23 for the latter.

(Exhibit 6 About Here)

12.9 Discussion

In terms of our findings regarding the time to stability, initial conditions are forgotten as a population moves to stability, which is consistent with ergodicity. However, given the results of our analysis (as summarized in Exhibit 4), it is clear that initial conditions (as represented by the initial Stability Index at the launch year) play a role well into the path to stability, up to and including, the quasi-stable point where ***S*** = .001. The average time to reach this point (across all 62 countries) is 173 years (with a standard deviation of 48.6). In this context, it is useful to note that the average time to reach stability is 490 years (with a standard deviation of 141 years). Thus, it appears that the initial conditions play a role in the path to stability up to the time that, on average, a country is approximately one-third of the way to stability. Further, as can be seen in Exhibit 4, the effects of initial conditions as measured by ***S*** at the launch year diminish as a country moves from its launch year to the year of quasi-stability where ***S*** = .001.

In terms of the components of change, fertility and net in-migration play a role all the way from launch to stability, which is when **S** =0.000000. As seen in the standardized coefficients found in Exhibit 4, the effect of fertility increases from launch to the point of quasi-stability where ***S*** = .0005, when it reaches value of -.4306. It then diminishes to -.4010 at the point of stability where ***S*** = zero. In terms of net in-migration, the standardized coefficient increases from launch to the point of where ***S*** =.001, when it reaches a value of .4895 and then diminishes to .4727 when ***S*** = .005 and finally, to .3884 when ***S*** = zero.

Life expectancy, our indicator for the mortality component, does not play a role until some point between ***S*** = .0005 and stability (when ***S*** = zero). As can be seen in Exhibit 4, the coefficient for this variable is not statistically significant until ***S*** = zero, where it has a value of .2325. Since the average time to ***S*** = .0005 is 204 years (with a standard deviation of 57 years), and the average time to stability is 490 years (with a standard deviation of 141 years), it appears that it takes a long time for the effects of mortality to come into play, on average.

Our analysis suggests that the initial value of ***S*** is the most important determinant on the path to stability up to the point when ***S*** = .005, which, on average, takes 103 years to achieve for our set of 62 countries (with a standard deviation of 31 years). The standardized coefficient for Initial ***S*** is much larger than those for fertility and net in-migration at both ***S*** = .01 and ***S*** = .005. However, by the time the point of ***S*** = .001 is reached, the initial value of ***S*** becomes the least important determinant in that its standardized coefficient (.2297) is exceeded (in the absolute sense) by both the standardized coefficient for net in-migration (.4895) and the standardized coefficient for fertility (-.3338). At the time when ***S*** = .005 is reached, the initial value of ***S*** is no longer statistically significant and the effects of fertility and net in-migration are about equal (in the absolute sense), with standardized coefficients of .4727 and-.4306, respectively. At this same point, the effect of mortality has not yet come into play.

Although we did not show the results of all of the regressions that were constructed, we did find that the initial rate of population change does not play a role in that this variable was not statistically significant in any of the multiple regressions. Similarly, no initial conditions (e.g., proportion of the population aged 0-4, proportion of the population aged 75 years and over, the difference between the proportion aged 0-4 and the proportion aged 75+) other than the initial value of ***S*** were statistically significant in any of the regressions we constructed. These findings serve to complement those generated by the traditional approach to examining the path to stability in which only fertility and mortality are considered as components of change (Cohen 1979a, Cohen 1979b, Keyfitz 1974, Kim and Schoen 1993a, Schoen 2006, Schoen and Kim 1991, Tuljapurkar 1982).

The results obtained here are most comparable to those of Kim and Schoen (1993a), who use an alternative measure of the rate of convergence and relate it to variation in the net-maternity function in a standard birth-death model Kim and Schoen (1993a) and Schoen (2006) argue that if the second largest eigenvalue is real (the solution will not hold if it is complex), then there is a "constant, ultimate force of convergence" given by:

$$\mathbf{h^* = 1- \{(|\lambda_2|/ \lambda_1|)\}^2} \qquad [19]$$

This measure, of course, suggests a negative exponential convergence upon stability. It differs from the damping ratio approach employed in this paper and is relevant to analyzing determinants of the rate of convergence whereas our use of the damping ratio (which would give a geometric approach to a predefined level of stability in which the ratio of the largest to second largest eigenvalue approaches 10) is utilized to compare results of a simulation approach to an asymptotic population model.

