A Theory of Premarital Investments and Marriage Timing

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Abstract

I extend Becker's static marriage matching model and study premarital investments and marriage timing decisions in a dynamic equilibrium framework. In my model, men and women face a college investment opportunity and a subsequent career reinvestment opportunity to improve their prospects in the labor market as well as in the marriage market, but these two investments delay marriage and affect the two genders' values in the marriage market asymmetrically. Unlike men, women may lose reproductive fitness and their associated value in the marriage market when they invest and delay marriage. Surprisingly, the recent global phenomenon that more women than men go to college arises in the unique equilibrium. The endogenous determination of marriage values plays a critical role in the result. The model not only explains college education and career advancement patterns but also deepens our understanding of marriage timing. In particular, I unify and extend previous theories to explain the shrinking gender gap in marriage age and the relationships between marriage age and personal income.

Keywords: premarital investments, differential fecundity, the college gender gap, marriage age-personal income relationships
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1 Introduction

In his seminal work "A Theory of Marriage," Gary Becker approaches marriage like any other economic transaction. A man and a woman marry to produce and share food, children, and companionship, just like workers cooperate to earn and divide monetary profits. A shadow market to find marriage partners operates just like a market to find trading partners. In this marriage market, everyone is selfish and finds the partner that achieves his or her maximum individual benefit. A rich set of implications follows from these simple economic principles. For example, marriage depends on not only physical attractions but also people's health, income, and education. The economic approach has revolutionized our view of marriage. Numerous papers have continued to explore the theoretical and empirical implications of this approach to this day.

However, in the original and most subsequent papers, people's marriage traits and the distribution of these traits in the population are exogenously given. They study the matching patterns of different traits, but do not consider how people come to possess these traits. Although traits like height and IQ are not changeable, many others are. People work hard to become high wage earners. People choose to become and stay smokers. Oftentimes people invest for the exact reason to boost their chances in courting and marriages. Teenagers go on diet or build muscles to be more presentable on dates. Parents hope that colleges present not only better job opportunities for their children, but also a bigger and better pool of marriage candidates. A successful career plays a key role to attract suitors and adds value to family life. When adults choose jobs, they often keep their marriage and family lives in mind.

Many significant investments also take significant amount of time. Weight loss and six packs require persistence. A quality college degree takes four years and an advanced degree takes additional three to five years. Climbing up a company's hierarchy requires meticulous planning and sometimes luck. Due to the time-consuming nature of these investments, people need to consider when they can marry and reap the benefits from these investments. An important marriage trait is closely tied to marriage age. It is women's reproductive capital. Their ability to bear children, an important source of marriage benefits, sharply declines with age. When women make college and career investments that improve their income, they delay marriage and face the risk of losing their reproductive capital and their associated attractiveness in the marriage market. Their biological clock is ticking.

In this paper, I integrate important premarital investments and marriage age considerations into the standard marriage theory to derive a set of new implications. I provide a dynamic investment-and-matching framework. Each individual faces a college investment opportunity and a subsequent career reinvestment opportunity. These investments are expected to improve their future wage earnings, and the improved wage earnings also boost people's marriage surpluses. As in Becker (1973), division of the marriage surpluses is endogenously determined by supply and demand of different traits in the marriage market. Making these investments delays marriage and childbearing, however. When women marry older, they may become reproductively unfit and can contribute less to marriage. The marriage delay therefore affects men's and women's marriage prospects and investment decisions differently.

Endogenizing premarital investments in a dynamic setting leads to many new predictions and explanations about higher education, work, and marriage age patterns. First, the theory sheds new lights on college investment patterns. In the model, a college education generates two previously understudied sources besides the wage gains in the labor market - the marriage market and the subsequent career reinvestment opportunity. People are more likely to go to college when the net benefits increase (Proposition 1). As women are less discriminated in the labor market and plan to have fewer children, and the social norm of women staying home weakens, their college enrollment rate would catch up with men's (Proposition 2). Most notably, when women face similar labor market opportunities as men and the social norm vanishes, but the reproductive consideration remains significant, more women than men go to college (Proposition 3). The reversal of the so-called college gender gap has been observed recently worldwide. The result could be surprising at first sight, as women continue to be disadvantaged and should have relatively less incentive to make investments. But, remember, the marriage market is competitive and the agents' payoffs are endogenously determined by the population's investment decisions and distributions of traits. Exactly because women face reproductive constraints, high wage reproductively fit women are more scarce and more valuable than high wage men and accrue higher payoffs in the marriage market. Consequently, more women go to college hoping to capture the higher gain in the marriage market.

Furthermore, the theory offers predictions consistent with current wage patterns and makes bold predictions about the future. Just as with the college investments, people make more career investments when the net benefits in the labor and marriage markets increase (Proposition 4). Women still earn less on average than men and are rare in top positions. These asymmetries have been improving as the net benefits increase more for women and the social norm about asymmetric gender roles weakens (Propositions 5 and 6). In the future when the work structure is more flexible so that women's fertility does not affect their career, "the grand gender convergence" envisioned by Goldin (2014) will arrive: the same proportions of men and women go to college, hold the same kinds of jobs, and earn the same wages (Proposition 7).

Finally, I identify new factors that could delay people's marriage and also explain salient marriage age patterns. Even when investments do not preclude people from marrying early, people may voluntarily delay marriage until they realize the returns from their investments (Proposition 8). The changes that incentivize college and career investments also delay marriage

(Propositions 9). As they make more investments, women marry later than before but still earlier than men (Proposition 10). Finally, the model explains the relationships between marriage age and personal income over time. The men who marry younger and who marry older earn less than those who marry between 25 and 30 (Propositions 11). A woman who marries later was more likely to earn higher income, but very recently the pattern in the United States has changed: the women who marry after 35 earn less than those who marry earlier (Proposition 12).

2 Contributions to Literature

This paper is one of the first to consider premarital investments in an equilibrium framework, and it extends previous related papers in several significant aspects. Cole et al. (2001) is the first to consider an investment phase before people match and share surpluses in Becker's marriage model. Iyigun and Walsh (2007) and Chiappori et al. (2009) are the first attempts to apply the model to study the interactions between premarital investments and the marriage market. I build on their two-period model in which agents can make a college investment in the first period and marry in the second period. I allow an additional career reinvestment opportunity in the second period and marriage in the third period of agents' life, and also incorporate stochastic returns from these investments, gender difference in reproductive length, and marriage timing consideration. To further demonstrate the importance of the investment return uncertainty, in a subsequent paper (Zhang, 2014), I dispense with the assumption that distributions of investments returns are fixed and investigate how agents "gamble," i.e. choose among investments with the same expected return but different levels of uncertainty. I find quite surprisingly that risk averse agents may choose very risky pre-matching investments.

I adopt the continuum version of the assignment model to represent the marriage market. The assignment model as originally set up in Shapley and Shubik (1971) and Becker (1973) consists of finite number of players with one-dimensional attributes. I borrow from Gretsky et al. (1992) in which each side of the two-sided matching market consists of a continuum of agents with possibly multi-dimensional attributes. The continuum setting preserves key results of the finite setting and brings additional advantages. First, each agent's individual investment decision has negligible impact on the distributions of marriage types and hence does not affect the marriage market payoffs of other agents. Second, agents' marriage market payoffs from the surplus division can be readily interpreted as their values in the marriage market and serve as competitive prices.

Interpreting the marriage market values as prices, I provide a new price-theoretic explanation to the college gender gap. Many papers have tried to empirically establish gender differences in benefits and costs to explain the college gender gap. Dougherty (2005) attempts to attribute the gender gap to women's higher college wage premium, but it has been debatable and refuted by Hubbard (2011). Goldin et al. (2006); Becker et al. (2010) attribute the gender gap to women's higher non-cognitive abilities and better preparations in high school as reflected in higher grades. The current explanation does not conflict with these explanations. Rather, I identify a universal economic channel which can explain the global reversal of the gender gap in college enrollment. Women's higher college marital premium predicted by the model is in line with the theoretical arguments in Chiappori et al. (2009) and the empirical findings of Chiappori et al. (2012). Women's higher college martial premium is partly due to the gender difference in reinvestments. This lifetime labor supply consideration has received increasing attention to explain women's rising college enrollment (Bronson, 2013; Reijnders, 2014).

Furthermore, I unify and extend the previous theories that try to account for the gender-specific relationships between marriage age and personal income. The Becker (1973)-Keeley (1974) theory predicts a negative relationship for males and a positive relationship for females. They argue that because higher wage men and lower wage women are more suited for household specialization, they can find mates more easily and marry earlier. On the other hand, Bergstrom and Bagnoli (1993) predict a positive relationship for men and no correlation for women. In their theory, men's wage-earning abilities are revealed later than women. Men with higher earning potential delay marriage to signal their wage-earning abilities but women have their marriage characteristics revealed right away and all marry early. Although these two prominent theories highlight crucial factors for marriage age variations, they cannot explain fully the marriage age-personal income relationships and their frameworks that a priori impose different gender roles in household are too restrictive to study other patterns. I also formalize a popular argument in sociology that uncertainty associated with the labor market can delay marriage (Oppenheimer, 1988).

The model relies on reproductive gender difference to derive the set of gender asymmetries, thus further demonstrating its significant impacts to the labor and marriage market patterns. Siow (1998) shows how differential fecundity affects marriage, divorce, and gender roles in the society. Díaz-Giménez and Giolito (2013) show that differential fecundity can explain the observed spousal age gap patterns in the United States. Low (2014) models the tradeoff between human capital gain and reproductive capital loss, and explains that as the importance of reproductive capital diminishes, highly educated women have married husbands with increasingly higher income in the United States. In addition, she experimentally verifies that men tend to prefer women who are perceived to be reproductively fit.

3 Model

This section describes the simplest theoretical framework that can capture the key economic insights. In the appendix, I present the general theoretical framework that relaxes several simplifying assumptions made in this section[1]. Main results continue to hold in the general framework.

3.1 Agents

There is an infinite number of discrete periods. At the beginning of each period, unit masses of males and females are born with heterogeneous abilities. In each period, the heterogeneous abilities $\theta_m \in \Theta_m = [0,1]$ and $\theta_f \in \Theta_f = [0,1]$ are distributed according to continuous and strictly increasing gender-specific mass distributions G_m and G_f.

Each agent lives for three periods, referred to as ages 1, 2, and 3. Each agent derives utility from wage (w_m, w_f) in the labor market plus payoff (U, V) in the marriage market and net any investment cost (c_m, c_f). All the agents are risk-neutral and there is no discounting.

3.2 Timing and Strategies

Each newborn learns his or her own ability θ and chooses whether to go to college (R_1) or not (A_1). An agent who does not go to college earns a low wage $(\underline{w}_m, \underline{w}_f)$ from the labor market and enters the marriage market immediately. On the other hand, an agent who goes to college delays entry to the marriage market[2] and pays a positive investment cost (c_m, c_f).

At the beginning of age 2, each ability θ agent who has made the college investment receives a high wage $(\overline{w}_m, \overline{w}_f)$ job offer with probability θ and a low wage offer otherwise. An agent who receives a high wage offer accepts the offer. One who receives a low wage offer can either accept (A_2) or reject (R_2) the offer. An agent who accepts the job offer earns the lifetime wage and enters the marriage market. An agent who rejects the job offer delays entrance to the marriage market and pays another investment cost (c_m, c_f) to receive another draw next period.

At the beginning of age 3, each ability θ agent who has rejected the job offer at age 2 receives a high wage offer with probability θ and a low wage offer otherwise. The agent at this point has no choice but to accept the offer and to enter the marriage market.

In summary, men and women face the same investment strategies. They each make two investment decisions at ages 1 and 2: $\sigma_m^1, \sigma_f^1 : [0,1] \to \{A_1, R_1\}$ and $\sigma_m^2, \sigma_f^2 : [0,1] \to \{A_2, R_2\}$.

[1]The simplifying assumptions I will relax are that agents do not discount, the labor market has discrete wage sets, agents draw from the same wage distributions from both the college investment and the career reinvestment, and reproductively unfit women produce zero marriage surplus.

[2]For now I assume that entering the marriage market while investing is not feasible. I show in Proposition 8 that the strategy even if feasible is dominated.

3.3 Marriage Market

The only gender asymmetry in the model is that a woman stays reproductively fit for shorter amount of time than men. A woman realizes her reproductive fitness after she enters the marriage market. She is fit ($\overline{r}$) with probability ϕ_a at age a and unfit ($\underline{r}$) with probability $1 - \phi_a$. Assume that a woman stays reproductively fit with declining probability as they age, and the rate of decline is increasing: $\phi_1 = 1 \geq \phi_2 \geq \phi_3$ and $\phi_1 - \phi_2 \leq \phi_2 - \phi_3$.

A couple's marriage surplus depends on the husband's wage but on both the wife's wage and reproductive fitness. A man with wage w_m and a woman with wage w_f and reproductive fitness r optimize their working time and produce total marriage output that consists of their wage earnings $t_m w_m + t_f w_f$ that contribute to their consumption and their output in the household $s(1 - t_m, 1 - t_f, w_m, w_f, r)$, including children and time spent on leisure and household chores,

$$Z(w_m, w_f, r) \equiv \max_{0 \leq t_m \leq 1, 0 \leq t_f \leq 1} t_m w_m + t_f w_f + s(1 - t_m, 1 - t_f, w_m, w_f, r).$$

People would work full time and consume their wages when they are unmarried. Therefore, a couple's marriage surplus is $S(w_m, w_f, r) \equiv Z(w_m, w_f, r) - w_m - w_f$. Assume a man and an unfit woman always produce zero surplus, $S(w_m, w_f, \underline{r}) = 0$. A high wage man and a high wage fit woman produce $S_{hh} \equiv S(\overline{w}_m, \overline{w}_f, \overline{r})$. A high wage man and a low wage fit woman produce $S_{hl} \equiv S(\overline{w}_m, \underline{w}_f, \overline{r})$. Similarly, a low wage man produces $S_{lh} \equiv S(\underline{w}_m, \overline{w}_f, \overline{r})$ and $S_{ll} \equiv S(\underline{w}_m, \underline{w}_f, \overline{r})$ with a high wage fit woman and with a low wage fit woman, respectively. Assume that the surplus is strictly increasing in wages, $S_{hh} > S_{hl}, S_{lh} > S_{ll}$. Finally, assume that the surplus is strictly supermodular, $S_{hh} + S_{ll} > S_{hl} + S_{lh}$.

Measures μ on $\{\underline{w}_m, \overline{w}_m\}$ and ν on $\{(\underline{w}_f, \overline{r}), (\overline{w}_f, \overline{r}), (\underline{w}_f, \underline{r}), (\overline{w}_f, \underline{r})\}$ describe the marriage market. Men and women frictionlessly match and bargain over division of their marriage surplus. An outcome of the marriage market specifies a feasible matching and stable marital payoffs. The matching function $\pi : \{\underline{w}_m, \overline{w}_m\} \times \{(\underline{w}_f, \overline{r}), (\overline{w}_f, \overline{r}), (\underline{w}_f, \underline{r}), (\overline{w}_f, \underline{r})\} \to \mathbb{R}_+$ describes the masses of matches between different types of men and women and is feasible if π has marginals μ and ν. The marital payoff functions $U : \text{supp}(\mu) \to \mathbb{R}_+$ and $V : \text{supp}(\nu) \to \mathbb{R}_+$ specify payoffs of men and women, respectively.

