Plotting Integration*

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Abstract

American metropolitan neighborhoods have become increasingly diverse over the past four decades. Yet many of the methods upon which researchers rely are more adept at showing segregation rather than *integration*. One consequence of being unable to visually explore the types of integration is that integrated neighborhoods are typically considered very similar. This paper demonstrates how barycentric plots can be used to demonstrate the diversity of different types of integrated neighborhoods. It provides an intuition for understanding how to read and create barycentric plots to visualize integration and racial change among two, three, and four racial groups. Finally, the paper demonstrates how studies relying on previous methods of classifying neighborhood integration can be visualized through barycentric plots.

Keywords. Racial integration; racial change; barycentric coordinates;

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Since the 1960s, metropolitan neighborhoods have become far more integrated. The passage of Civil Rights and immigration reform legislation created new opportunities for minorities in the U.S. Studies using Census data since the 1960s show a consistent trend of declining segregation, to the point that the 2010 Census arguable shows that integrated neighborhoods are now the most common type of neighborhood (Logan and Zhang, 2010; Glaeser and Vigdor, 2012).

While increasing racial integration is a positive sign of a more inclusive society, studying integration presents problems that previous generations of researchers did not encounter. In the past, the small number of racial and ethnic groups and extensive racism limited the prospects for integrated neighborhoods. Therefore, researchers focused almost exclusively on studying segregated neighborhoods, or neighborhoods where all residents identified as a single racial or ethnic group (e.g., Duncan and Duncan, 1957; **?**). Knowing that a group was present in the neighborhood above some minimal threshold allowed researchers to consider that neighborhood integrated. Integration of any degree was so rare that researchers lumped all neighborhoods with multiple groups into the class of integrated neighborhoods regardless of the proportion of neighborhood residents each group made up.

Despite the growing number of integrated neighborhoods, particularly over the last two decades, the analysis of neighborhood racial and ethnic change has remained focused on segregation. Analysis is often limited to identifying which racial and ethnic groups are present in neighborhoods across space or time, or both space and time. Definitions of integration based only on whether a racial or ethnic group is present prevent studies examining the variation among integrated neighborhoods. This limits the extent to which we can study differences within or among integrated neighborhoods as they have become the most common type of neighborhood in the U.S.

As the country became more diverse and racial discrimination became both less acceptable and less common, integration has become far more common. In fact, by the latest Census in 2010, the vast majority of American metropolitan neighborhoods are now integrated in some form or another. But this also means that integrated neighborhoods might not share much in common. Different combinations of groups can now make up different types of integrated neighborhoods. Even when the same two groups share a neighborhood, the share of each could be very different and changes in the share of groups could influence how the shares of groups change.

The problem is that most tools used to study racial and ethnic neighborhood composition were developed to study segregation and, therefore, are not well-suited to study forms and patterns of *integration*. This is true of both exploratory techniques to describe integration as well as analytical tools to distinguish common forms of neighborhood change. This paper focuses on exploratory techniques that can be used to examine the level of integration among multiple racial and ethnic groups simultaneously. One substantial obstacle to this problem is the lack of tools that offer complete and efficient visualization of data to conduct exploratory data analysis. In this article, I show how barycentric coordinate systems can be used to explore patterns of integration over time. I demonstrate how barycentric coordinates relate to the more commonly used entropy index and transition matrices. Finally, I show how using barycentric coordinates can help describe the findings of previous studies.

1 Brief Exploration of Integration Measures (Outline)

1. Transition matrices

2. Group growth/decline
3. Measures of integration (entropy)
4. Description of what these measures do not allow

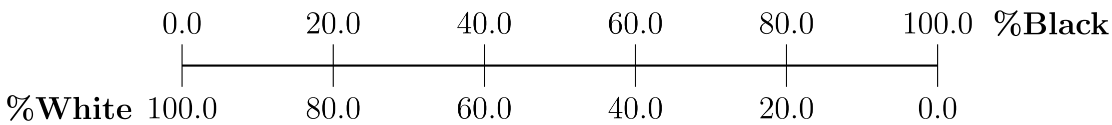
2 Description of Barycentric Coordinate Plots

One way to overcome the challenges presented by studying integration is to use barycentric coordinate plots. In mathematical terms, barycentric coordinates represent the center of mass of a simplex when weights are placed on the vertices of the simplex. It is the point at which the simplex – a line, or a triangle, or a tetrahedron – would balance when the weights are placed on each point. Barycentric coordinates are used for many applications including software graphics for computer games.