On this basis, Kim and Schoen (1993a) further suggested that if the net-maternity function in a standard birth-death model of population dynamics is parameterized in the fashion suggested by Keyfitz (1977) - using a normal curve - then the moments of the distribution of Lotka's stable net maternity function may be used to analyze variation in the speed of convergence as:

$$\mathbf{h^* \approx 1 - \exp[-4n\pi^2 \sigma^2 / \mu^3]} \qquad [20]$$

Specifically, Kim and Schoen (1993a) suggest that the rate of convergence measured by h* should be inversely proportional to the mean of the stable net maternity function. Greater variability in the stable net maternity function and a lower age at mean childbearing, under the model of Kim and Schoen (1993a) convergence should occur more rapidly. This idea corresponds to findings of Coale (1972), who also suggested that greater variance in age at reproduction would converge upon stability more rapidly. Utilizing data from 177 populations originally analyzed by Keyfitz and Flieger (1968), Kim and Schoen (1993a) illustrated that a strong relationship between the h* shown in Equation [20] and the h* measure calculated using the first and second largest eigenvalues as in

Equation [19] (R^2 of 0.98). They also found that alternative measures of convergence, such as population entropy (Tuljapurkar 1982) also co-vary in the same direction with net-maternity functions.

In this study, we find that levels of TFR are negatively related to the years required to reach convergence upon stability in demographic projections. Schoen and Kim (1993a) found that the level of net maternity did not affect their results—only the mean age at childbearing. Since NRR and TFR should be positively related (because NRR is simply the TFR schedule discounted for survivorship), we might have expected to find more similar results. In this study, the effect of increasing levels of the TFR is negative. For each one unit increase in TFR, a reduction in time to stability of approximately 68 years is observed. For each one unit increase in the average CCR of the 20-34 year old age intervals, we observe a nearly 1,000 year increase in the time to stability. This suggests that net in-migration has a much stronger and more pronounced effect upon variation in the time required to reach stability than does fertility in a births-deaths only model such a those examined by Coale (1972) or Kim and Schoen (1993a). This is not surprising, given that shifts in fertility are known to have more pronounced effects on age-structure than those in mortality when births-deaths only models are considered (Coale and Trussell 1974, and Caswell 2001). However, we are surprised by the orders of magnitude difference observed in this study when migration effects are included. To our knowledge, this finding is a novel one in the literature on stable population dynamics.

We believe that these findings in regard to the role played by initial conditions (i.e., age structure) as measured by ***S*** in regard to the path to stability are novel. While the effect of varying initial age structures can be seen in simulations (e.g., Caswell 2001: 12-13), it appears that they have not been quantified. As such, our regression analyses provide a starting point for developing an analytical description of this process. Also useful as a point of departure for further analysis is the finding that the mean length of time to stability using the CCR approach with our 62 country dataset is 489.92 years. This length of time likely reflects at least partially the fact that CCRs can be greater than 1.0, owing to the effect of migration. Continuing this line, our analysis suggests that the time to stability is increased as net in-migration increases. The preceding points regarding migration also lead to the realization that the age structure of a stable population found using the CCR approach can look very different than that found using the traditional approach. Due to the effects of migration being incorporated into the CCRs, the former may have, for example, more people in a given age group than found in a preceding age group, something not found in the latter.

In regard to migration, it is important to note that because CCRs are always greater than zero and can encompass both net in-migration and net out-migration, they can be used with a Leslie Matrix with assurance that a given population will converge to stability. This is not the case with other approaches that have looked at accommodating migration as part of the process to convergence in that they only allowed for net in-migration in order to provide assurance that a given population would converge (e.g., Espenshade 1986, Sivamurthy 1982). As such, the CCR Leslie Matrix approach is more general in regard to accommodating migration. While we believe that our use of CCRs provides new insights and capabilities, we also note that nothing extraordinary is found in their use in regard to the mathematical foundation of the Leslie Matrix approach. That is CCRs are always greater than zero. Thus, when one is using CCRs as the process to stability in the context of a Leslie Matrix, the matrix is "positive," which means the population in question will converge (Caswell 2001: 79, Schoen 2006: 26-29).

Turning to the topic of estimating intrinsic r (***r***) from initial conditions, we confirm the finding of Swanson and Tedrow (2013) that ***r*** can be estimated from initial measures of population change. The model (Equation [13]) they constructed using the natural rate of increase (NRI) from 67 populations selected from Keyfitz and Flieger (1968) is similar to the model constructed using the initial rate of increase (IRI) from the 62 populations used here (Equation [14]). In the earlier model, the coefficient of determination was .899 and in the current model it is .881. Given that the model found in Equation [13] uses NRI and that the one found in Equation [14] uses IRI, the regression coefficients in both models are somewhat different at 1.0532 and 1.778, respectively. When the individual components of change are added as predictor variables, the model for estimating ***r*** is improved even more in that R^2 is .952. We provided an idea of the relative importance of the predictor variables by looking at their standardized coefficients. The highest one, .7345, is found for the fertility variable, TFR, while the second highest one, .4805, is found for the net in-migration variable, MeanCCR20-34. The standardized coefficient for IRI is the third highest at .2366 while the lowest, .1086, is found for the mortality variable, which we operationalized as e_0.[7]