An outcome (π, U, V) is stable if π solves the primal problem

$$\sup \left\{ \int S d\tilde{\pi} | \tilde{\pi} \text{ is feasible} \right\},$$

and U and V solve the dual problem

$$\inf\left\{\int \tilde{U} d\mu + \int \tilde{V} d\nu | \tilde{U} \geq 0, \tilde{V} \geq 0\right\}.$$

The primal problem and the dual problem have solutions and there is no gap between the solutions (Gretsky et al. 1992, Theorem 1), so a stable outcome exists. The solutions satisfy the stability conditions, $U(w_m) + V(w_f, r) \geq S(w_m, w_f, r)$ for any $(w_m, w_f, r) \in \text{supp}(\mu) \times \text{supp}(\nu)$ and $U(w_m) + V(w_f, r) = S(w_m, w_f, r)$ if $\pi(w_m, w_f, r) > 0$ (Gretsky et al. 1992, Lemma 3).

Assume that those who produce zero surplus are matched. Then the stable matching π is uniquely determined. Finally, if there are unequal masses of men and women, assume that everyone with the same matching characteristic has the same probability of being unmatched, and that an agent unmatched in the current period enters the marriage market in the subsequent period if alive.

4 Stationary Equilibrium

In this section, I define and characterize the stationary equilibrium of this dynamic economy. The equilibrium consists of strategies and a marriage market outcome. The strategies maximize agents' expected lifetime utilities, and the marriage market outcome is stable and is consistent with the equilibrium strategies the agents play. We say (σ_m, σ_f) induce (μ, ν) if the marriage market is (μ, ν) when everyone plays according to (σ_m, σ_f).

For every marriage market (μ, ν), stable payoff functions U and V are defined only for types in the supports $\text{supp}(\mu)$ and $\text{supp}(\nu)$. If an agent takes an action and becomes a type outside the support, then his or her payoff is undefined. I extend the stable payoff functions to be defined on $\{\underline{w}_m, \overline{w}_m\}$ and $\{\underline{w}_f, \overline{w}_f\} \times \{\underline{r}, \overline{r}\}$ whenever necessary. For any $w_m \notin \text{supp}(\mu)$, define w_m's payoff as the maximal payoff he could get by matching with one of the existing partners, namely, $U(w_m) \equiv \sup_{(w_f, r) \in \text{supp}(\nu)}[S(w_m, w_f, r) - V(w_f, r)]$. V is similarly extended: for any $(w_f, r) \notin \text{supp}(\nu)$, $V(w_f, r) \equiv \sup_{w_m \in \text{supp}(\mu)}[S(w_m, w_f, r) - U(w_m)]$. Call the redefined outcome (π, U, V) an extended stable outcome of (μ, ν).

Definition 1. $(\sigma_m^*, \sigma_f^*, \pi^*, U^*, V^*)$ is a stationary equilibrium if the strategies (σ_m^*, σ_f^*) are optimal with respect to (π^*, U^*, V^*) and (π^*, U^*, V^*) is an extended stable outcome of the marriage market (μ^*, ν^*) induced by (σ_m^*, σ_f^*).

Assume that there are always positive masses of men and women investing in equilibrium. It is sufficient to restrict the investment costs to be sufficiently low ($c_m < \overline{w}_m - \underline{w}_m$ and $c_f <$

$\overline{w}_f - \underline{w}_f + \phi_2 S_{lh} - S_{ll}$). In the remainder of the section, I solve the equilibrium by characterizing the candidate equilibrium strategies and stable outcome of the marriage market.

Lemma 1 (Stable Outcome of the Marriage Market)**.** *Men and fit women are matched positively assortatively. The stable marriage payoffs are increasing in wages, and all unfit women get zero. Namely,* $\overline{U} \equiv U(\overline{w}_m) \geq \underline{U} \equiv U(\underline{w}_m)$, $\overline{V} \equiv V(\overline{w}_f, \overline{r}) \geq \underline{V} \equiv V(\underline{w}_f, \overline{r})$, *and* $\overline{v} \equiv V(\overline{w}_f, \underline{r}) = \underline{v} \equiv V(\underline{w}_f, \underline{r}) = 0$.

The stable marriage payoffs are sufficient statistics of the marriage market for the agents to make optimal investment decisions. Express the marriage payoff differences as $\Delta U \equiv \overline{U} - \underline{U} \geq 0$ and $\Delta V \equiv \overline{V} - \underline{V} \geq 0$, and the wage differences as $\Delta w_m \equiv \overline{w}_m - \underline{w}_m \geq 0$ and $\Delta w_f \equiv \overline{w}_f - \underline{w}_f \geq 0$.

4.1 Optimal Strategies

Optimal strategies can be characterized by cutoffs. Any male with ability below a certain cutoff never invests, and any male with ability above the cutoff always invests facing a low wage offer. Any female with ability above a cutoff makes the initial college investment, but only those above another higher cutoff reinvest facing a low wage offer at age 2. Let's solve these cutoffs.

A college investment bears a cost and generates benefits from expected utility gain in the labor market, expected utility gain in the marriage market, and future expected utility gain from reinvestment in case the college investment fails. An ability θ_m male pays investment cost c_m, and generates an expected gain of $\theta_m \Delta w_m$ in the labor market, an expected gain of $\theta_m \Delta U^*$ in the marriage market, and an indirect benefit from the opportunity of reinvestment. The reinvestment opportunity yields an expected benefit of $\theta_m^* \Delta w_m + \theta_m^* \Delta U^* - c_m$ when the college investment fails to yield a high wage offer. Therefore, any ability $\theta_m > \theta_m^* \equiv (\Delta w_m + \Delta U^*)/c_m$ male reinvests after a failed college investment. An ability θ_m^* male gets zero benefit from reinvestment and also a zero benefit $\theta_m^* \Delta w_m + \theta_m^* \Delta U^* - c_m = 0$ from a college investment. In summary, ability $\theta_m < \theta_m^*$ males do not invest at all as they do not yield a positive net benefit from any investment, ability $\theta_m > \theta_m^*$ males go to college and always reject a low wage offer at age 2, and θ_m^* males are indifferent between investing and not investing. I make the assumption that agents invest whenever indifferent. This assumption is innocuous since the ability distribution is atomless. Lemma 2 summarizes males' optimal strategies.

Lemma 2 (Males' Optimal Strategies)**.** *Suppose the marriage market outcome is* (π, U, V). *Let*

$$\theta_m^* = c_m/(\Delta w_m + \Delta U). \tag{1}$$

Males' optimal strategies are

$$(\sigma_m^{1*}(\theta_m), \sigma_m^{2*}(\theta_m)) = \begin{cases} (R_1, R_2) & \textit{if } \theta_m \geq \theta_m^*, \\ (A_1, A_2) & \textit{if } \theta_m < \theta_m^*. \end{cases}$$

In contrast to males, Females face the additional reproductive loss. At age 1, a θ_f female who goes to college pays cost c_f and accrues an expected labor market gain of $\theta_f \Delta w_f$, an expected marriage market gain of $\theta_f \phi_2 \Delta V - (1-\phi_2)\underline{V}$, and an expected gain of $\max\{0, \theta_f(\Delta w_f + \phi_3 \Delta V) - c_f - (\phi_2 - \phi_3)\underline{V}\}$ from the reinvestment opportunity. An ability $\theta_{f2}^* = [c_f + (\phi_2 - \phi_3)\underline{V}]/(\Delta w_f + \phi_3 \Delta V)$ female gets no benefit from reinvestment. Because the fitness probability declines at an increasing rate, a $\theta_{f1}^* = [c_f + (1-\phi_2)\underline{V}]/(\Delta w_f + \phi_2 \Delta V)$ female who is indifferent about making the initial college investment will not reinvest. Therefore, females with abilities below θ_{f1}^* never invest, females with abilities between θ_{f1}^* and θ_{f2}^* will only make the college investment, and those with abilities above θ_{f2}^* make the college investment as well as the career reinvestment. Lemma 3 summarizes females' optimal strategies.

Lemma 3 (Females' Optimal Strategies)**.** *Suppose the marriage market outcome is* (π, U, V)*. Let*

$$\theta_{f1}^* = [c_f + (1-\phi_2)\underline{V}]/(\Delta w_f + \phi_2 \Delta V), \tag{2}$$

and

$$\theta_{f2}^* = [c_f + (\phi_2 - \phi_3)\underline{V}]/(\Delta w_f + \phi_3 \Delta V). \tag{3}$$

Females' optimal strategies are

$$(\sigma_f^{1*}(\theta_f), \sigma_f^{2*}(\theta_f)) = \begin{cases} (R_1, R_2) & \textit{if } \theta_f \geq \theta_{f2}^*, \\ (R_1, A_2) & \textit{if } \theta_{f1} \leq \theta_f < \theta_{f2}^*, \\ (A_1, A_2) & \textit{if } \theta_f < \theta_{f1}^*. \end{cases}$$

The strategies specified in the two propositions are the unique optimal strategies for all but the measure zero $\theta_m^*, \theta_{f1}^*, \theta_{f2}^*$ agents who are indifferent between investing and not investing. These optimal strategies induce unique stationary wage distributions in the marriage market, characterized by (4)-(9) below.

4.2 Induced Marriage Market Distributions

High wage males marry at either age 2 or age 3. The mass of the high wage males who enter the marriage market at age 2 is $\int_{\theta_m^*}^1 \theta dG_m(\theta)$, and the mass of the high wage males who enter the marriage market at age 3 is $\int_{\theta_m^*}^1 (1-\theta)\theta dG_m(\theta)$. Therefore, the mass of high wage males in the marriage market is

$$\overline{\mu}^* \equiv \mu^*(\overline{w}_m) = \int_{\theta_m^*}^1 (2-\theta)\theta dG_m(\theta). \tag{4}$$

Two types of males earn a low wage: those who do not make the college investment, and those who fail both investments at age 1 and at age 2. The mass of low wage males is then

$$\underline{\mu}^* \equiv \mu^*(\underline{w}_m) = G_m(\theta_m^*) + \int_{\theta_m^*}^1 (1-\theta)^2 dG_m(\theta). \tag{5}$$

Similar to high wage males, the high wage fit females are either age 2 or age 3,

$$\overline{\nu}^* \equiv \nu^*(\overline{w}_f, \overline{r}) = \phi_2 \int_{\theta_{f1}^*}^1 \theta dG_f(\theta) + \phi_3 \int_{\theta_{f2}^*}^1 (1-\theta)\theta dG_f(\theta). \tag{6}$$

The low wage fit females in the marriage market can be any of the three ages. The females with abilities between θ_{f1}^* and θ_{f2}^* enter the marriage market at age 2 regardless of their labor market outcome. Therefore, besides age 1 females who never invest and age 3 females who fail two investments, there are also age 2 low wage fit females in the marriage market.

$$\underline{\nu}^* \equiv \nu^*(\underline{w}_f, \overline{r}) = G_f(\theta_{f1}^*) + \phi_2 \int_{\theta_{f1}^*}^{\theta_{f2}^*} (1-\theta) dG_f(\theta) + \phi_3 \int_{\theta_{f2}^*}^1 (1-\theta)^2 dG_f(\theta). \tag{7}$$

The unfit females make up the rest of the marriage market,

$$\nu^*(\overline{w}_f, \underline{r}) = (1-\phi_2) \int_{\theta_{f1}^*}^1 \theta dG_f(\theta) + (1-\phi_3) \int_{\theta_{f2}^*}^1 (1-\theta)\theta dG_f(\theta), \tag{8}$$

$$\nu^*(\underline{w}_f, \underline{r}) = (1-\phi_2) \int_{\theta_{f1}^*}^{\theta_{f2}^*} (1-\theta) dG_f(\theta) + (1-\phi_3) \int_{\theta_{f2}^*}^1 (1-\theta)^2 dG_f(\theta). \tag{9}$$

There could be a bigger mass of high wage men, a bigger mass of high wage fit women, or equal mass of them in the marriage market. When the mass of high wage men exceeds that of high wage fit women, there are three different cases: a bigger mass of high wage men, a bigger mass of fit women, and equal mass of them. The stable marriage payoffs differ in these five possible scenarios. They are delineated in the appendix.

4.3 Equilibrium Uniqueness and Efficiency

In fact, there always exists a unique stationary equilibrium. Even the marriage payoffs are uniquely determined.

Theorem 1 (Equilibrium Uniqueness)**.** *There exists a unique stationary equilibrium.*

Moreover, the unique stationary equilibrium is socially efficient. More precisely, in the equilibrium, the average lifetime utility of each cohort of men and women is maximized. In other words, even a benevolent social planner who can dictate the investments of the population to maximize the population's total welfare would choose the privately chosen equilibrium investments.

Theorem 2 (Equilibrium Efficiency)**.** *The unique stationary equilibrium is socially efficient.*

Efficiency of private investments has been shown in many similar settings (Cole et al., 2001; Peters and Siow, 2002; Iyigun and Walsh, 2007; Chiappori et al., 2009; Dizdar, 2013; Nöldeke and Samuelson, 2014; Hatfield et al., 2014). However, the current result does not directly follow from the previous results in the settings in which investments yield deterministic returns. When the investments yield deterministic returns, a man and a woman who would match after investing can contract on their pre-matching investments. In this setting, such contracts cannot be signed pairwise, because the investment outcome not only changes one's own marriage type but also possibly changes one's partner. Nonetheless, the underlying economic principles are comparable. In the deterministic settings, stable payoffs internalize the social gains. In this setting, the rationally expected stable payoffs internalize the *expected* social gains. Moreover, socially inefficient equilibria in Cole et al. (2001); Dizdar (2013); Nöldeke and Samuelson (2014) do not arise in the current setting. Inefficiency occurs when men and women mis-coordinate. I have assumed that in equilibrium a positive mass of men and a positive mass of women invest. Therefore, no mis-coordination is possible.

5 College Investments and Career Reinvestments

In this section, I analyze the costs and benefits of a college education. In the current model, a college education's benefits come from better job prospects immediately out of college, better marriage prospects immediately out of college, and the future investment opportunities enabled by a college degree. I pay special attention to the benefits associated with marriage and those associated with the reinvestment opportunity. These considerations have often been ignored or mentioned superficially in previous analyses. The combination of these two additional considerations can explain an array of college investment and career investment patterns. In particular, it offers an explanation to the most important but also the most puzzling pattern of college investments in the past several decades: women have not only caught up with men in the college enrollment rate, but they have also surpassed men in most of the developed countries and many of the developing countries.

5.1 College Investments

A college investment is costly in many ways beyond the tuition fee. The investment cost also includes the cost of financing the tuition, the interests paid on top of loans and the psychological cost associated with financing burden for example. The average American college graduate has a debt of $29,400. This amount is a quite significant burden as it is almost the average annual income. The investment cost also includes the energy spent preparing for college entrance and the energy and disutility spent studying in college. College preparation can go back as far as before a person is even born. Preschool investments, high school SAT preparations, extracurricular resume-padding activities - all of these can cause significant discomfort. Finally, a significant concern is foregone earnings and other opportunity cost from spending time in college. A four-year college prevents an individual from holding a full-time job and also delays job skill accumulation for the same period of time. The opportunity cost of wage earnings has been considered a major tradeoff for the marginal students who decide between a high school diploma and a college education and who are probably not in high socioeconomic families who can afford college. For women, especially those in developing countries, college also prevents childbearing at an early age.

Despite the multitude of costs associated with a college investment, it also generates many benefits. It generates an enormous gain in wage earnings. In the United States, the average wage of those who go to college is about 60% higher than those who finish high school but do not go to college (Hubbard, 2011). The wage earnings bring about better health, higher life expectancy, and also gains from marriage. The marriage gains include higher probability of marriage, a better partner in the marriage market, and more efficient household production and more enjoyable consumption with marriage partner, and more and higher quality children. Finally, the reinvestment opportunity provides another chance to realize the gains from the labor market and in the marriage market. This reinvestment is much easier with a college degree and sustained skills and knowledge gained through a college education.