For our purposes, rather than thinking of masses as physical entities weighing on vertices, we can instead imagine that a racial group increases in mass the more it is represented by the population in the neighborhood. Although confusing in mathematical terms, it is perhaps easier to consider an easy and intuitive example.

Suppose that a neighborhood is made up of only two groups, Whites and Blacks. With that being the case, then the sum of their respective proportions of the population must equal one, that is $P^{\text{White}} + P^{\text{Black}} = 1$. We can then imagine a line, such as the one draw in Figure 1 below, where the the percentages of Whites and Blacks in the neighborhood sums to one at all points. For any point, P, we can determine how many Whites and Blacks exist in the neighborhood relative to one another. If only Whites live in the neighborhood, then the center of mass will be drawn to the left-most point of the line. Alternatively, if only Blacks live in the neighborhood, then the center of the line will be pulled to

Figure 1: One dimensional representation of three two-group combinations of racial composition of three racial groups



the right end of the line. If an equal number of Whites and Blacks live in the neighborhood, then both "masses" would be equal and the point would sit in the center of the line.

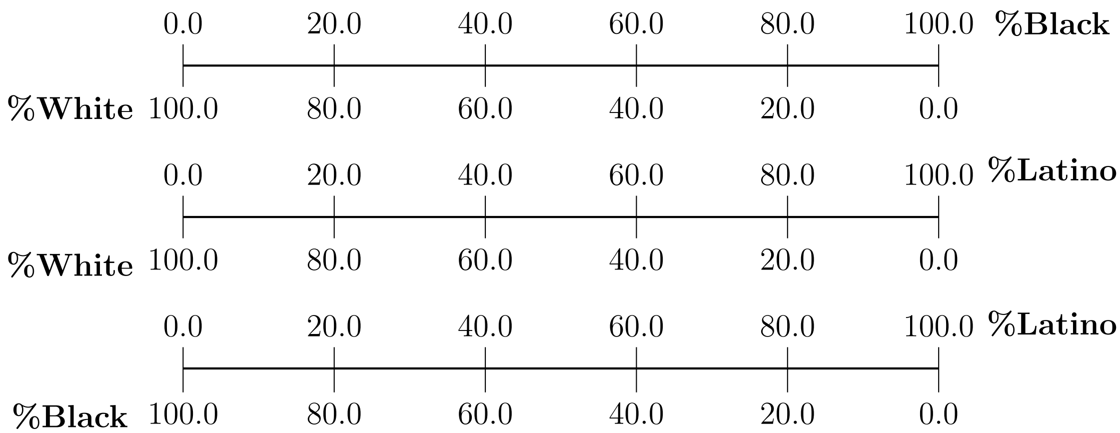
Barycentric coordinates allow me to represent the racial composition of two groups in a single dimension. We could represent all neighborhoods made up of only Whites and Blacks on this line because it is true that if we know the racial composition of Whites, we also know the racial composition of Blacks (which is just $1 - P^{\text{White}}$).

Applying the same principle to three racial groups, say we add Latinos to Whites and Blacks, it is possible to represent those three racial groups in two dimensions. Let us start by conceiving now of three separate lines, one measuring Whites and Blacks (just as we did above), one measuring Whites and Latinos, and one measuring Blacks and Latinos. We can represent these configurations as three separate lines as I do in Figure 2.

A problem arises, however, because these lines are no long mutually exclusive. Take, for example, a neighborhood that is 90 percent White, 5 percent Black, and 5 percent Latino. To represent that point, the it would fall 5.3 percentage points in on both of the first two lines,[1], and at the midpoint of the Latino/Black line (because both groups are equally weighted, going back to the idea of the center of mass).

[1]Because among Whites and Blacks, Whites make up 90 percent of the 95 percent of the neighborhood that Whites and Blacks make up together $[90 \div (90 + 5)] = 94.7$, the same is true for Whites and Latinos.