As was the case regarding our findings on the path to stability, we believe that our findings regarding the estimation of ***r*** from a measure of initial population change (i.e., both NRI and IRI) complement: (1) those generated by the more traditional approach to examining the path to stability in which only fertility and mortality are considered as components of change (Cohen 1979a, Cohen 1979b, Keyfitz 1974, Kim and Schoen 1993a, Preston 1986, Schoen and Kim 1991, Tuljapurkar 1982); and (2): a continuous approach (as opposed to our discrete approach using CCRs), such as found in Schoen (2006: 71). Importantly, we also find that estimates of ***r*** and time to stability generated by the CCR Leslie Matrix approach are consistent with estimates developed from the analytic approach.[8]

In summary, Cohort Change Ratios (CCRs) appear to us to be useful as a tool for examining the idea of a stable population. The consistency found between CCR-based estimates of both time to stability and ***r*** and those using the analytic approach suggest that the former is consistent with the theoretical foundation of stable population theory. Given this consistency, a major benefit of the CCR approach is the ability to easily deal with both sexes and all of the components of change, including migration. In this regard, a recent analysis of the path to stability for India that used the traditional framework by looking at the reproduction and survivorship of females would have been more realistic if it had employed the CCR approach, which would have accommodated all of the components of change (including migration) in regard to both males and females (Nair and Nair 2010). In addition, the CCR approach could have easily been implemented within the Leslie Matrix framework.

In conjunction with regression models and variables representing fertility, migration, and mortality, we believe that our examination of the CCR approach and the use of the ***S*** Index has yielded some useful insights on the effect of variables on the path to stability, insights not fully available from existing analytic methods, but yet consistent with ergodicity.[9] It is worthwhile to note again here that ergodicity (in either the form of the strong or week theorems) states that initial conditions are forgotten and that “vital statistics”(which can be generalized to include CCRs) are the determinants of the stable age structure. Our findings suggest that the initial Stability Index plays a role about one-third of the way on the temporal path to stability while fertility and migration play a role along the entire path and mortality only does so toward the end of the temporal path to stability. This finding could lead to a refinement of the concept of ergodicity.

It is important to note that if different definitions were used in place of those we used to operationalize the predictor variables for initial conditions and the components of population change, it is likely that the regression models resulting from them would vary from ours. It may also be the case that if different age groupings (e.g., 0-4, 5-9,…, 85-89, 90+) and forecast cycle lengths (e.g., ten years), the results would be different. However, it is likely the case that there would be findings common to them as well and these common findings would serve to point the way to increased understanding of the path to stability.

In terms of future research, one area might be the use of different points of quasi-stability. Recall that in this study we use ***S*** =.01, ***S*** = .005, ***S*** = .001, and ***S*** = .0005 because they generally encompass portions of the path to stability that in terms of time are rapid (.01), somewhat less rapid (005), slow, (.001) and very slow (.0005). It may be the case that different points yield different insights.

Another area for future research is to examine CCRs in conjunction with ideas promulgated by Keyfitz (1974) for examining stable processes across two (or more) interacting populations, ideas explored by, among others, Keyfitz (1980), Kim and Schoen (1993b), and Liaw (1980). Because it can deal with both sexes and migration quite handily, the CCR approach appears to be more tractable in regard to examining the path to stability in such populations. Another area, which we mentioned earlier, would be to develop formal statements like the strong and weak forms of the ergodicity theorem that specify the effect of both initial conditions and all three of the components of population change on the path to stability. Yet another area could involve decomposing CCRs into their survivorship and migration components and examining the effects of these two components of change directly.

In conclusion, we know that regression models are generally not as satisfying as analytical expressions in regard to describing relationships. It would be much more elegant to express the time to stability in terms of an analytic expression that incorporates the initial Stability Index (and possibly other information about initial conditions) and components of change than it is to express the relationship in the form of a regression model.[10] The same can be said about the relationship between the initial rate of increase in a given population and its intrinsic rate of increase. However, we also note that regression analysis has already been successfully employed in conjunction with stable population analysis, to include the Bourgeois-Pichat method for estimating intrinsic r from the proportional age distribution of a given population (Keyfitz and Flieger 1968:49, United Nations 1968), McCann's (1973) method for estimating mean generation length from a trial value of the intrinsic rate of increase, and the generation of model life table families and from them, stable populations (Coale and Demeny 1966).