The world has experienced a boom in college education in the recent decades. Every country has recorded higher proportion of college graduates among their 30-34 year-olds in 2010 than in 1970. The increase has been as dramatic in the lower income countries as in the higher income ones. From 1970 to 2010, The college enrollment rate has grown from less than 3% to 11% in the countries with below median per capita GDP, and from 12% to 27% in the countries with above median per capita GDP (Lutz et al., 2007; Becker et al., 2010). In the United States, the college enrollment rate has gone up from 21% to 33% during that period (Figure 1). The rapid growth of higher education is unarguable no matter the measure used (e.g. number of applications, enrollment in four-year institutions, enrollment in two-year institutions, number of graduates, BA degrees granted)[3].

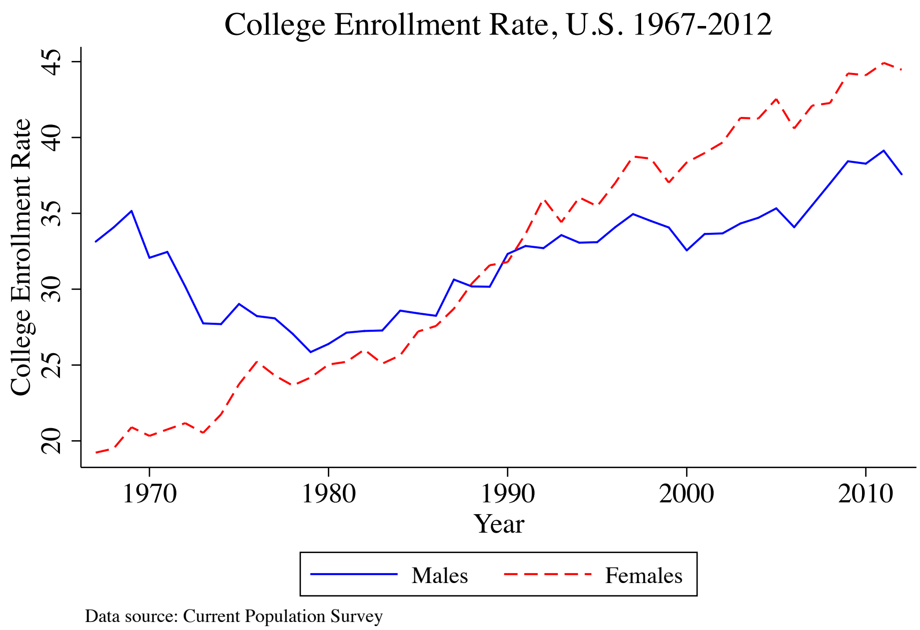

Figure 1: College enrollment rate in the United States, 1967-2012. Data source: Current Population Survey.

Why has this worldwide boom in higher education occurred? In the model, proportion $G_m^{R_1} \equiv 1 - G_m(\theta_m^*)$ of males and proportion $G_f^{R_1} \equiv 1 - G_f(\theta_{f1}^*)$ of females go to college. The model predicts that more men and women would go to college when their investment costs decrease, when their expected labor market gains increase, or when their marriage market gains increase.

Proposition 1 (Changes in College Investments)**.** *The proportion of men making college investments, $G_m^{R_1}$, increases when their investment cost c_m decreases, wage gain Δw_m increases, labor market opportunities increase (G_m first order stochastically dominantly shifts), or a low wage fit woman generates increasingly more marriage surplus with a high wage man than with a low wage man ($S_{hl} - S_{ll} \uparrow$).*

The proportion of women making college investments, $G_f^{R_1}$, increases when their investment cost

[3]See Goldin et al. (2006) for college education patterns of the United States over the last one hundred years. See Becker et al. (2010) for major college education patterns around the world.

c_f decreases, wage gain Δw_f increases, labor market opportunities increase (G_f first order stochastically dominantly shifts), age two fitness probability ϕ_2 increases, a high wage man generates increasingly more surplus with a high wage fit woman than with a low wage fit woman ($S_{hh} - S_{hl} \uparrow$), or a low wage couple's marriage surplus S_{ll} decreases.

Probably the increase in demand for more skilled labor has been one of the primary reasons for the boom in higher education. Higher demand drives up the competitive wage for high skilled workers more than that for low skilled workers. It has been well documented how college enrollment responds to changes in wages (Katz and Murphy, 1992). The college wage premium, the difference between the wages earned by those who go to college and those who do not, has increased steadily for both men and women (Dougherty, 2005; Hubbard, 2011).

The changes in the family structure and the marriage market are important causes too, especially for women. Medical technology has advanced, effectively lengthening and improving women's overall reproductive fitness. The decline in total fertility rate has been conspicuous. The total fertility rate has dropped drastically on every continent, and the world's average has halved from 5 in 1960 to 2.5 in 2012. The fertility decline and the introduction of household items such as dishwashers and laundry machines contribute to women's higher labor force participation and declining importance of reproductive fitness in marriage. The increased reproductive fitness at early age, the increased labor force participation, the decreased relative importance of reproductive fitness in marriage, and changes in the social norm in gender roles all contribute to women's higher college enrollment.

These changes in the family structure and the marriage market are perhaps the most important reason to explain the gender difference in rate of increase in college enrollment. Figures 2 and 3 show that the countries with lower total fertility rate tend to have relatively more female college graduates. Women's college enrollment rate in the United States has doubled from 17% in 1970 to 34% in 2010, while men's enrollment has been fluctuating around 30% since 1975 (Figure 1). Women have caught up with men in college enrollment around the world. Goldin and Katz (2002) find supporting evidence that the introduction of birth control pill raised females college enrollment rate in the United States. The model predicts that the college gender gap $\Delta G^{R_1} \equiv G_m^{R_1} - G_f^{R_1}$ shrinks when female investment cost decreases, female wage differential increases, the probability of fitness at age two increases, or women face less discrimination in the labor market.

Proposition 2 (The Shrinking College Gender Gap)**.** *Women catch up to men in college enrollment rate ($\Delta G^{R_1} \downarrow$) when women's investment cost c_f decreases, wage gain Δw_f increases, labor market opportunities increase (G_f first oder stochastically dominantly shifts), age two fitness probability ϕ_2 increases, a low wage fit couple's marriage surplus S_{ll} decreases, or a high wage man generates* *increasingly more marriage surplus with a high wage fit woman than with a low wage fit woman* $(S_{hh} - S_{hl} \uparrow)$.

Perhaps more puzzling is a sign reversal in the college gender gap from male-dominated to female-dominated. In most developed countries and many developing countries, significantly more women than men go to college. In 1970, only five out of 120 countries in the world had more 30-34-year-old college women than college men. In 2000, that number has leaped to 50. By 2010, the number has increased to 67. The United States witnessed the gender gap reversal in the early 1990s and the gap has been growing steadily. Out of the seventeen OECD countries with consistent higher education data, four had a female-dominated college enrollment rate in 1985, but fifteen did by 2002 (Goldin et al., 2006). Every habited continent has countries with more female students than male students enrolled in college now.

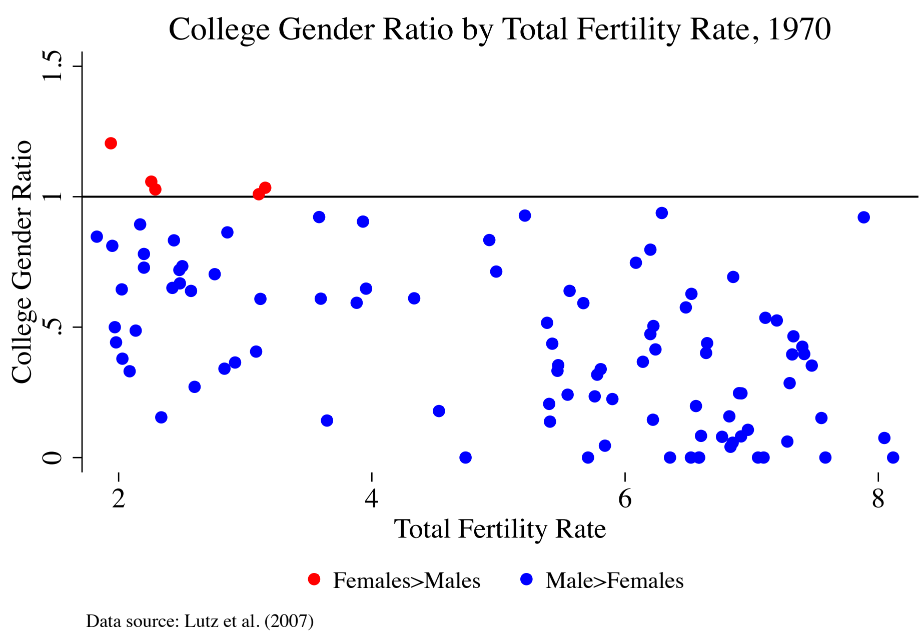

Figure 2: College gender ratio among 30-34 year-olds by total fertility rate, 120 countries in 1970. Data source: Lutz et al. (2007).

Previous explanations of the college gender gap reversal hinge on the changes in some market conditions from less favorable to more favorable to females. Chiappori et al. (2009) propose that more women go to college because the college wage premium may be higher for women, educated women's value in marriage increases, or the social norm that women should stay at home weakens because of technological and social advances. However, in a setting in which men and women have equal investment opportunities, more women than men go to college only when females have lower investment cost or higher college wage premium than males.

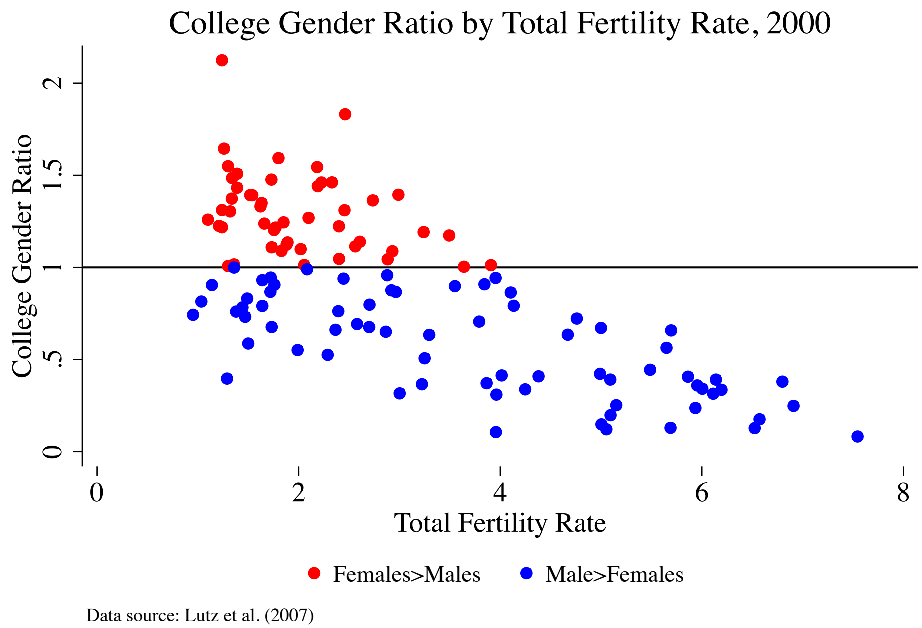

Figure 3: College gender ratio among 30-34 year-olds by total fertility rate, 120 countries in 2000. Data source: Lutz et al. (2007).

Hubbard (2011) shows that females do not have higher college wage premium in the United States. Explanations of Becker et al. (2010) rely on the genders' asymmetric distributions of their non-cognitive abilities. Females' non-cognitive abilities are on average better than males, and their abilities are less varied than men's. When the cost of going to college was high, only the extremely able students attended. Since males' abilities distribution is more widespread, more males were in the upper tail of the abilities distribution. High ability men outnumbered high ability women so men outnumbered women in college enrollment. As the cost of going to college declines, however, many more women have similar abilities and are well-suited for college. The rate of increase to college enrollment has been higher for women than for men, as many women of similar abilities attend colleges.

In the current model, more women than men go to college even if all the aforementioned conditions are gender-symmetric and women contribute to experience reproductive fitness decline. We say a setting is gender-symmetric if their investment costs, wage gains, chances of success in the labor market, and the contributions to household production are the same for both genders while women may remain significantly disadvantaged reproductively. That is, the primitives of the model are the same for both genders: $G_m = G_f \equiv G$, $c_m = c_f \equiv c$, $\Delta w_m = \Delta w_f \equiv \Delta w$, and $S(w_m, w_f, r) = S(w_f, w_m, r)$.

Proposition 3 (Reversal of the College Gender Gap)**.** *In a gender-symmetric setting (*$G_m = G_f \equiv$

G, $c_m = c_f \equiv c$, $\Delta w_m = \Delta w_f \equiv \Delta w$, and $S(w_m, w_f, r) = S(w_f, w_m, r)$), when women may become unfit only later in life ($\phi_2 = 1$ and $\phi_3 < 1$), more women than men go to college in equilibrium ($\Delta G^{R_1} < 0$).

In general, when women do not face a serious tradeoff between marriage and the initial college investment, that is, when ϕ_2 is sufficiently close to 1, more women than men attend college.

Proposition 3' (Reversal of the College Gender Gap, Generalized)**.** *In a gender-symmetric setting ($G_m = G_f \equiv G$, $c_m = c_f \equiv c$, $\Delta w_m = \Delta w_f \equiv \Delta w$, and $S(w_m, w_f, r) = S(w_f, w_m, r)$), when women stay reproductively fit with sufficient high probability at age two ($\phi_2 > [c(S_{hl} - S_{ll}) + S_{ll}(\Delta w + S_{hl} - S_{ll})]/[c(S_{hh} - S_{lh}) + S_{ll}(\Delta w + S_{hl} - S_{ll})]$ and $\phi_3 < 1$), more women than men go to college in equilibrium ($\Delta G^{R_1} < 0$).*

The result could be surprising at first sight, as women are nominally disadvantaged and should have relatively less incentive to invest. The endogenously determined marriage market values play a key role in the result. The marriage matching market is a competitive market where supply and demand determine the agents' values and payoffs. Exactly because women face penalty if they marry late, fewer women than men reinvest and fulfill their labor market potential. This makes high wage fit women relatively more scarce than high wage men, and raises their relative marriage value. When the chance of losing fitness at age two is sufficiently low, more women than men take the initial college investment in order to capture the higher marriage gain.

Since the distribution of abilities is the same for men and women, abilities of the marginal college investors are compared to determine the college enrollment rate. A marginal male investor of θ_m^* ability pays investment cost c and generates expected labor market benefit of $\theta_m^* \Delta w$ and expected marriage market benefit of $\theta_m^* \Delta U^*$. A marginal ability θ_{f1}^* female investor pays cost c and generates expected labor market benefit of $\theta_{f1}^* \Delta w$ and marriage market benefit of $\theta_{f1}^* \Delta V^*$ when $\phi_2 = 1$. Both marginal college investors generate zero reinvestment benefit as the reinvestment benefit is weakly lower than the college investment. Therefore, $\theta_m^* = c/(\Delta w + \Delta U^*) < \theta_{f1}^* = c/(\Delta w + \Delta V^*)$ if and only if $\Delta U^* < \Delta V^*$. Intuitively, the equilibrium marriage payoff difference between high wage fit women and low wage fit women is larger than that between high wage and low wage men. The proof proceeds by contradiction.

The model poses a plausible and significant channel for women's higher college marriage premium compared to men's. Women still earn less than men on average, partly because at some point in life they need to start to carry reproductive responsibilities that interfere with their job prospects. The current job structure is not flexible enough for them to achieve full potential in work and family, as supported by Bertrand et al. (2010) and Goldin (2014). As a result, educated, high wage, and reproductively desirable women who can be successful in both the labor market and the marriage market remain scarce and consequently are valued highly. More women go to college rationally expecting the higher marriage gain.