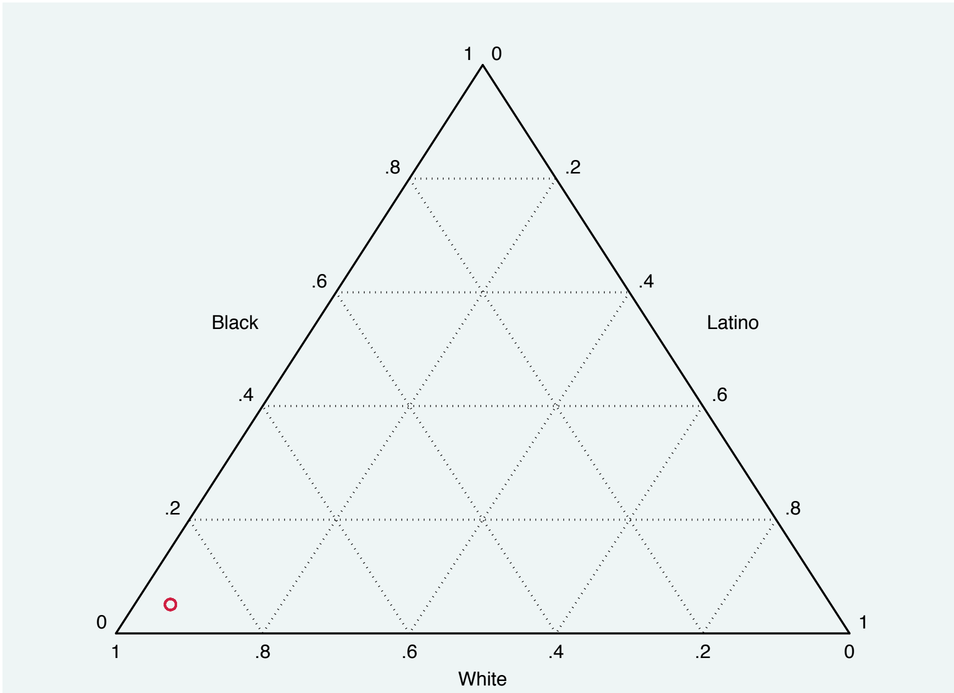
Figure 2: One dimensional representation of three two-group combinations of racial composition of three racial groups



Rather than thinking of these as three two-way combinations of racial groups, we can think about these as representing one three-way combination. Just as we could represent the composition of two groups along a single dimension, we can use barycentric coordinates to represent three groups in two dimensions. To express this two dimensional form, we can start by taking the left-most points of the first two lines in Figure 2 and connecting those to one another since the represent the same racial composition (100 percent White). We can do the same for the right-most points of the bottom two lines since both represent neighborhoods that are 100 percent Latino. Finally, we can connect the right-most point of the top line to the left-most line of the bottom line (where Blacks make up 100 percent of the population). The result is a triangle like that shown in Figure 3.

Now, moving to two dimensions, we can see how the vertices that we just connected represent neighborhoods that are composed exclusively of a single group: neighborhoods with 100 percent White racial composition will be represented on the left vertex, 100 percent Latino on the right vertex, and 100 Black on the top vertex. What this also means is that neighborhoods where one group is entirely

Figure 3: Ternary plot with point representing 90 percent White, 5 percent Black, and 5 percent Latino plotted



absent will reduce to the lines that we drew before, but are now represented on each of the sides of the triangle. Neighborhoods with no Latinos will fall along the left side of the triangle, those without Whites will fall along the right side, and those without Whites will fall along the bottom. The closer to equal representation the neighborhood attains among each of the three groups, the closer to the middle of the triangle the point representing the neighborhood will fall.

As an example, Figure 3 shows the point representing the neighborhood we attempted to plot before, the neighborhood that was 90 percent White, 5 percent Black, and 5 percent Latino. One can see that it is very close to the left vertex, meaning it is close to being all-White (which is true). One can also see that drawing a line from the vertex through the point to the right side of the triangle will end up crossing the midpoint of the right side. This demonstrates that Blacks and Latinos have share an equal proportion of the population of the neighborhood (and would correspond to the point being at the midpoint on the Black/Latino line in Figure 2 above).