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Endnotes

1. The input data used to generate cohort change ratios need to be separated by a time interval that is consistent with the age groups used in the input data. For example, if the data are in five year age groups (up to the terminal, open-ended age group), the time interval should be either five years or ten years. If the data are in 10-year age groups then the time interval will need to be ten years. If the data are in single-year age groups, then the time interval should be one year. Fortunately, most population data are provided in five-year age groups.

Although we do provide a proof here, it is easy to show that using CCRs to move apopulation through time is consistent with the fundamental demographic equation. This consistency is important for two reasons. First, as noted by Land (1986) any quantitative approach to forecasting is constrained to satisfy various mathematical identities, and a demographic approach should ideally satisfy demographic accounting identities, which is summarized in the fundamental demographic equation. The second reason is based on the argument by Vaupel and Yashin (1985) that a demographic forecasting method needs to be consistent with the fundamental demographic equation in order to minimize the potential errors associated with hidden heterogeneity.

2. If one has a life table, the CCRs for a given population could be compared to their corresponding survival rates and the effects of migration could be separated from the effects of mortality. This would be similar to using a life table to estimate net migration by age using the Forward Life Table Survival Method. Again, an important assumption is that differential net undercount by age is absent or at least very minimal.

3. Often, the Index of Dissimilarity is expressed as a percentage, whereby the formula shown in equation [16] is multiplied by 100. In our use of this Index, we define "zero" to six significant digits. That is, when ***S*** is equal to "0.000000," we define this stability. If fewer or more significant digits were used, the point at which stability is reached would, of course, be different.

   It is worthwhile to note here that Keyfitz and Flieger (1968: 23 and 24-41) display a "dissimilarity" score between a current population age distribution and the age distribution for the corresponding stable population. The index is the sum of positive differences between the two distributions. This index is only one simple step from the Index of Dissimilarity. However, even so, it is neither employed by Keyfitz and Flieger (1968) to define a stable population nor used to estimate time to stability. However, Keyfitz (1968: 47) does use it to define the distance to stability and other measures of this distance are found in Caswell (2001), Cohen (1979b), Schoen (2006), Schoen and Young (1991), and Tuljapurkar (1982).

   Also, as noted in the text and described by Keyfitz (1968: 47), the Index of Dissimilarity could be used in conjunction with the relative age distribution at stability and at the initial launch point. As an example of this use, the highest value found in the 62 country data set is for Hong Kong, which has a Dissimilarity Index of .399073; the lowest is found for Guatemala, with a Dissimilarity Index of .05001. Thus, Hong Kong's age distribution at origin is furthest from its stable age distribution while Guatemala's is closest. As would be expected, Hong Kong's time to stability (740 years) is much longer than Guatemala's (250 years). These two respective indices also provide an easy-to-interpret measure of how different the initial population age structure is from the age structure at stability. For Hong Kong, 39.91 percent of the initial population needs to be re-allocated to match its relative age distribution at stability while for Guatemala only 5 percent needs to be reallocated. Due to the specific dynamics underlying a country's path to stability, the Index is not the sole determinant of time to stability, however. For example, Hong Kong does not take the longest time to reach stability of the 62 countries (Singapore does, at 890 years) and Guatemala does not take the shortest time to reach stability (El Salvador does, at 225 years).

4. The Leslie Matrix was implemented as a "macro" in Excel using Excel's coding language, VBA. The code as well as a "template" excel file with instructions on

how to implement the Leslie Matrix are available on request from the authors. Also available from the authors are the files for all 62 countries as well as the summary file containing life expectancy at birth, the total fertility rate, and the mean CCR for ages 20-24, 25-29, and 30-34.

There are different ways in which the Leslie Matrix could be implemented in terms of the constant ASFRs and CCRs. For example, once one developed the ASFRs for both sexes (as we have done) and set them up in a forecast cycle (which in our case here is for a five year period), the ASFRs could then be adjusted for infant mortality. One also could determine the mid-cycle populations of child-bearing ages (15-19, 20-24,…, 45-49) and then apply the either the unadjusted ASFRS or mortality-adjusted ASFRs to them. We implemented the ASFRs without an adjustment for mortality and applied them to the population at the beginning of the forecast cycle. In the long run to stability, the different implementations are not likely to create substantial differences in the time to stability, but they could make a difference if one were attempting to develop realistic forecasts with much shorter horizons (e.g., 10 years, 20, years and even 50 years).