The difference between average college marital premium and marginal college martial premium should be noted. Chiappori et al. (2012) empirically confirm that American women's average college marriage premium has been consistently higher than men's. However, it is not direct evidence to support the current model. Although the higher average marital premium is highly correlated with higher college enrollment rate, it is not sufficient for more women to go to college, however. The college marital premium is $\theta(\Delta w_f + \Delta V^*) - c_f$ for $\theta \in [\theta^*_{f1}, \theta^*_{f2}]$ women and is $(2-\theta)[\theta(\Delta w_f + \phi_2 \Delta V^*) - c_f] - (1-\theta)[\theta(\phi_2 - \phi_3)\Delta V^* + (1-\phi_3)\underline{V}^*]$ for $\theta_f \geq \theta^*_{f2}$ women. On the other hand, it is $(2-\theta)[\theta(\Delta w_m + \Delta U^*) - c_m]$ for all $\theta > \theta^*_m$ men. Each college graduate male can expect to reap an additional reinvestment benefit, but only some females can reap the reinvestment benefit. Therefore, women's average college martial premium is not necessarily higher than men's.

Furthermore, surplus supermodularity is a crucial assumption for the result. If the surplus function is not strictly supermodular, there cannot be more women than men in college (shown in the appendix). The possible transition of the role of jobs from being substitutes to complements in marriages posits us another reason for the college gender gap reversal. In the recent decades, as technological advances free women from daily chores, the role of the household may have transitioned from a unit of labor specialization to one of cooperation. The marriage surplus, resulting from an underlying intra-household optimization problem, may have changed from being submodular to supermodular in the couple's wage earnings. The college gender gap reversal consequently accompanies the change of husband's and wife's wages from substitutes to complements in household production.

The modularity transition occurs not necessarily when people's preferences change but when people spend time differently. A couple in the United States on average spent eight hours a week less on household chores in 2011 compares to 1965. On the other hand, much more time is spent more to take care of children: 4 hours extra for women and 4.5 hours extra for men. 62% of parents find childcare very meaningful but only 43% find housework meaningful. Because less time is required for and spent for household chores and more time is spent on more pleasurable time of childcare, men and women may prefer partners who can share their happiness and sadness. Men and women may gain from college a better lifelong partner and also better quality in family life.

5.2 Career Reinvestments

A person reinvests if he or she fails to succeed on the first try after college. While ability $\theta \geq \theta_m^*$ men and ability $\theta \geq \theta_{f1}^*$ women make college investments, all $\theta \geq \theta_m^*$ men will reinvest, but only $\theta \geq \theta_{f2}^* > \theta_{f1}^*$ women reinvest. In each cohort, proportion $G_m^{R_2} \equiv \int_{\theta_m^*}^{1}(1-\theta)dG_m(\theta)$ of men and proportion $G_f^{R_2} \equiv \int_{\theta_{f2}^*}^{1}(1-\theta)dG_f(\theta)$ of women make career reinvestments. In general, the changes that encourage more college investments also encourage more career reinvestments.

Proposition 4 (Changes in Career Reinvestments)**.** *The proportion of men making career reinvestments $G_m^{R_1}$ increases when their investment cost c_m decreases, wage gain Δw_m increases, labor market opportunities increase (G_m first order stochastically dominantly shifts), or a low wage woman generates increasingly more marriage surplus with a high wage man than with a low wage man ($S_{hl} - S_{ll} \uparrow$).*

The proportion of women making career reinvestments $G_f^{R_1}$ increases when their investment cost c_f decreases, wage gain Δw_f increases, market opportunities increases (G_f first order stochastically dominantly shifts), age two fitness probability ϕ_2 decreases, age three fitness probability ϕ_3 increases, a low wage fit couple's marriage surplus S_{ll} decreases, or a high wage man generates increasingly more marriage surplus with a high wage fit woman than with a low wage fit woman ($S_{hh} - S_{hl} \uparrow$).

The increasing labor force participation, fertility decline, weakened social norm on gender labor division, and increasing surplus complementarity contributes to more women making more career advancements relatively to men. Denote the gender gap in career reinvestments by ΔG^{R_2}.

Proposition 5 (The Shrinking Gender Gap in Career Reinvestments)**.** *Women catch up to men in career reinvestments ($\Delta G^{R_2} \uparrow$) when their investment cost c_f decreases, wage gain Δw_f increases, labor market opportunities increase (G_f first order stochastically dominantly shifts), age two fitness probability ϕ_2 decreases, age three fitness probability ϕ_3 increases, or a high wage fit woman generates increasing more marriage surplus with a high wage man than with a low wage man ($S_{hh} - S_{hl} \uparrow$).*

These changes also shrink the gender gap in wages. In the United States, women's average wage was only half of men's in the 1970s, but today that ratio has risen to 77%. The biggest reason for the gap is the lack of continued investments by women. Bertrand et al. (2010) find that among young MBAs who have just graduated to take their first post-graduation jobs, men and women earn roughly the same. However, ten years after graduation, the gender gap between wages is huge. In the model, the average wage of men is $\mathbb{E}w_m^* = \underline{w}_m + \overline{\mu}^* \Delta w_m$, and the average wage of women is $\mathbb{E}w_f^* = \underline{w}_f + \nu^*(\overline{w}_f, \cdot)\Delta w_f$. The model predicts the changes on $\Delta w^* = \mathbb{E}w_m^* - \mathbb{E}w_f^*$.

Proposition 6 (The Shrinking Gender Gap in Wages)**.** *The gender gap in wages decreases ($\Delta w^* \downarrow$) when women's investment cost decreases ($c_f \downarrow$), wages $\underline{w}_f, \overline{w}_f$ increase, labor market opportunities increase (G_f first order stochastically dominantly shifts), age two fitness probability ϕ_2 increases, age three fitness probability ϕ_3 increases, or a high wage man generates increasingly more marriage surplus with a high wage fit woman than with a low wage fit woman ($S_{hh} - S_{hl} \uparrow$).*

Goldin (2014) suggests that although the society has experienced gender convergence in education, work, and other aspects, in order to reach the "last chapter" of a "grand gender convergence," work compensations and promotions should not be heavily based on longer hours the workers put in, because the structure hurts women more. In this model, the only fundamental gender difference is that reproduction gets in the way of career advancement. If the technology has advanced to the level that age of marriage and pregnancy affect neither the quantity nor the quality of offsprings, the work is more flexible to accommodate working mothers, and men do not significantly value reproductive fitness or youth in the marriage market, we could experience the envisioned grand gender convergence.

Proposition 7 (Grand Gender Convergence)**.** *When the setting becomes gender-symmetric ($G_m = G_f \equiv G$, $c_m = c_f \equiv c$, $\Delta w_m = \Delta w_f \equiv \Delta w$, and $S(w_m, w_f, r) = S(w_f, w_m, r)$) and reproductive fitness is no longer a concern in the labor and marriage markets ($\phi_2 = \phi_3 = 1$), male and female college investments and career investments converge.*

Under this convergence, there should be equal proportions of male and female college graduates. After the phase of a shrinking college gender gap in which women catch up to men in college enrollment, and the current phase of a reversal of the college gender gap in which women surpass men in college enrollment, we should experience in perhaps the near future a phase of vanishing college gender gap in which the same amount of men and women go to college. Of course, biological and genetic gender differences might prevent the gender gap to vanish altogether. These biological differences can be quite significant obstacles that make the grand gender convergence an unfulfillable dream. Nonetheless, as more women can balance career and family, the increasing abundance and thus declining marriage values of the high wage fit women will result in a smaller college gender gap compared to today.

6 Marriage Age Patterns

In this section, I identify factors that affect people's timing of marriage, and explain observed patterns about marriage age. In particular, the model is consistent with the overall marriage delay in the population, the shrinking gender gap in marriage age, the relationships between marriage age and personal income around the world, and the relationships' evolution across time in the United States. Other marriage age matching patterns are also discussed.

6.1 Marriage-Delaying Factors

People delay marriages for different voluntary and involuntary reasons at different stages of their lives. Time-intensive human capital investments, the uncertainties that accompany the investments, and failure from the investments delay marriages. In contrast, the factors that deter the investments (e.g. high investment cost, low wage benefits) and success from the investments expedite marriage. Women's reproductive constraint expedites their marriages and is the key gender difference that makes women marry younger than men in general. Let me elaborate on these factors and relate to the previous literature.

First, young agents who have higher chance of success take time-intensive college investments and delay marriage to gain from both the labor market and the marriage market. In the model, ability $\theta_m \geq \theta_m^*$ males and ability $\theta_f \geq \theta_{f1}^*$ females go to college at age 1 and delay marriage to age 2 for this reason. Empirically, we observe that more highly educated tend to marry later. For example, among the 40-44-year-old men in the United States in 2012, over 50% of those who did not go to college had married by age 24, but only 20% of the college educated had married by age 24.

In the basic model, the marriage delay due to investment has been forced: people who invest cannot enter the marriage market in the period of investment. In real life, people could choose to marry while investing. However, marrying while investing, even if a feasible strategy, is not an attractive option in the model. Especially for men, entering the marriage market while investing is weakly dominated by entering the marriage market after realizing the investment return.

Suppose that the strategy of marrying and investing at the same time is feasible for men. If an ability θ_m man chooses to enter the marriage market without resolving the uncertainty from investment, he enters as a matching type $\theta_m \circ \overline{w}_m + (1-\theta_m) \circ \underline{w}_m$. He provides marriage surplus as a $\overline{w}_m$ male with probability θ_m and as a $\underline{w}_m$ male otherwise. Any female of type (w_f, r) and the agent produce and divide the expected marriage surplus of $S(\theta_m \overline{w}_m + (1-\theta_m)\underline{w}_m, w_f, r) \equiv \theta_m S(\overline{w}_m, w_f, r) + (1-\theta_m) S(\underline{w}_m, w_f, r)$. A male of type $\theta_m \circ \overline{w}_m + (1-\theta_m) \circ \underline{w}_m$ has an expected marriage payoff of $U(\theta_m \circ \overline{w}_m + (1-\theta_m) \circ \underline{w}_m)$. Suppose the agents match and split the expected surpluses in a stable way such that $U(\theta_m \circ \overline{w}_m + (1-\theta_m) \circ \underline{w}_m) + V(w_f) \geq S(\theta_m \overline{w}_m + (1-\theta_m)\underline{w}_m, w_f)$[4]. It turns that entering the marriage market while investing is always weakly dominated.

[4]Refer to Borch (1962); Wilson (1968); Chiappori and Reny (2006) for detailed treatments of matching market with similar income uncertainty.

Proposition 8 (Investment Uncertainty Delays Marriage)**.** *A man prefers to marry after the investment return is realized over to marry before the investment return is realized.*

A person who decides whether or not to entering the marriage market before realizing the investment compares the expected utility he or she receives in the marriage market as a type $\theta_m \circ \overline{w}_m + (1-\theta_m) \circ \underline{w}_m$ and the expected utility that he receives by entering the market as a $\overline{w}_m$ with probability θ_m and as a type $\underline{w}_m$ with probability $1-\theta_m$. The only difference is in expected marriage payoffs. Suppose a type $\theta_m \circ \overline{w}_m + (1-\theta_m) \circ \underline{w}_m$ male is stably assigned to a type (w_f, r) partner, so that

$$U(\theta_m \circ \overline{w}_m + (1-\theta_m) \circ \underline{w}_m) = \theta_m S(\overline{w}_m, w_f, r) + (1-\theta_m) S(\underline{w}_m, w_f, r) - V(w_f, r).$$

By stability, $U(\overline{w}_m) \geq S(\overline{w}_m, w_f, r) - V(w_f, r)$ and $U(\underline{w}_m) \geq S(\underline{w}_m, w_f, r) - V(w_f, r)$. Combining the two inequalities,

$$\theta_m U(\overline{w}_m) + (1-\theta_m) U(\underline{w}_m) \geq \theta_m S(\overline{w}_m, w_f, r) + (1-\theta_m) S(\underline{w}_m, w_f, r) - V(w_f, r)$$

and the right hand side exactly equals $U(\theta_m \circ \overline{w}_m + (1-\theta_m) \circ \underline{w}_m)$. Therefore,

$$\theta_m U(\overline{w}_m) + (1-\theta_m) U(\underline{w}_m) \geq U(\theta_m \circ \overline{w}_m + (1-\theta_m) \circ \underline{w}_m).$$

I will show in the appendix that the effect holds even if the matching attributes are multi-dimensional. In essence, a man is matched to the partner that maximizes his marriage payoff. This claim directly follows from stability. A type x man is matched to type y woman, and $U(x) = S(x, y) - V(y)$, and for any other $y' \neq y$, $U(x) \geq S(x, y') - V(y')$. A man may be stably and optimally assigned to different partners based on different wage realizations. A man who marries without realizing the investment returns is assigned to a wife that may not be the wife he would be matched to if he enters the marriage market after he realizes his investment return. Due to this stable reassignment effect, waiting to marry until the return from investment realizes weakly dominates marrying before the return from investment realizes.

The marriage-delaying stable reassignment effect is fundamentally different from the signaling effect in Bergstrom and Bagnoli (1993) that relies on imperfect information. In Bergstrom and Bagnoli (1993), a person's true ability is only perfectly observable in the second period of life, and people are matched based on observable abilities. If it is costly to wait, then only people with ability higher than a certain threshold delay their marriage. It is costly to reveal information, and only people with high enough ability are willing to pay such cost of delay. In contrast, in the current investment, people delay marriage even when every matching related characteristic is observable. The stable reassignment consideration arises from return uncertainty from human capital investment that affects subsequent marriage matching outcome, whereas the signaling consideration does not change an agent's innate attribute. The two effects correspond to the two different interpretations of education as human capital investment or a Spence (1973) signaling device. Nonetheless, both the stable reassignment incentive and the signaling incentive deter high ability agents to marry early.

On the other hand, some marriage delays are involuntary. In particular, ability $\theta_m \geq \theta_m^*$ males and ability $\theta_f \geq \theta_{f2}^*$ females who receive a low wage offer at age 2 would have not delayed marriage to age 3 if they have received a high wage offer instead. Therefore, the labor market search friction results in an imperfect labor market outcome and delays these agents from marrying early as the marriage market also values agents' earning abilities. The labor market friction has a similar effect to the search friction mentioned in Becker (1973) and formalized in Keeley (1974). Becker (1973) argues that marriage forms household that serves as small production unit in which men more likely specialize in labor market work and women are responsible for household work and childbearing. As a result, male high wage earners are more valued than low wage earners. High wage earners more easily find mates and marry earlier, whereas low wage earners delay their marriages as a result of marriage market friction explicitly modeled in Keeley (1974). Effectively different from the signaling and stable reassignment effects that delay marriage of more able agents, Becker-Keeley marriage market search friction and the labor market uncertainty both delay marriage of the less able agents.

To recap, the factors that delay marriage in the model include higher benefits and lower costs of the investments, uncertainty from the investments and the associated stable reassignment effect, and labor market friction. Although these factors apply to both men and women, gradual potential loss of reproductive desirability deters women from marrying late. Although women gain wages from investment that boosts their expected labor return and marriage return, they may lose their marriage desirability as they invest. Significant loss of desirability interferes with their investment and marriage incentives, creating gender asymmetry.

In the model, the average age at marriage for a cohort of males is

$$\mathbb{E}a_m^* = 1 + \int_{\theta_m^*}^{1} \theta dG_m(\theta) + 2\int_{\theta_m^*}^{1} (1-\theta) dG_m(\theta) = 1 + \int_{\theta_m^*}^{1} (2-\theta) dG_m(\theta),$$

and the average age at marriage for a cohort of females is

$$\mathbb{E}a_f^* = 1 + \int_{\theta_{f1}^*}^{\theta_{f2}^*} dG_f(\theta) + \int_{\theta_{f2}^*}^{1} (2-\theta) dG_f(\theta).$$

Most of the changes that encourage investments also delay marriage.