Figure **??** shows how such plots can help us interpret the changing nature of racial composition from 1970 to 2000 in Chicago metropolitan neighborhoods [**please note vertices change in these graphs**]. In these graphs, all-White neighborhoods fall on the right vertex, all-Black on the left, and all-Latino on the top. One can see in the plot representing neighborhoods in 1970, the large white space in the center meant that there were very few multiethnically integrated neighborhoods. By 2000, there were many more neighborhoods integrated among Whites, Blacks, and Latinos. The plots also show that most White/Black integrated neighborhoods have very few Latino (the points fall along the bottom of the plot), but that there are a fair number of Blacks in predominantly White/Latino neighborhoods. In addition to plotting the composition over time, one may also represent how the racial composition of individual neighborhoods change over time as well. Plots showing these changes can be found in Figure 5.

Finally, one can apply the same principles to extend the plot to four racial groups in three dimensions. An example of the change between 1970 and 2010 among neighborhoods classified as "stable White" in a growth mixture model among New York, Los Angeles, Chicago, and Houston metropolitan neighborhoods can be viewed in Figure 6. In these plots, each vertex still represents a neighborhood with a single racial group (Whites on the upper-right, Blacks on the upper-left, Latinos on the upper-near, and Asians on the bottom) and each line still represents neighborhoods made up of only two racial groups. Now, however, each face of the tetrahedron represents neighborhoods made up of only three groups and the center volume represents neighborhoods.

Figure 4: Ternary plots of White, Black, and Latino composition of Chicago metropolitan neighborhoods from 1970 to 2000.