5. Given that the path to stability is non-linear, we also explored regression models in which the time (number of years) to stability was transformed using natural logarithms. However, we found that other than the change in the regression coefficients to accommodate the transformation, these models were not substantially different than their non-transformed counterparts. For example, the model that corresponds to the provided in equation [12] has an R^2 of .61 and an adjusted R^2 of .59 and the rank-order of the standardized coefficients is the same as found for the model shown in Exhibit 3 for time to $\boldsymbol{S}$ = zero, which are those associated with equation [12] . It is useful to note here that the NCSS regression procedure employs Huber's method when skewed residuals are encountered. As such, it is a robust approach and its results will vary from those found using OLS methods which do not employ this method when skewed residuals are encountered.

It is worthwhile to note that some of the effects of the predictor variables found in equation [12], may also be non-linear on their own and interactive. We have not explored these possibilities here, but they may prove useful in future work.

It also is worthwhile to mention here work by Preston (1986) in which he found that there is a close approximation between the intrinsic growth rate of a population and the mean of age-specific growth rates below age T, the mean length of a generation. He concluded therefore that where a disparity exists between the intrinsic growth rate and the actual growth rate of a population (whether or not net migration is included in both rates), it must be attributable to an unusual growth rate of the population block above age T.

6. While it appears that regression analysis has not been used to estimate intrinsic r from an initial r, Bourgeois-Pichat employed it to estimate intrinsic r from the

proportional age distribution of a given population (see Keyfitz and Flieger 1968: 40).

7. Based on comments by Barry Edmonston, we also constructed models for estimating r using the initial Stability Index (***S***). In the model in which all three components of change were included along with initial r, we found that e_0 was not statistically significant. We then eliminated this predictor variable and re-ran the model with the other two components of change, Initial ***S***, and initial r, and found a model with an adjusted R^2 of .948 and predictor variables that were all statistically significant. These results suggest that the difference between the initial age distribution and the stable age distribution may be a factor in the difference between initial r and ***r***, a suggestion provided by Barry Edmonston. This idea may also account for the difference between the regression model for estimating ***r*** from initial r that was constructed using the Keyfitz and Flieger (1968) data and the model for estimating r from initial r that was constructed using the Census Bureau's International Data Base. That is, the differences found between initial population age structures and the stable ones for each of the 67 populations taken from Keyfitz and Flieger (1968), on the one hand, may vary from the differences found for each of the 62 populations taken from the US Census Bureau's International Data Base, on the other. This is a topic for future research.

8. There may be approaches other than the one we employ (Caswell 2001: 95-97) to compare with the estimate of time to stability generated from our CCR approach using regression and the Stability Index (***S***). For example, it may be possible to substitute a variation of the Kullback Distance (Nair and Nair 2010, Schoen 2006: 29-33) for the Stability Index as an independent variable in a regression model such as we employed. With appropriate modifications, the Kullback Distance potentially could be used with cohort change ratios and its results compared with both those generated by the CCR method and the analytic approach we used. It is useful to note that the Kullback Distance declines monotonically during the process of convergence (Schoen 2006: 31), which is similar to the behavior of ***S***, where the initial decline may not be monotonic, but becomes so at some point and overall, is monotonic or nearly so. Also, like ***S***, the Kullback Distance possesses a number of desirable properties (Schoen 2006: 31). However, the Kullback Distance also may generate different values than the method we used and, as such, yield different summary statistics in a comparison with the CCR approach. Similarly, there variations on the analytic approach we used to estimate ***r***, which is taken from Caswell (2001: 74-75). Descriptions of variations that potentially could be used can be found in Barclay (1958: 216-222) Coale (1957, 1972), Dublin and Lotka (1925), Keyfitz and Flieger (1968), Lotka (1907), Pressat (2009: 318-328), Preston et al. (2001:138-170), and United Nations (1968). Again, as we noted in regard to time to stability, these approaches may generate different values of ***r*** than the method we used and, as such, yield different summary statistics in a comparison with the values of ***r*** generated by the CCR approach.

9. Caswell (2001: 572) notes that demographers have addressed the "two-sex" problem since the 1940s, but that much of the literature focuses on the "consistency" problem: how to make estimates of intrinsic r based on male and female life tables agree. Although he notes that those studies that deal with demographic dynamics in any detail have focused on models lacking age structure, examples of studies using age can be found in Schoen (1988).

10. In regard to the usefulness of empirical findings, we note that in discussing the exploration of Kim and Sykes (1976) on stable population concepts, Cohen (1979a: 286) observed that their numerical experiments uncovered empirical regularities that invite theoretical explanation.