Proposition 9 (Changes in Marriage Age)**.** *Men's average age at marriage $\mathbb{E}a_m^*$ increases when their investment cost c_m decreases, wage gain Δw_m increases, labor market opportunities increase (G_m first order stochastically dominantly shifts), or a high wage man generates increasingly more marriage surplus with a low wage fit woman than a low wage man does ($S_{hl} - S_{ll} \uparrow$).*

Women's average age at marriage $\mathbb{E}a_f^*$ increases when their investment cost c_f decreases, wage gain Δw_f increases, labor market opportunities increase (G_f first order stochastically dominantly shifts), age three fitness probability (ϕ_3) increases, a low wage couple's marriage surplus S_{ll} decreases, a high wage fit woman generates increasing more marriage surplus with a high wage man than with a low wage man ($S_{hh} - S_{hl} \uparrow$).

People have generally married later. In the United States, the median ages at marriage have been rising for both genders. The median ages at marriage were 20.3 for women and 22.8 for men in 1960, but have increased to 26.1 for women and 28.2 for men in 2010. In 1960, over 60% American men and over 70% American women had married by age 25, and a relatively small portion of people stayed unmarried passing 35. By 2010, only 30% men and 35% women have marry by age 25, while large proportions of men and women marry late or stay unmarried.

6.2 The Gender Gap in Marriage Age

There are several salient global patterns regarding the gender gap in marriage age. First, the gender gap is positive; men on average marry later than women in every country in the world (United Nations, 1990, 2000). The gender marriage age gap tends to be bigger for developing countries than for developed countries. Within a country, the gap tends to shrink over time. In the model, the gender marriage age gap is

$$\Delta\mathbb{E}a^* = \int_{\theta_m^*}^{1} (2-\theta) dG_m(\theta) - \int_{\theta_{f\,1}^*}^{\theta_{f\,2}^*} dG_f(\theta) - \int_{\theta_{f\,2}^*}^{1} (2-\theta) dG_f(\theta),$$

and it shrinks when the conditions move to favor investments by females.

Proposition 10 (The Shrinking Gender Gap in Marriage Age)**.** *The gender gap in marriage age Δa^* shrinks when women's investment cost c_f decreases, wage gain Δw_f increases, labor market opportunities increase (G_f first order stochastically dominantly shifts), age three fitness probability ϕ_3 increases, a low wage couple's marriage surplus S_{ll} decreases, a high wage fit woman generates increasing more marriage surplus with a high wage man than with a low wage man ($S_{hh} - S_{hl} \uparrow$).*

The declining demand for fertility explains at least partly the marriage age distributions and the gender gap in median age at marriage. If women expect to have fewer children over their lifetime, they do not have the urgency to plan ahead and marry early. As a result, they can invest more in terms of education and career, further delaying their marriage age - not only from time spent in lumpy investments such as colleges and graduate schools, but also from the expected delay of marriage due to negative labor shocks and further investments. In developing countries, fecundity constraint tends to be more significant because of worse medical technology, conservative social norms, and demand for quantity of children as opposed to demand for quality. Furthermore, the college wage premiums may not be as high in developing countries.

Bergstrom and Bagnoli (1993) attribute the gender marriage age gap to gender-specific social roles. They set up a restrictive framework in which females cannot participate in labor market. The social roles in this model arise endogenously instead. Because of limited fecundity, females do not make as much human capital investment as males and consequently have more confined social roles. The current framework builds on a biological gender difference, is more flexible to investigate the changes in marriage age gap and other patterns over time.

6.3 Relationships between Marriage Age and Personal Income

There are systematic relationships between age at first marriage and personal income earned later in life. The relationship for males has been persistently inverse-U shaped around the world: those who marry earlier and later earn significantly less than those who marry around a median age (Bergstrom and Schoeni, 1996). On the other hand, the correlation between marriage age and personal income for females has been positive: women who marry later tend to earn more, and the unmarried earn the most (Keeley, 1979). However, recently in the United States, the marriage age - personal income relationship for females tends towards inverse-U shaped. Although the overall positive correlation continues to hold, the women who marry in their late thirties or will not marry at all, the group of people who previously earned significantly more, now earn less than those who marry earlier. Figures 4-11 illustrate these relationships for 40-44-year-old Americans over the last fifty years (1960, 1980, 2008, 2012). Figures 12 and 13 illustrate the relationships for 40-44-year-old Canadians in 1981 and Brazilians in 1991, respectively.

The model captures these salient patterns. The marriage age - personal income relationship for males is persistently inverse-U shaped. All of those who marry at age 1 earn low wage $\mathbb{E}w^*_{m1} = \underline{w}_m$, and all of those who marry at age 2 earn high wage $\mathbb{E}w^*_{m2} = \overline{w}_m$. Those who marry at age 3 have successfully or unsuccessfully realized a good outcome in the labor market. Their average wage is

$$\mathbb{E}w^*_{m3} = \frac{\int_{\theta^*_m}^{1}(1-\theta)[\theta\overline{w}_m + (1-\theta)\underline{w}_m]dG_m(\theta)}{\int_{\theta^*_m}^{1}(1-\theta)dG_m(\theta)},$$

strictly between $\underline{w}_m$ and $\overline{w}_m$. If men are in excess supply, the lowest earners do not marry. Therefore, $\mathbb{E}w^*_{m\infty} \leq \mathbb{E}w_{m1} < \mathbb{E}w^*_{m3} < \mathbb{E}w^*_{m2}$ and the relationship is inverse-U shaped.

Proposition 11 (Males' Marriage Age - Personal Income Relationship)**.** *Males' marriage age - personal income relationship is persistently inverse-U shaped.*

The upward sloping portion of the relationship comes from the stable reassignment effect. The downward sloping portion of the curve from age 2 to age 3 formalizes two effects casually remarked in Bergstrom and Schoeni (1996): *"Some of these men who marry very late in life or not at all may be persons whose successes in life have not met the expectations that led them to postpone marriage and who continue to postpone marriage until their true worth is recognized. There may also be a considerable number of males who are such poor marriage material, that any female whom they would wish to marry would prefer being single to marrying one of these males."*

On the other hand, the females could have positive or inverse-U shaped equilibrium relationships. Only low ability women marry at age 1 and earn a low wage of $\mathbb{E}w^*_{f1} = \underline{w}_f$. Those who marry at age 2 consist of all the females with ability θ_f between θ^*_{f1} and θ^*_{f2}, and the high wage females with ability $\theta_f \geq \theta^*_{f2}$,

$$\mathbb{E}w^*_{f2} = \frac{\underline{w}_f \int_{\theta^*_{f1}}^{\theta^*_{f2}} (1-\theta)dG_f(\theta) + \overline{w}_f \int_{\theta^*_{f1}}^{1} \theta dG_f(\theta)}{\int_{\theta^*_{f1}}^{\theta^*_{f2}} (1-\theta)dG_f(\theta) + \int_{\theta^*_{f1}}^{1} \theta dG_f(\theta)}.$$

The women who marry at age 3 have failed the second period's labor market, but they have very high ability and are very likely to achieve a high income from their second investment,

$$\mathbb{E}w^*_{f3} = \frac{\int_{\theta^*_{f2}}^{1} (1-\theta)[\theta\overline{w}_f + (1-\theta)\underline{w}_f]dG_f(\theta)}{\int_{\theta^*_{f2}}^{1} (1-\theta)dG_f(\theta)}.$$

Whether $\mathbb{E}w^*_{f2}$ or $\mathbb{E}w^*_{f3}$ is bigger depends on the relationship between ϕ_2 and ϕ_3. If ϕ_2 and ϕ_3 are relatively close, θ^*_{f1} is relatively close to θ^*_{f2}. Among the women who marry at age, not many will earn low wage; to be exact mass $\int_{\theta^*_{f1}}^{\theta^*_{f2}} (1-\theta)dG_f(\theta)$ is smaller relative to $\int_{\theta^*_{f1}}^{1} \theta dG_f(\theta)$, so the average age 2 wage tends to $\overline{w}_f$. Take the extreme grand gender convergence case. When females do not face any reproductive disadvantage ($\phi_2 = \phi_3 = 1$), $\theta^*_{f1} = \theta^*_{f2}$, males and females have the same optimal strategies and they should exhibit the same qualitative marriage age-personal income relationship. On the other hand, when the probabilities of desirability decline sharply, the relationship may be different. Those who marry at age 2 tend to be low wage earners. When θ^*_{f2} increases, only very high ability unlucky women invest into the third period, resulting in an average wage close to $\overline{w}_f$.

Proposition 12 (Females' Marriage Age - Personal Income Relationship)**.** *Females' marriage age*

- personal income relationship is positive when the fitness loss is significant ($1 - \phi_2 << \phi_2 - \phi_3$)*, and is inverse-U shaped when the fitness loss is not significant* ($1 - \phi_2 \approx \phi_2 - \phi_3$)*.*

Take the United States. The total fertility rate sharply declined from early 1960s to early 1980s. Although it has been steady around 2, the change in the ages of the moths has been dramatic. Fertility rates among ages groups of 30-34, 35-39, 40-44 grew exponentially, while fertility rates among the younger groups all dropped, supporting the evidence that fertility constraint is becoming less binding. In terms of the parameters of the model, we can take $\phi_2 = 1$ and $\phi_3 < 1$, but ϕ_3 has been increasing in the recent years. Figures 14, 15, and 16 numerically illustrates the persistent male relationship and the changing female relationships as ϕ_3 increases.

The two prominent theories of Becker-Keeley and Bergstrom-Bagnoli highlight crucial marriage - delaying factors but incompletely predict marriage age-personal income relationships. They do not fully account for these observed gender-specific, non-monotonic marriage age-personal income relationships or any changes over time. Both theories take the premise that women specialize in housework and men specialize in market work. The Becker-Keeley theory predicts a negative correlation between men's personal income and age at first marriage and a positive correlation between women's, because higher wage men and lower wage women benefit more from marriage and household specialization. The Bergstrom-Bagnoli theory predicts that marriage age and personal income are positively correlated for men and uncorrelated for women, because men participate in the labor market that reveals their wage earning abilities relatively late while women's marriage characteristics are revealed earlier. The frameworks are too restrictive to extend. Keeley (1977) considers a partial equilibrium marriage search market. Bergstrom and Bagnoli (1993) rely on gender-asymmetric information revelation process and cannot explain any marriage age variation, or correlation between personal income and marriage age for females because they all marry at the same earliest time.

Our model is consistent with more intricate empirical observations related to the relationship between marriage age and personal income. Controlling for education, Keeley (1979) finds a negative marriage age-personal income relationship for men and a negative one for women. Given that agents make the initial investment, those men who marry at age 2 earn more on average than those who marry at age 3. When the fertility decline is serious, women who marry early earn less. Zhang (1995) finds that the marriage age-personal income relationship is negative among the men who have non-working wives and positive among those with working wives. By positive assortative matching, those with working wives tend to be highly educated men who work themselves, and among them, the high earners tend to marry later as their continued investment delays marriage.

7 Concluding Remarks

This paper considers the interactions of premarital investments and the subsequent marriage market. Namely, college investment and career reinvestment can improve people's wage earnings and consequently their prospects in the marriage market, but the investments delay marriage and lower women's reproductive capital. I incorporate into the previous gender-symmetric investment-and-matching frameworks repeated investments, stochastic investment returns, differential fecundity, and associated intertemporal considerations.

These extensions prove fruitful to shed new lights on important patterns in higher education, work and marriage age. Notably, I justify the puzzle that more women than men would make college investments even when their reproductive disadvantage tampers with their investment incentives. Furthermore, the model unifies and extends the theories on marriage timing. In particular, it explains the relationships between marriage age and personal income. Finally, all of the gender differences in college education, in wage earnings, and in social roles derive from an unarguable biological difference, suggesting the key role differential fecundity plays in the society.

The model is very parsimonious. The parsimony has so far demonstrated simple economic channels clearly, but it possesses limitations. Fertility decisions are not explicitly modeled. The relative importance of reproductive capital in household production is exogenously specified by the marriage surplus function. An important change mentioned is the sharp decline of total fertility rate, so perhaps it is more important to understand the change with respect to fertility decision more intricately. Furthermore, cohabitation and divorce, two increasingly common phenomena, are not incorporated into the model. These possibilities also influence people's investment decisions.

Nonetheless, I hope that I have demonstrated the analytical tractability and economic usefulness of the investment-and-matching framework. I believe that this class of models can play an instrumental role in future explorations of more issues related to investments for the marriage market as well as other matching markets such as the labor market. The model is ready for structural estimations and calibration exercises to quantify relevant impacts and to deliver policy suggestions.

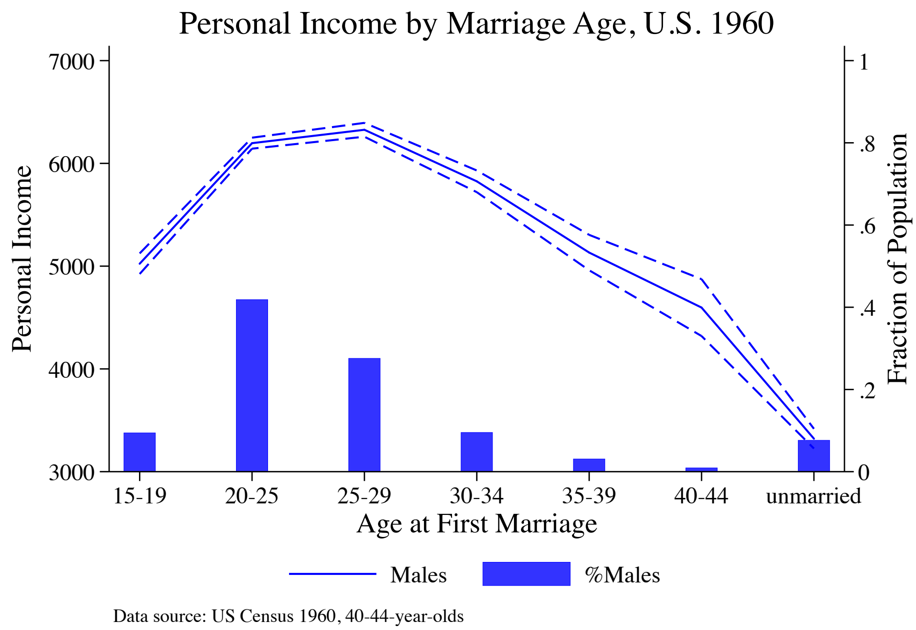

Figure 4: Marriage age distributions and marriage age-personal income relationships, 40-44-year-old male Americans in 1960.

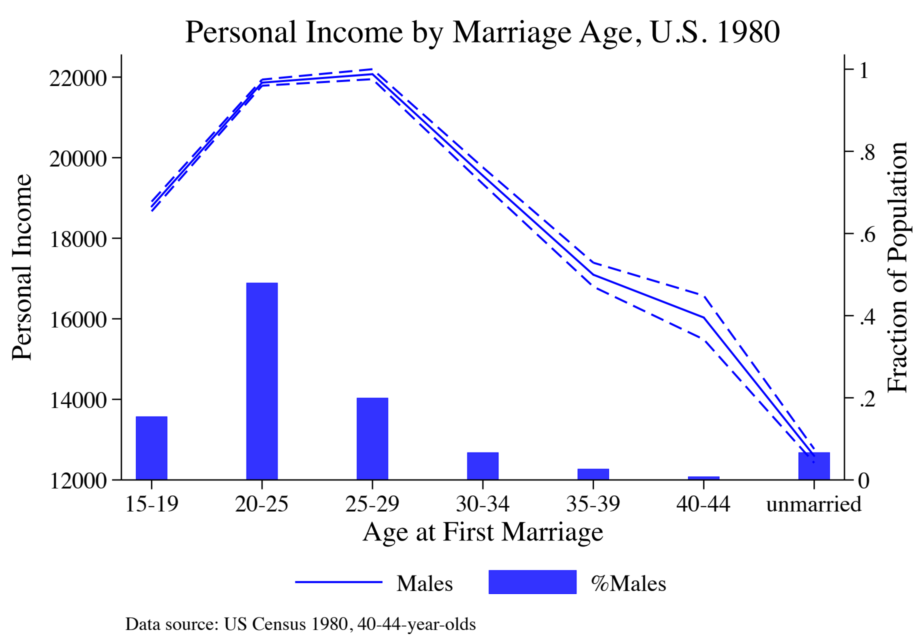

Figure 5: Marriage age distributions and marriage age-personal income relationships, 40-44-year-old male Americans in 1980.