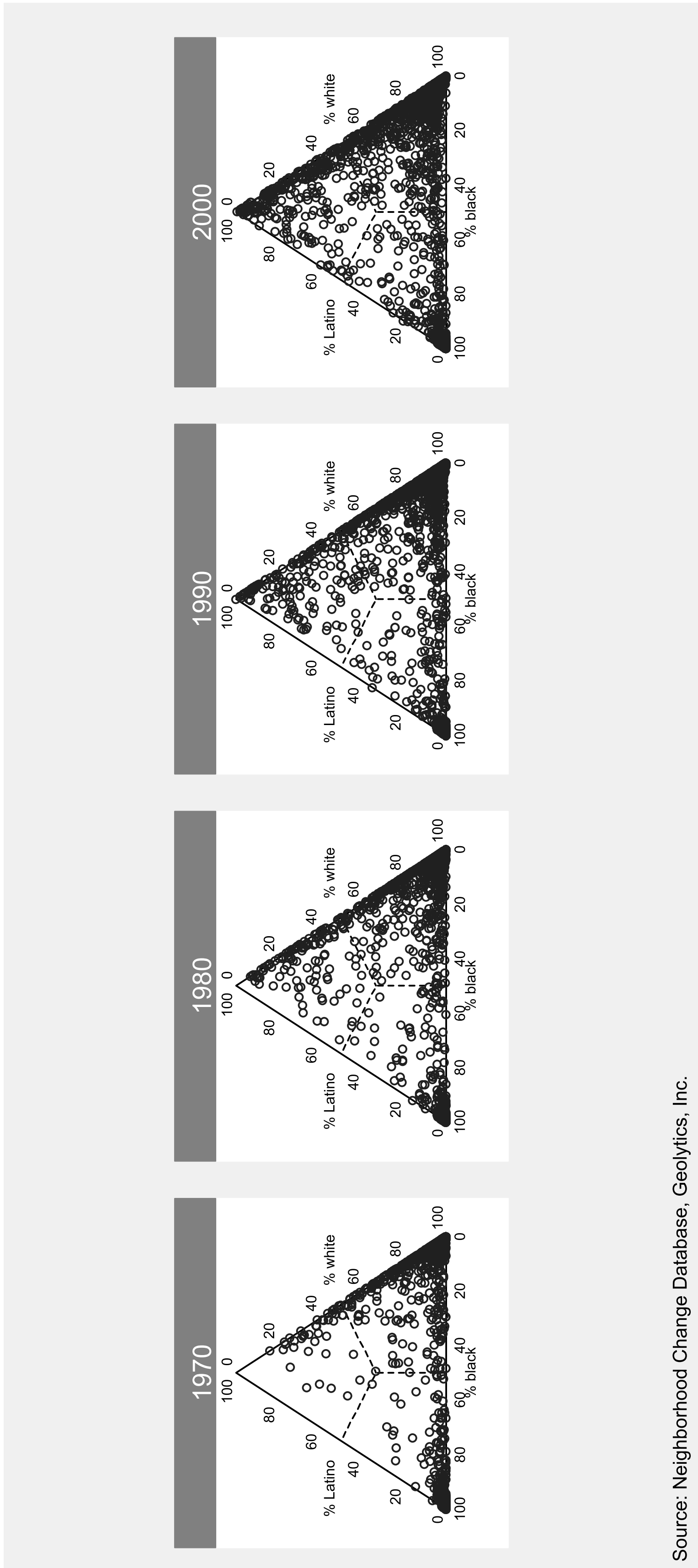


Figure 5: Ternary plots showing change in racial composition by starting racial composition category of Chicago metropolitan neighborhoods from 1970 to 2000.