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TABLE 1. A 2010 HAMILTON-PERRY PROJECTION OF AUSTRIA USING 2000-2005 CCRs & FERTILITY DATA*				
	2000	2005	CCR	2010 FORECAST
Total Population: 0 to 4 years	416,996	373,688	MID-POINT ASFR**	412,804
Total Population: 5 to 9 years	474,442	424,606	1.01825	380,508
Total Population: 10 to 14 years	468,613	482,338	1.01664	431,673
Total Population: 15 to 19 years	486,243	478,246	1.02056	492,253
Total Population: 20 to 24 years	470,256	500,377	1.02907	492,148
Total Population: 25 to 29 years	570,381	485,939	1.03335	517,065
Total Population: 30 to 34 years	704,616	578,844	1.01484	493,149
Total Population: 35 to 39 years	715,158	706,090	1.00209	580,055
Total Population: 40 to 44 years	615,389	712,287	0.99599	703,255
Total Population: 45 to 49 years	521,659	609,722	0.99079	705,728
Total Population: 50 to 54 years	499,456	513,512	0.98438	600,200
Total Population: 55 to 59 years	494,015	486,579	0.97422	500,273
Total Population: 60 to 64 years	419,019	475,689	0.96290	468,529
Total Population: 65 to 69 years	344,843	395,638	0.94420	449,146
Total Population: 70 to 74 years	331,663	312,967	0.90756	359,067
Total Population: 75 years and over	580,664	648,169	0.71046	682,846
Total Population	8,113,413	8,184,691	1.00879	8,268,696

*SOURCE DATA: US CENSUS BUREAU'S INTRNATIONAL DATA BASE
(http://www.census.gov/population/international/data/idb/informationGateway.php)
** The age-specific fertility rates in the source data are female dominant and for a single year.
They were averaged to represent 2002.5 and adjusted to the total population (both males and females)
and to represent a five-year period to correspond with the forecast cycle.
The final values are, by age group:

<= 19	20 - 24	25 - 29	30 - 34	35 - 39	40 - 44	>= 45
0.037	0.211625	0.26725	0.174125	0.067875	0.01375	0.001375

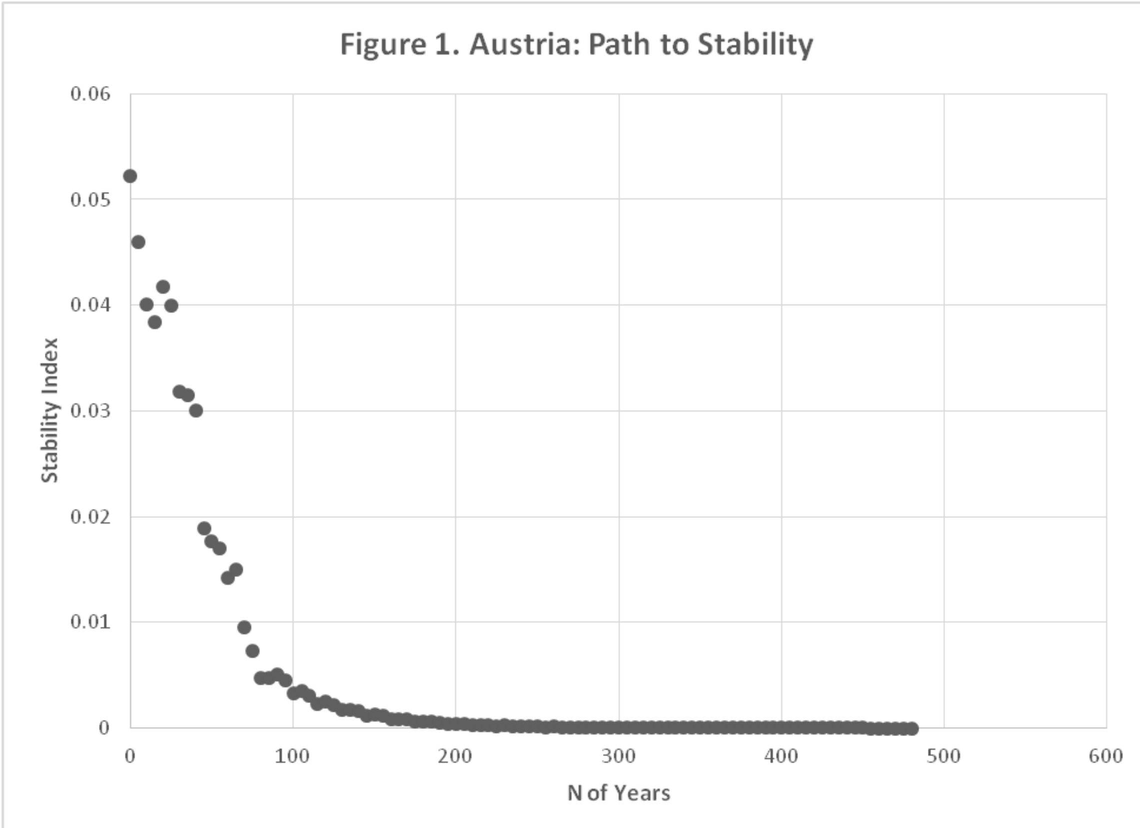
Figure 1. Austria: Path to Stability
Stability Index
0.06
0.05
0.04
0.03
0.02
0.01
0
0
100
200
300
400
500
600
N of Years