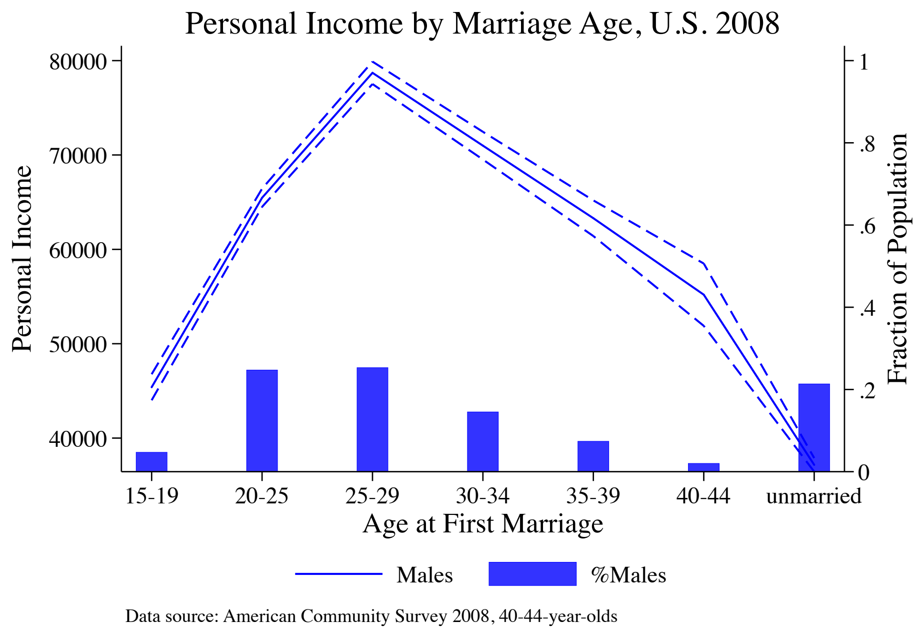

Figure 6: Marriage age distributions and marriage age-personal income relationships, 40-44-year-old male Americans in 2008.

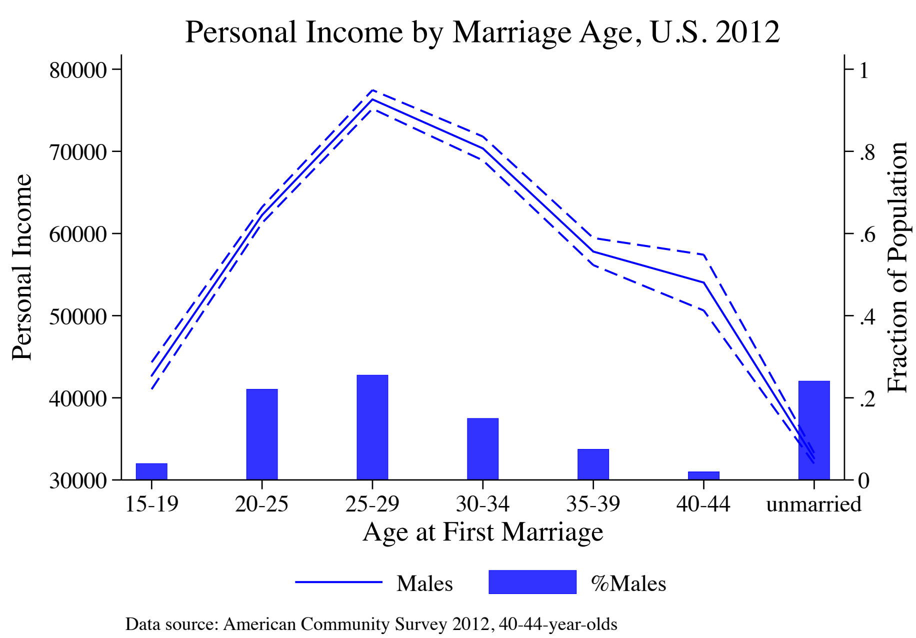

Figure 7: Marriage age distributions and marriage age-personal income relationships, 40-44-year-old male Americans in 2012.

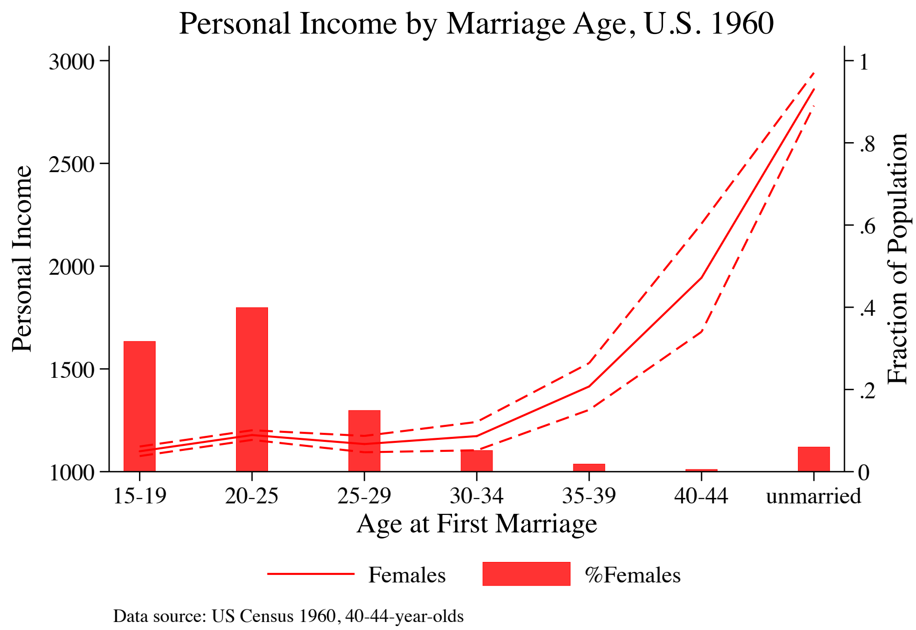

Figure 8: Marriage age distributions and marriage age-personal income relationships, 40-44-year-old female Americans in 1960.

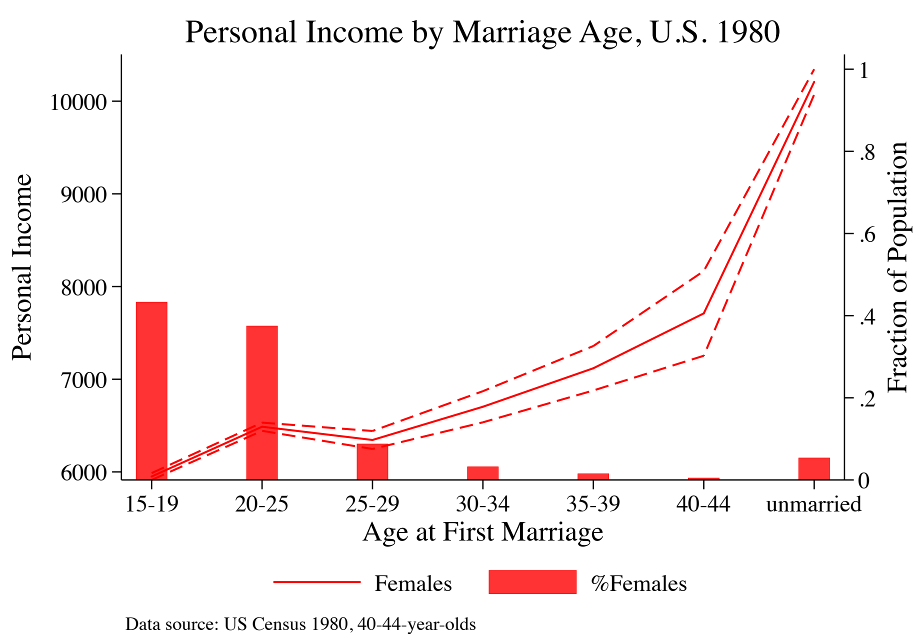

Figure 9: Marriage age distributions and marriage age-personal income relationships, 40-44-year-old female Americans in 1980.

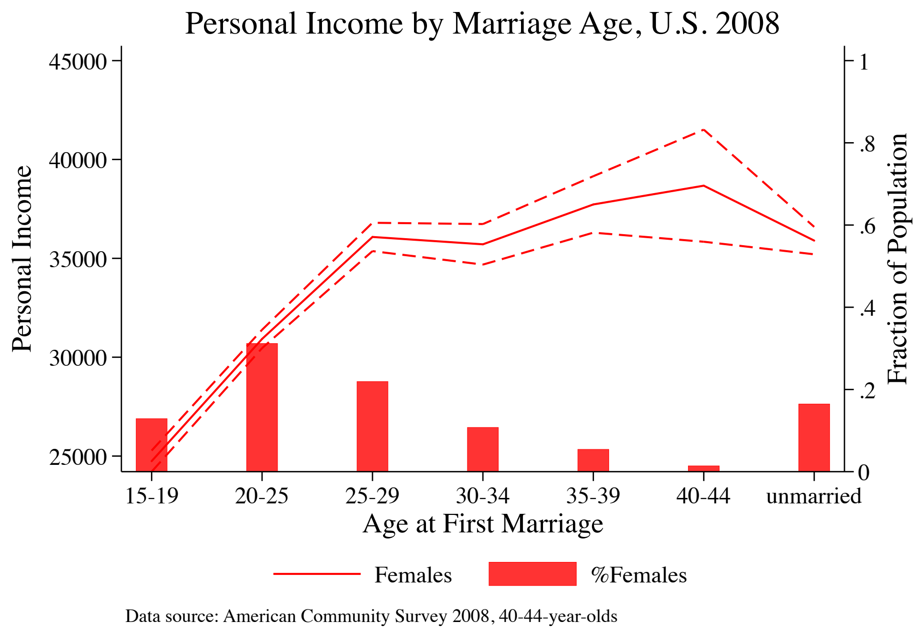

Figure 10: Marriage age distributions and marriage age-personal income relationships, 40-44-year-old female Americans in 2008.

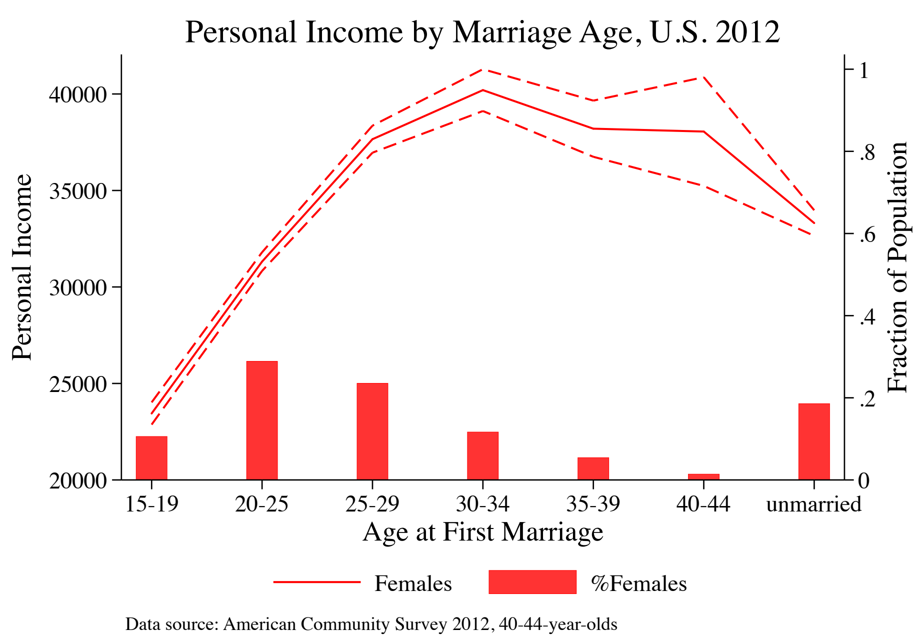

Figure 11: Marriage age distributions and marriage age-personal income relationships, 40-44-year-old female Americans in 2012.

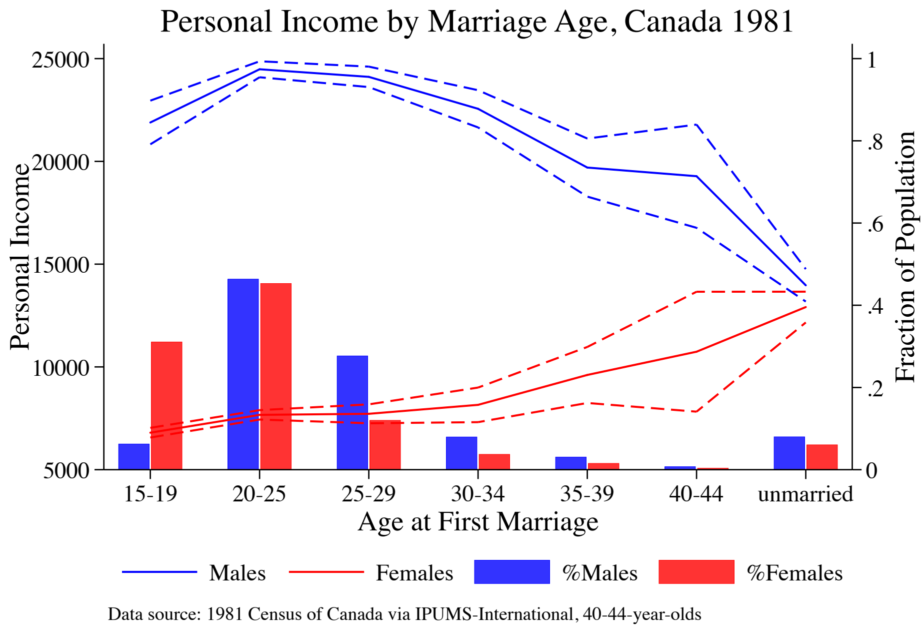

Figure 12: Marriage age distributions and marriage age-personal income relationships, 40-44-year-old Canadians in 1981.

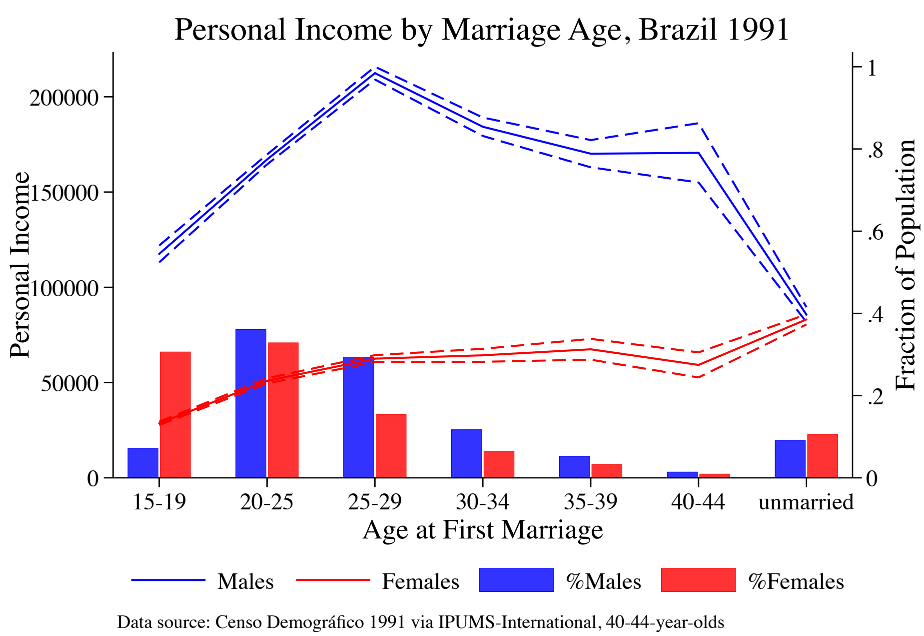

Figure 13: Marriage age distributions and marriage age-personal income relationships, 40-44-year-old Brazilians in 1991.