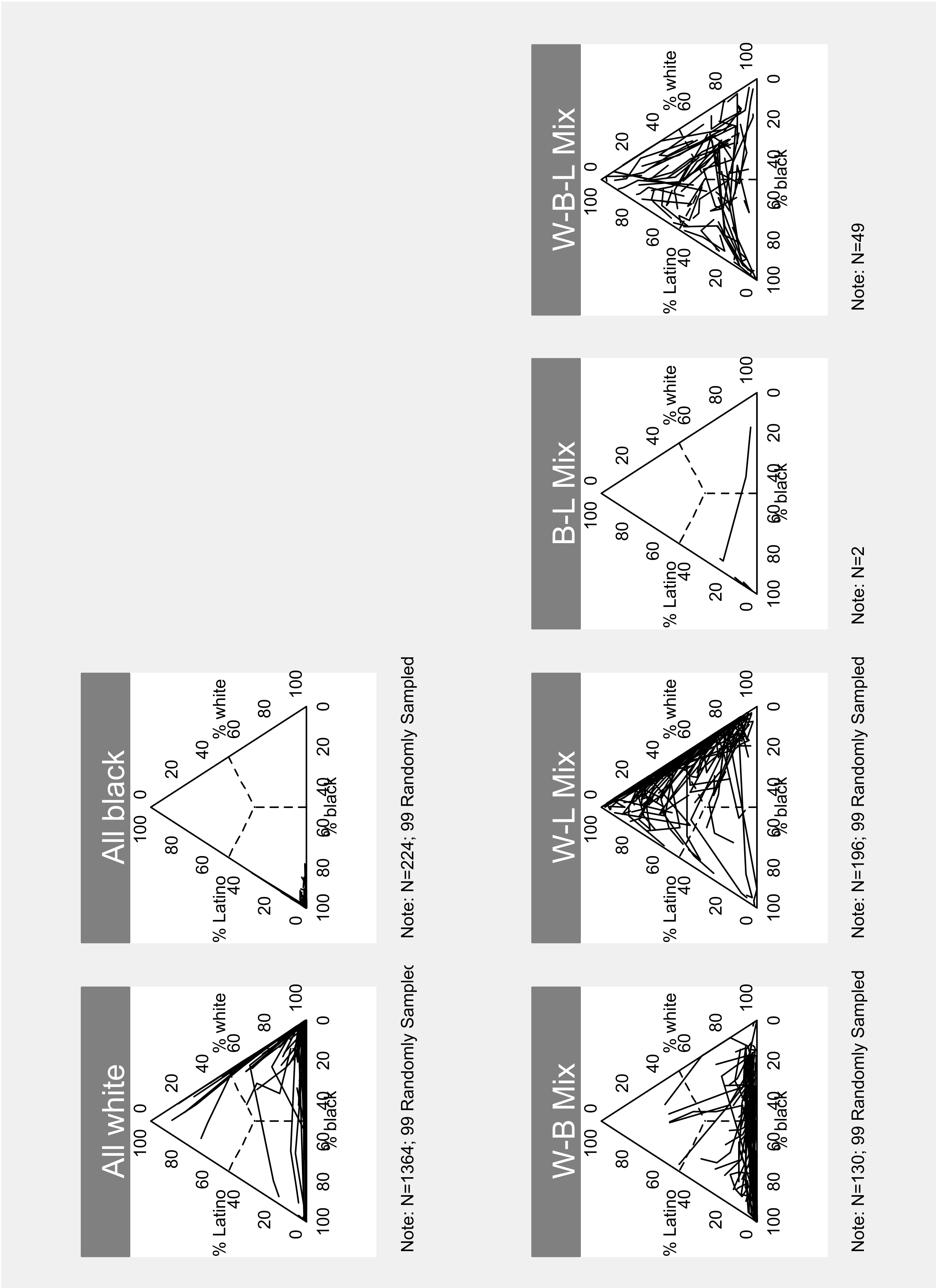
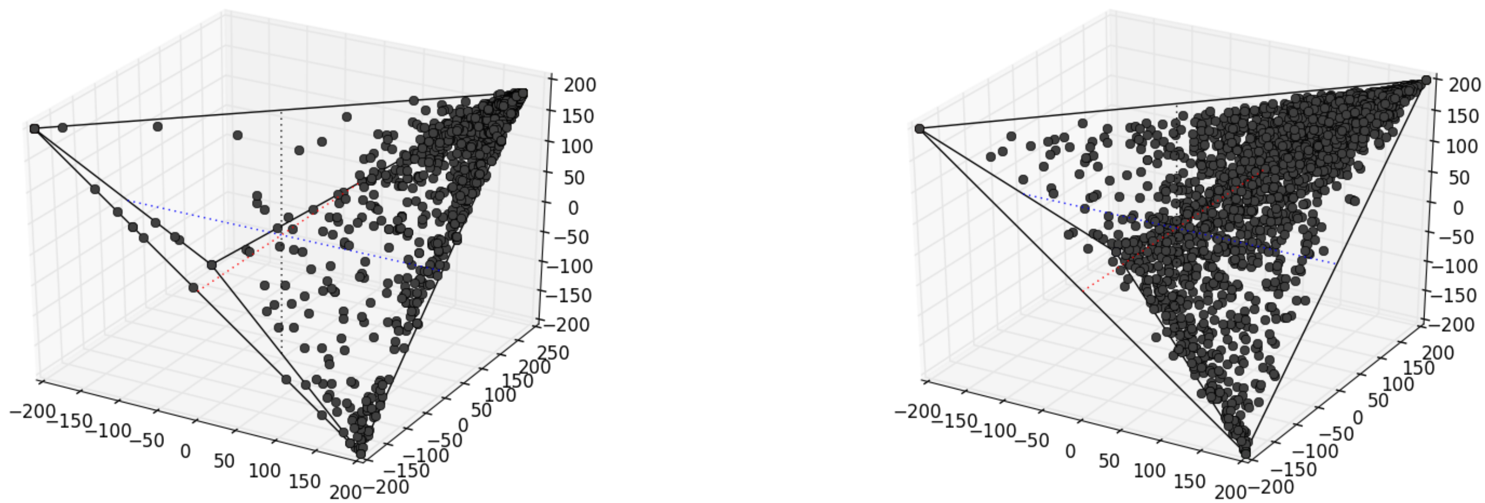


Figure 6: Three-dimensional tetrahedrons representing neighborhood racial composition among Whites, Blacks, Latinos, and Asians in 1970 and 2010 in "stable White" neighborhoods in the New York, Los Angeles, Chicago, and Houston metropolitan areas

(a) 1970 **(b)** 2010



3 Relationship to Existing Methods

This section will describe, using frequently cited papers, how barycentric coordinate plots can be used to visualize existing methods of research and what they can reveal beyond what is available in those methods.

1. Transition matrices
   - Logan & Zhang
   - Friedman
2. Group growth/decline
   - Ellen
   - Denton & Massey
3. Entropy
   - Farrell & Lee

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