Exhibit 1. List of 62 Countries.

N	COUNTRY	REGION	IDB POPULATION 2000*	IDB POPULATION 2005*
1	CANADA*	N. AMERICA	31,376,736	32,656,679
2	COSTA RICA	N. AMERICA	3,882,581	4,208,691
3	CUBA	N. AMERICA	11,071,849	11,198,439
4	EL SALVADOR	N. AMERICA	5,849,822	5,956,221
5	GUATEMALA	N. AMERICA	11,085,025	12,182,548
6	JAMAICA*	N. AMERICA	3,837,878	4,089,964
7	USA	N. AMERICA	282,162,411	295,516,599
8	CHILE	S. AMERICA	15,174,571	15,979,150
9	URUGUAY	S. AMERICA	3,219,793	3,264,911
10	VENEZUELA	S. AMERICA	23,492,753	25,269,177
11	ARMENIA	ASIA	3,100,045	3,084,084
12	AZERBAIJAN	ASIA	8,463,076	8,825,439
13	HONG KONG*	ASIA	6,714,968	6,955,186
14	GEORGIA	ASIA	4,818,805	4,790,009
15	ISRAEL	ASIA	6,114,570	6,742,915
16	JAPAN	ASIA	126,775,612	127,715,356
17	KAZAKHSTAN	ASIA	15,687,251	16,122,665
18	KYRGYZSTAN	ASIA	4,937,128	5,164,248
19	SAUDI ARABIA	ASIA	21,311,904	23,642,207
20	SINGAPORE*	ASIA	4,169,481	4,713,561
21	TAJIKISTAN	ASIA	6,229,697	6,814,791
22	TURKMENISTAN	ASIA	4,385,485	4,664,155
23	UZBEKISTAN	ASIA	25,041,821	26,539,888
24	ALBANIA	EUROPE	3,158,352	3,024,533
25	AUSTRIA	EUROPE	8,113,413	8,184,691
26	BELARUS	EUROPE	10,033,392	9,806,452
27	BELGIUM	EUROPE	10,263,618	10,364,388
28	BOSNIA-HERZGOVNIA	EUROPE	3,805,512	3,893,097
29	BULGARIA	EUROPE	7,818,495	7,450,349
30	CROATIA	EUROPE	4,410,830	4,495,904
31	CZECH REPUBLIC	EUROPE	10,268,899	10,266,923
32	DENMARK	EUROPE	5,337,416	5,432,335
33	ESTONIA	EUROPE	1,379,835	1,332,893
34	FINLAND	EUROPE	5,168,595	5,223,442
35	FRANCE	EUROPE	61,255,363	63,059,742
36	GERMANY	EUROPE	82,183,670	82,439,417
37	GREECE	EUROPE	10,559,110	10,668,354
38	HUNGARY	EUROPE	10,147,425	10,057,624
39	IRELAND*	EUROPE	3,872,700	4,309,024
40	ITALY	EUROPE	57,784,373	59,037,808
41	LATVIA	EUROPE	2,376,178	2,290,237
42	LITHUANIA	EUROPE	3,654,387	3,596,617
43	MACEDONIA/FORMER YUGOSLAVIA	EUROPE	2,014,512	2,045,262
44	MOLDOVA	EUROPE	4,180,215	3,948,261
45	MONTENEGRO	EUROPE	732,302	699,259
46	NETHERLANDS	EUROPE	15,930,181	16,299,097
47	NORWAY	EUROPE	4,492,400	4,624,875
48	POLAND	EUROPE	38,654,164	38,557,964
49	PORTUGAL	EUROPE	10,335,597	10,568,212
50	ROMANIA	EUROPE	22,447,353	22,197,164
51	RUSSIAN FEDERATION	EUROPE	147,053,966	143,319,518
52	SERBIA	EUROPE	7,604,335	7,502,197
53	SLOVAKIA	EUROPE	5,400,320	5,431,363
54	SLOVENIA	EUROPE	2,010,557	2,011,070
55	SPAIN	EUROPE	40,589,004	43,704,367
56	SWEDEN	EUROPE	8,924,354	9,082,561
57	SWITZERLAND	EUROPE	7,277,250	7,448,224
58	UKRAINE	EUROPE	49,005,222	46,959,420
59	UNITED KINGDOM*	EUROPE	59,374,727	60,846,809
60	AUSTRALIA*	OCEANIA	19,294,257	20,489,472
61	FIJI*	OCEANIA	810,728	843,945
62	NEW ZEALAND*	OCEANIA	3,837,878	4,089,964