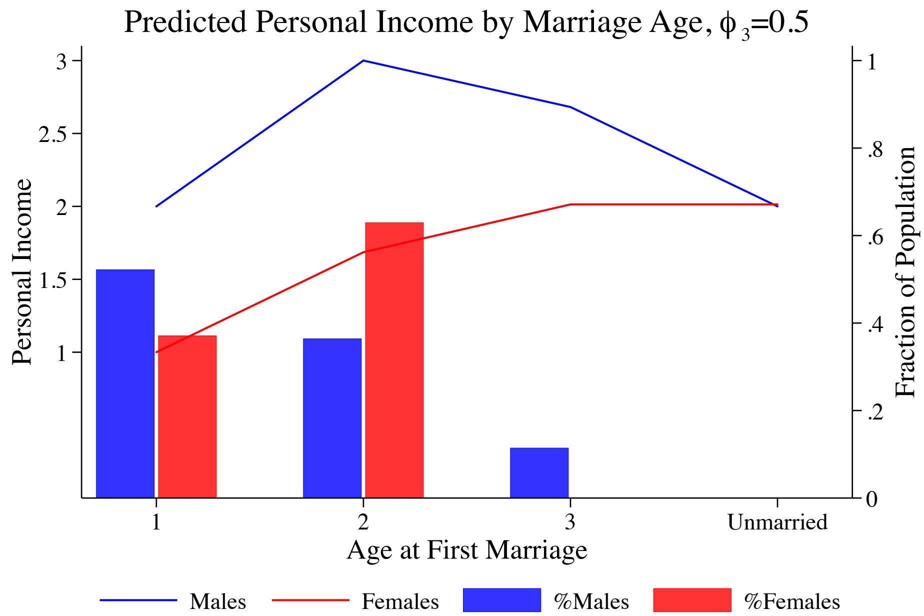

Figure 14: Predicted marriage age-personal income relationships when $\phi_3 = 0.5$. ($\phi_2 = 1$, $G_m = G_f = U[0,1]$, $c_m = c_f = 1.3$, $\overline{w}_m = 3$, $\underline{w}_m = 2$, $\overline{w}_f = 2$, $\underline{w}_f = 1$, $S(w_m, w_f, \overline{r}) = w_m w_f$).

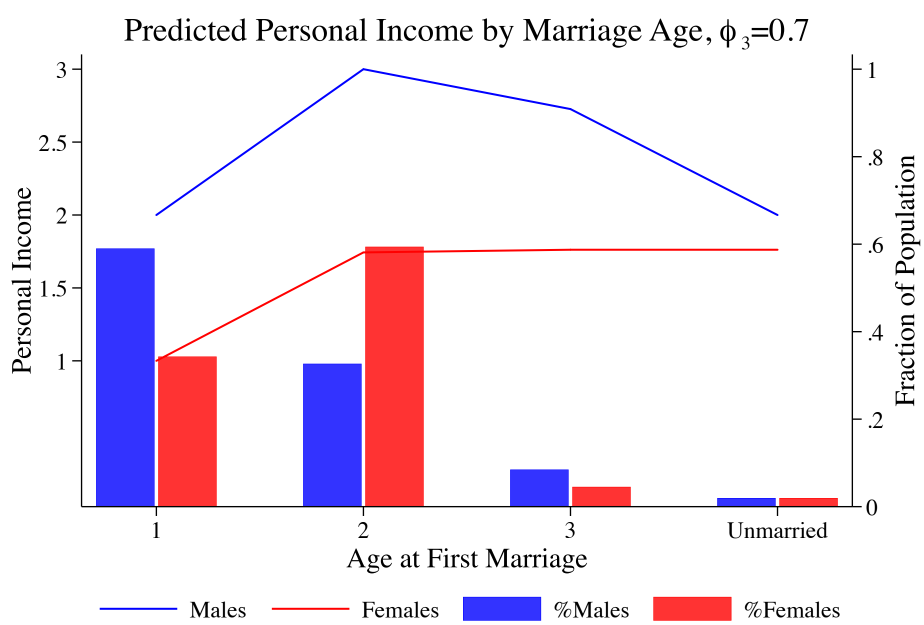

Figure 15: Predicted marriage age-personal income relationships when $\phi_3 = 0.7$ ($\phi_2 = 1$, $G_m = G_f = U[0,1]$, $c_m = c_f = 1.3$, $\overline{w}_m = 3$, $\underline{w}_m = 2$, $\overline{w}_f = 2$, $\underline{w}_f = 1$, $S(w_m, w_f, \overline{r}) = w_m w_f$).

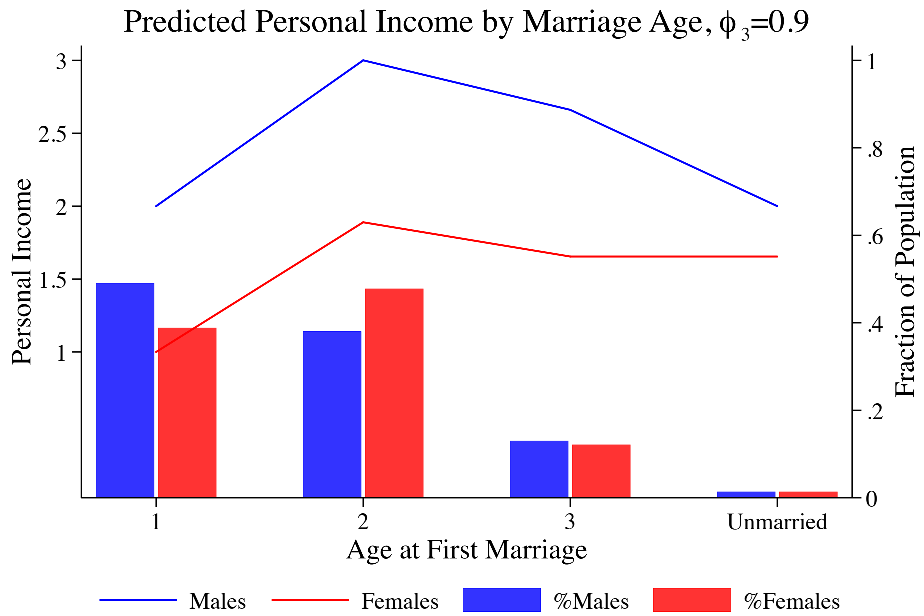

Figure 16: Predicted marriage age-personal income relationships when $\phi_3 = 0.9$. ($\phi_2 = 1$, $G_m = G_f = U[0, 1]$, $c_m = c_f = 1.3$, $\overline{w}_m = 3$, $\underline{w}_m = 2$, $\overline{w}_f = 2$, $\underline{w}_f = 1$, $S(w_m, w_f, \bar{r}) = w_m w_f$).

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Appendix

A Proofs

A.1 Proof of Lemma 1 (Stable Marriage Market Outcome)

First, I prove positive assortative matching of men and fit women by contradiction. Suppose men and fit women are not positively assortatively matched in the stable outcome. There are several ways they are not positively assortatively matched. First, suppose that some positive masses of men and fit women are unmatched. They get zero payoff $U(w_m) = V(w_f, \bar{r}) = 0$. However, the payoffs cannot be stable, because $U(w_m) + V(w_f, \bar{r}) = 0 < S(w_m, w_f, \bar{r})$, violating the stability condition. Second, similarly, if a positive mass of men is matched with unfit women while a positive mass of fit women is unmatched, the match cannot be stable. $U(w_m) = V(w_f, \bar{r}) = 0 < S(w_m, w_f, \bar{r})$. Finally, suppose there are a positive mass ω of matches between $\overline{w}_m$ and $(\underline{w}_f, \bar{r})$ and a positive mass ω' of matches between $\underline{w}_m$ and $(\overline{w}_f, \bar{r})$. $\overline{U} + \underline{V} = S_{hl}$ and $\underline{U} + \overline{V} = S_{lh}$. By stability, it must be true that $\overline{U} + \overline{V} \geq S_{hh}$ and $\underline{U} + \underline{V} \geq S_{ll}$. However, $\overline{U} + \underline{U} + \overline{V} + \underline{V} = S_{hl} + S_{lh} < S_{hh} + S_{ll}$ by strict supermodularity of $S(\cdot, \cdot, \bar{r})$. In summary, since stable outcome exists, men and fit women must be positively assortatively matched.

Next, I show that U and V are increasing in wages. Take a type w_m man. A positive mass of w_m men can be unmatched. If it is the case, then the stable marriage payoff of an unmarried man is $U(w_m) = 0$. Since U is non-negative, $U(w'_m) \geq U(w_m)$ for all $w'_m > w_m$. Suppose otherwise that measure zero of w_m man is unmatched. There is a positive mass of matched between w_m and say type (w_f, r) women, and they share their surplus $U(w_m) + V(w_f, r) = S(w_m, w_f, r)$. By stability, for any w'_m, $U(w'_m) + V(w_f) \geq S(w'_m, w_f)$. For $w'_m > w_m$, $U(w'_m) - U(w_m) \geq S(w'_m, w_f, r) - S(w_m, w_f, r) > 0$ because surplus function S is increasing in w_m. Monotonicity of the females' stable marriage payoff function in both arguments can be similarly proven. Unfit women produce zero surplus, so their marriage payoff is zero. By individual rationality, fit women get non-negative marriage payoffs. Therefore, women's marriage payoff increases in reproductive fitness. ■

A.2 Proof of Lemma 2 (Males' Optimal Strategies)

Given (π, U, V), a θ_m male invests in college if and only if

$$\underline{w}_m + \underline{U} < \theta_m(\overline{w}_m + \overline{U}) - c_m + (1 - \theta_m) \max\{\underline{w}_m + \underline{U}, \theta_m(\overline{w}_m + \overline{U}) + (1 - \theta_m)(\underline{w}_m + \underline{U}) - c_m\}.$$

The conditions reduces to

$$0 < \theta_m(\Delta w_m + \Delta U) - c_m + (1-\theta_m)\max\{0, \theta_m(\Delta w_m + \Delta U) - c_m\}.$$

A θ_m male rejects age 2 low wage offer if and only if

$$\underline{w}_m + \underline{U} < \theta_m(\overline{w}_m + \overline{U}) + (1-\theta_m)(\underline{w}_m + \underline{U}) - c_m,$$

or

$$0 < \theta_m(\Delta w_m + \Delta U) - c_m.$$

Therefore, a $\theta_m^* = c_m/(\Delta w_m + \Delta U)$ male is indifferent between investing and not investing at age 1, and accepting and rejecting a low wage offer at age 2. ■

A.3 Proof of Lemma 3 (Females' Optimal Strategies)

Given (π, U, V), a θ_f female invests in college if and only if

$$\underline{w}_f + \underline{V} < \theta_f(\overline{w}_f + \phi_2\overline{V}) - c_f + (1-\theta_f)\max\{\underline{w}_f + \phi_2\underline{V}, \theta_f(\overline{w}_f + \phi_3\overline{V}) + (1-\theta_f)(\underline{w}_f + \phi_3\underline{V}) - c_f\}.$$

The condition reduces to

$$0 < \theta_f(\Delta w_f + \phi_2\Delta V) - c_f - (1-\phi_2)\underline{V} + (1-\theta_f)\max\{0, \theta_f(\Delta w_f + \phi_3\Delta V) - c_f - (\phi_2 - \phi_3)\underline{V}\}.$$

A θ_f female rejects age 2 low wage offer if and only if

$$\underline{w}_f + \phi_2\underline{V} < \theta_f(\overline{w}_f + \phi_3\overline{V}) + (1-\theta_f)(\underline{w}_f + \phi_3\underline{V}) - c_f,$$

or equivalently,

$$0 < \theta_f(\Delta w_f + \phi_3\Delta V) - c_f - (\phi_2 - \phi_3)\underline{V}.$$

Therefore, $\theta_{f2}^* = [c_f + (\phi_2 - \phi_3)\underline{V}]/(\Delta w_f + \phi_3\Delta V)$. Since $\phi_2 \geq \phi_3$ and $1 - \phi_2 \leq \phi_2 - \phi_3$,

$$\theta_f[\Delta w_f + \phi_2\Delta V] - c_f - (1-\phi_2)\underline{V} \geq \theta_f[\Delta w_f + \phi_3\Delta V] - c_f - (\phi_2 - \phi_3)\underline{V}.$$

The age 1 cutoff ability is $\theta_{f1}^* = [c_f + (1-\phi_2)\underline{V}]/(\Delta w_f + \phi_2\Delta V)$. ■

A.4 Proof of Theorem 1 (Equilibrium Uniqueness)

First, I apply Kakutani's fixed point theorem to prove equilibrium existence.

Let Φ_θ be the mapping from vectors of marriage payoffs $(\overline{U}, \underline{U}, \overline{V}, \underline{V})$ to vectors of optimal strategy cutoffs $(\theta^*_m, \theta^*_{f1}, \theta^*_{f2})$ as specified in (1)-(3). Let Φ_M be the mapping from vectors of optimal strategy cutoffs to marriage market masses $(\underline{\mu}, \overline{\mu}, \underline{\nu}, \overline{\nu})$ as specified in (4)-(7). Let $\mathcal{B}$ be the mapping from the marriage market masses to sets of stable marriage payoffs. By Gretsky et al. (1992, Theorem 7), $\mathcal{B}$ is a correspondence and is upper-hemicontinuous. Since the functions Φ_θ and Φ_M are continuous and the correspondence $\mathcal{B}$ is upper-hemicontinuous, the composite mapping $\mathcal{B} \circ \Phi_M \circ \Phi_\theta$ is upper-hemicontinuous. The set of feasible marriage payoffs is compact and convex. Therefore, by Kakutani's fixed point theorem, there exists a fixed point, i.e. a vector of stable marriage payoffs $(\overline{U}^*, \underline{U}^*, \overline{V}^*, \underline{V}^*)$ that satisfies $(\overline{U}, \underline{U}, \overline{V}, \underline{V}) \in \mathcal{B}(\Phi_M(\Phi_\theta(\overline{U}, \underline{U}, \overline{V}, \underline{V})))$. The vector and $\underline{v}^* = \overline{v}^* = 0$ specify equilibrium marriage payoffs. The equilibrium strategies and equilibrium marriage market masses are derived from (1)-(7). The equilibrium matching function π^* is derived from the equilibrium masses.

Next, I prove equilibrium uniqueness. I do so by completely characterizing an equilibrium and showing that it is the unique equilibrium.

Positive masses of men and women invest by assumption. Positive masses of them achieve high wages and positive masses of them become unfit. Therefore, there are positive masses of every possible marriage type in the marriage market. Furthermore, since it is assumed that even agents who generate zero surplus are matched, in each period there are mass 1 of males and mass 1 of females in the marriage market. It has been assumed that $\overline{\mu}^* < \overline{\nu}^* + \underline{\nu}^*$. There are three possible equilibrium marriage market distributions: (1) $\overline{\mu}^* < \overline{\nu}^*$, (2) $\overline{\mu}^* = \overline{\nu}^*$, (3) $\overline{\mu}^* > \overline{\nu}^*$, as shown in Figure 17.