* FOR AUSTRALIA, CANADA, FIJI, HONG KONG,IRELAND, JAMAICA
NEW ZEALAND, SINGAPORE, & THE UNITED KINGDOM, THE YEARS
SELECTED ARE 2001 AND 2006, NOT 2000 AND 2005, RESPECTIVELY.

Exhibit 2. The Leslie Matrix for Austria

0	0	0	0.037	0.211625	0.26725	0.174125	0.067875	0.01375	0.001375	0	0	0	0	0	0
1.018249576	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1.016642709	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1.020556408	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1.02906777	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1.03335	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1.014837	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1.002092	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0.995985503	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0.990791	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0.984383	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0.974218	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0.962904	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0.944201	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0.907564	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.710457	0

Exhibit 3. Summary Statistics for the Variables used in the Study.

Variable	Mean	Std. dev.	maximum	minimum
Initial S	0.04506	0.01545	0.08378	0.01919
e_0	75.04	4.80	81.90	63.90
Mean CCR20-34	1.00	0.06	1.29	0.84
TFR	1.7804	0.6430	4.2298	0.9078
Initial r	0.00419	0.00840	0.02569	-0.01114
Intrinsic r (***r***)	-0.00468	0.01078	0.01995	-0.02518
N of Years to				
S = .01	75.83	35.53	162.66	22.84
S = .005	102.82	31.20	216.06	45.27
S = .001	173.05	48.55	340.00	91.76
S = .0005	204.24	56.76	395.00	103.48
S = zero	489.92	140.91	890.00	225.00

Exhibit 4. The Effect of the Initial ***S*** Score and the Components of Change on Selected points on the Path to Stability.

Standardized Coefficient**

Variable	Time to ***S***=.01	Time to ***S***=.005	Time to ***S***=.001	Time to ***S***=.0005	Time to ***S***=zero
Initial S	.6175	.4787	.2297	N/A*	N/A*
Mean CCR20-34	.1263	.2815	.4895	.4727	.4286
TFR	-.2246	-.2296	-.3338	-.4306	-.3327
e_0	N/A*	N/A*	N/A*	N/A*	.2376
ADJ R^2	.52	.41	.50	.51	.58

*Not statistically significant ($\alpha = .05$)
** The coefficients shown are for models for which only the statistically significant predictor variables are present. Models were, of course, constructed in which non-significant variables were present, but when such models were found, they were re-run without the non-significant variables, the results of which are shown here.

Exhibit 5. Summary of the comparison of estimates of time to stability found using the Leslie Matrix Approach and an Analytic Approach for the 62 countries*

	Estimate of Time to Stability (***S*** = zero)				
	CCR Leslie Matrix Approach (1)	Analytic Approach (2)	Algebraic Difference (2) – (1)	Percent Difference Algebraic	Absolute
Mean	97.85	102.84	- 5.01	- 4.22	6.99
Median	92.50	93.69	- 2.64	- 2.92	5.91
Std Dev	27.97	33.09	N/A	N/A	N/A
C.V.	0.29	0 .32	N/A	N/A	N/A

* The time to stability for the CCR approach as shown in Exhibit 3 was divided by 5 (the width of the age groups and the length of the forecast interval used in the CCR approach was five years) to compare the time to stability found using the analytic approach. For example, the mean time to stability found in Exhibit 3 is 489.92, which is equal to mean shown here (97.85) multiplied by 5.

Exhibit 6. Summary of the comparison of estimates of r found using the Leslie Matrix Approach and an Analytic Approach for the 62 countries

	Estimate of ***r***				
	CCR Leslie Matrix Approach (1)	Analytic Approach (2)	Algebraic Difference (2) – (1)	Percent Difference Algebraic	Absolute
Mean	-0.0050	-0.0047	0.00030	-4.05%	8.67%
Median	-0.0047	-0.0045	0.00020	-3.20%	4.49%
Std Dev	0.0106	0.0105	N/A	N/A	N/A
C.V.	-2.12	-2.23	N/A	N/A	N/A