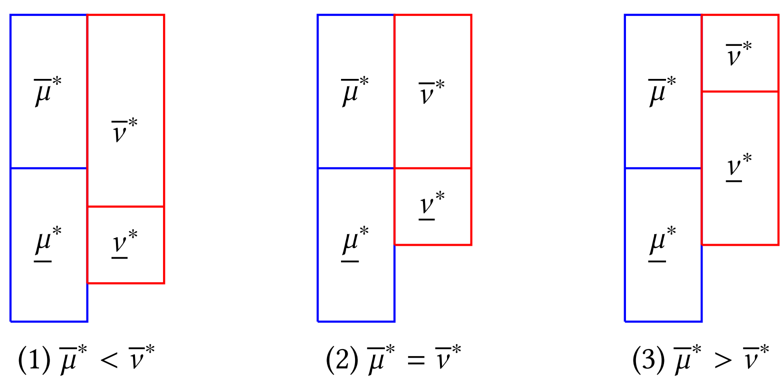

Figure 17: Possible equilibrium marriage market distributions.

In each case, a positive mass of unfit women and low wage men is matched. Since unfit women produce zero surplus, $\overline{v}^* = \underline{v}^* = \underline{U}^* = 0$ in every equilibrium. Furthermore, there is a positive mass of matches between $\underline{w}_m$ and $(\underline{w}_f, \overline{r})$, so $\underline{U}^* + \underline{V}^* = S_{ll}$. $\underline{V}^*$ is uniquely determined to be $\underline{V}^* = S_{ll}$. The equilibrium payoffs $\overline{U}^*$ and $\overline{V}^*$ depend on the equilibrium distributions.

Since a positive mass of high wage men and high wage fit women is always matched in equilibrium, $\overline{U}^* + \overline{V}^* = S_{hh}$. Coupled with $\underline{U}^* + \underline{V}^* = S_{ll}$,

$$\Delta U^* + \Delta V^* = (\overline{U}^* + \overline{V}^*) - (\underline{U}^* + \underline{V}^*) = S_{hh} - S_{ll}.$$

Define

$$\Delta V(k) \equiv k(S_{lh} - S_{ll}) + (1-k)(S_{hh} - S_{hl}) \tag{10}$$

and

$$\Delta U(k) \equiv k(S_{hh} - S_{lh}) + (1-k)(S_{hl} - S_{ll}) \tag{11}$$

so that $\Delta U(k) + \Delta V(k) = \Delta U^* + \Delta V^*$. $\Delta U(k)$ is increasing in k and $\Delta V(k)$ is decreasing in k. A k characterizes equilibrium marriage payoffs. In distribution (1) $\overline{\mu}^* < \overline{\nu}^*$, $k^* = 1$. In distribution (3) $\overline{\mu}^* > \overline{\nu}^*$, $k^* = 0$. In distribution (2) $\overline{\mu}^* = \overline{\nu}^*$, payoffs with any $k^* \in [0,1]$ can be stable.

Define the optimal cutoffs when the agents face the marriage payoffs characterized by k,

$$\begin{aligned}
\theta_m^*(k) &\equiv c_m/[\Delta w_m + \Delta U(k)],\\
\theta_{f1}^*(k) &\equiv [c_f + (1-\phi_2)S_{ll}]/[\Delta w_f + \phi_2 \Delta V(k)],\\
\theta_{f2}^*(k) &\equiv [c_f + (\phi_2 - \phi_3)S_{ll}]/[\Delta w_f + \phi_3 \Delta V(k)].
\end{aligned}$$

$\theta_m^*(k)$ is decreasing in k, and $\theta_{f1}^*(k)$ and $\theta_{f2}^*(k)$ are increasing in k.

Define the induced marriage market distributions $\overline{\mu}(k)$, $\overline{\nu}(k)$, and $\underline{\nu}(k)$ when the strategies are determined by $\theta_m^*(k)$, $\theta_{f1}^*(k)$, and $\theta_{f2}^*(k)$,

$$\begin{aligned}
\overline{\mu}(k) &\equiv \int_{\theta_m^*(k)}^1 \theta(2-\theta)dG_m(\theta),\\
\overline{\nu}(k) &\equiv \phi_2 \int_{\theta_{f\,1}^*(k)}^{\theta_{f\,2}^*(k)} \theta dG_f(\theta) + \phi_3 \int_{\theta_{f\,2}^*(k)}^1 (2-\theta)\theta dG_f(\theta),\\
\underline{\nu}(k) &\equiv G_f(\theta_{f1}^*(k)) + \phi_2 \int_{\theta_{f\,1}^*(k)}^{\theta_{f\,2}^*(k)} (1-\theta)dG_f(\theta) + \phi_3 \int_{\theta_{f\,2}^*(k)}^1 (1-\theta)^2 dG_f(\theta).
\end{aligned}$$

$\overline{\mu}(k,\gamma)$ is strictly increasing in k and strictly decreasing in γ when $\overline{\mu}(k) > 0$, $\overline{\nu}(k)$ is strictly decreasing in k when $\overline{\nu}(k) > 0$, and $\underline{\nu}(k,\gamma)$ is strictly increasing in k when $\underline{\nu}(k) > 0$.

An equilibrium can be succinctly represented by a pair k^*. The equilibrium marriage payoffs are $\overline{U}^* = \overline{U}(k^*)$, $\overline{V}^* = S_{ll} + \Delta V(k^*)$, $\underline{V}^* = S_{ll}$, and $\underline{U}^* = \overline{\upsilon}^* = \underline{\upsilon}^* = 0$. The equilibrium cutoffs are $\theta_m^* = \theta_m(k^*)$, $\theta_{f1}^* = \theta_{f1}(k^*)$, and $\theta_{f2}^* = \theta_{f2}^*(k^*)$. The equilibrium induced marriage market distributions are $\overline{\mu}^* = \overline{\mu}(k^*)$, $\overline{\nu}^* = \overline{\nu}(k^*)$, and $\underline{\nu}^* = \underline{\nu}(k^*)$.

Define $\overline{\delta}(k) \equiv \overline{\mu}(k) - \overline{\nu}(k)$

$$= \int_{\theta_m^*(k)}^1 \theta(2-\theta)dG_m(\theta) - \phi_2 \int_{\theta_{f\,1}^*(k)}^1 \theta dG_f(\theta) - \phi_3 \int_{\theta_{f\,2}^*(k)}^1 (1-\theta)\theta dG_f(\theta).$$

$\overline{\delta}(k)$ is increasing strictly in k. There are three cases: (1) $\overline{\delta}(0) < \overline{\delta}(1) < 0$, (2) $\overline{\delta}(0) < 0 < \overline{\delta}(1)$, and (3) $0 < \overline{\delta}(0) < \overline{\delta}(1)$.

Case 1: $\overline{\delta}(0) < \overline{\delta}(1) < 0$.
In other words, $\overline{\mu}(k) < \overline{\nu}(k)$ for all $k \in [0,1]$. Even if the male marriage gain ΔU is at maximum possible and the female marriage gain ΔV is at minimum possible, the mass of high wage fit women still exceeds that of high wage men. The equilibrium marriage market distributions must satisfy $\overline{\mu}^* < \overline{\nu}^*$, as demonstrated by (1) in Figure 17.
The equilibrium is characterized by $k^* = 1$.
The equilibrium payoffs are $\overline{U}^* = S_{hh} - S_{lh}$, $\underline{V}^* = S_{ll}$, $\overline{V}^* = S_{lh}$, and $\underline{U}^* = \overline{v}^* = \underline{v}^* = 0$.
The equilibrium strategies are characterized by

$$\begin{aligned} \theta_m^* &= c_m/[\Delta w_m + S_{hh} - S_{lh}] \\ \theta_{f1}^* &= [c_f + (1-\phi_2)S_{ll}]/[\Delta w_f + \phi_2(S_{lh} - S_{ll})] \\ \theta_{f2}^* &= [c_f + (\phi_2 - \phi_3)S_{ll}]/[\Delta w_f + \phi_3(S_{lh} - S_{ll})]. \end{aligned}$$

Case 2: $\overline{\delta}(0) < 0 < \overline{\delta}(1)$
In other words, $\overline{\mu}(0) < \overline{\nu}(0)$ and $\overline{\mu}(1) > \overline{\nu}(1)$. No equilibrium with $\overline{\mu}^* < \overline{\nu}^*$ or $\overline{\mu}^* > \overline{\nu}^*$ can be supported. The equilibrium marriage market distributions must satisfy $\overline{\mu}^* = \overline{\nu}^*$, as demonstrated by (2) in Figure 17. The equilibrium is characterized by the unique k^* where $\overline{\delta}(k^*) = 0$. The solution is unique because $\overline{\delta}(k)$ is strictly increasing.
The equilibrium payoffs are $\overline{U}^* = k^*(S_{hh} - S_{lh}) + (1-k^*)(S_{hl} - S_{ll})$, $\underline{V}^* = S_{ll}$, $\overline{V}^* = k^* S_{lh} + (1 - k^*)(S_{hh} - S_{hl} + S_{ll})$, and $\underline{U}^* = \overline{v}^* = \underline{v}^* = 0$.
The equilibrium strategies are characterized by

$$\begin{aligned} \theta_m^* &= c_m/[\Delta w_m + k^*(S_{hh} - S_{lh}) + (1-k^*)(S_{hl} - S_{ll})] \\ \theta_{f1}^* &= [c_f + (1-\phi_2)S_{ll}]/[\Delta w_f + \phi_2(k^*(S_{lh} - S_{ll}) + (1-k^*)(S_{hh} - S_{hl}))] \\ \theta_{f2}^* &= [c_f + (\phi_2 - \phi_3)S_{ll}]/[\Delta w_f + \phi_3(k^*(S_{lh} - S_{ll}) + (1-k^*)(S_{hh} - S_{hl}))]. \end{aligned}$$

Case 3: $0 < \overline{\delta}(0) < \overline{\delta}(1)$.
$\overline{\mu}(k) > \overline{\nu}(k)$ for all $k \in [0,1]$. Even if the male marriage gain ΔU is at minimum possible and the female marriage gain ΔV is at maximum possible, the mass of high wage men still exceeds that of high wage fit women. The equilibrium marriage market distributions must satisfy $\overline{\nu}^* < \overline{\mu}^* < \overline{\nu}^* + \underline{\nu}^*$, as demonstrated (3) in Figure 17.
The equilibrium is characterized by $k^* = 0$.
The equilibrium payoffs are $\overline{U}^* = S_{hl} - S_{ll}$, $\underline{V}^* = S_{ll}$, and $\overline{V}^* = S_{hh} - S_{hl} + S_{ll}$, and $\underline{U}^* = \overline{v}^* = \underline{v}^* = 0$.
The equilibrium strategies are characterized by

$$\begin{aligned}
\theta_m^* &= c_m/[\Delta w_m + S_{hl} - S_{ll}] \\
\theta_{f1}^* &= [c_f + (1-\phi_2)S_{ll}]/[\Delta w_f + \phi_2(S_{hh} - S_{hl})] \\
\theta_{f2}^* &= [c_f + (\phi_2 - \phi_3)S_{ll}]/[\Delta w_f + \phi_3(S_{hh} - S_{hl})].
\end{aligned}$$

Therefore, the equilibrium is always uniquely determined. ■

A.5 Proof of Theorem 2 (Equilibrium Efficiency)

I characterize the socially efficient investments and show that they always coincide with the equilibrium investments.

The socially efficient investments and sorting yields the maximum total social welfare $\Sigma \equiv \sum S + \sum w_m + \sum w_f - \sum c_m - \sum c_f$, the total marriage surplus plus wages net any investment costs. The socially efficient investment strategies are characterized by the cutoffs θ_m^{**}, θ_{f1}^{**}, and θ_{f2}^{**} as the equilibrium investment strategies, since it is always weakly more efficient for the more able agents to invest. The distributions in the marriage market are as specified in (4)-(9). The total marriage surplus $\sum S$ is

$$\begin{aligned}
& 1_{\overline{\mu} \le \overline{\nu}}[\overline{\mu}(S_{hh} - S_{lh}) + \overline{\nu}S_{lh} + \underline{\nu}S_{ll}] \\
+ \quad & 1_{\overline{\nu} < \overline{\mu} \le \overline{\nu} + \underline{\nu}}[\overline{\mu}(S_{hl} - S_{ll}) + \overline{\nu}(S_{hh} - S_{hl} + S_{ll}) + \underline{\nu}S_{ll}] \\
+ \quad & 1_{\overline{\mu} > \overline{\nu} + \underline{\nu}}[\overline{\nu}S_{hh} + \underline{\nu}S_{hl}]
\end{aligned}$$

The total wage of males $\sum w_m$ is

$$\underline{w}_m \int_{\theta_m^{**}}^{1} (1-\theta)^2 dG_m(\theta) + \overline{w}_m \int_{\theta_m^{**}}^{1} \theta(2-\theta) dG_m(\theta),$$

and females' total wage $\sum w_f$ is

$$
\begin{aligned}
& \underline{w}_f \left[G_f(\theta^{**}_{f1}) + \int_{\theta^{**}_{f1}}^{\theta^{**}_{f2}} (1-\theta) dG_f(\theta) + \int_{\theta^{**}_{f2}}^{1} (1-\theta)^2 dG_f(\theta) \right] \\
+ \ & \overline{w}_f \left[\int_{\theta^{**}_{f1}}^{1} \theta dG_f(\theta) + \int_{\theta^{**}_{f2}}^{1} (1-\theta)\theta dG_f(\theta) \right]
\end{aligned}
$$

The total investment cost of males $\sum c_m$ is

$$c_m \int_{\theta^{**}_m}^{1} (2-\theta) dG_m(\theta)$$

and the total investment cost of females $\sum c_f$ is

$$c_f \left[G_f(\theta^{**}_{f2}) - G_f(\theta^{**}_{f1}) + \int_{\theta^{**}_{f2}}^{1} (2-\theta) dG_f(\theta) \right].$$

For any induced marriage market distributions (μ, ν), express $\sum S$ as the sum of males' stable marriage payoffs $\sum U$ and females' stable marriage payoffs $\sum V$ for some stable outcome (π, U, V) in (μ, ν).

The derivatives are

$$\frac{\partial \Sigma}{\partial \theta_m} / (2-\theta_m) g_m(\theta_m) = c_m - \theta_m [\Delta w_m + 1_{\overline{\mu} \leq \overline{\nu}} (S_{hh} - S_{lh}) + 1_{\overline{\nu} < \overline{\mu} \leq \underline{\nu} + \overline{\nu}} (S_{hl} - S_{ll})]$$

$$
\begin{aligned}
\frac{\partial \Sigma}{\partial \theta_{f1}} / g_f(\theta_{f1}) \ &= \ c_f + (1-\phi_2)(1_{\overline{\mu} \leq \underline{\nu} + \overline{\nu}} S_{ll} + 1_{\overline{\mu} > \underline{\nu} + \overline{\nu}} S_{hl}) \\
& \quad - \theta_{f1} [\Delta w_f + \phi_2 [1_{\overline{\mu} \leq \overline{\nu}} (S_{lh} - S_{ll}) + 1_{\overline{\mu} > \overline{\nu}} (S_{hh} - S_{hl})]
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\partial \Sigma}{\partial \theta_{f2}} / (1-\theta_{f2}) g_f(\theta_{f2}) \ &= \ c_f + (\phi_2 - \phi_3)(1_{\overline{\mu} \leq \underline{\nu} + \overline{\nu}} S_{ll} + 1_{\overline{\mu} > \underline{\nu} + \overline{\nu}} S_{hl}) \\
& \quad - \theta_{f2} [\Delta w_f + \phi_3 [1_{\overline{\mu} \leq \overline{\nu}} (S_{lh} - S_{ll}) + 1_{\overline{\mu} > \overline{\nu}} (S_{hh} - S_{hl})].
\end{aligned}
$$

When the primitives of the model fall under Case 1 or 3, θ^*_m, θ^*_{f1}, and θ^*_{f2} satisfy the first order conditions $\partial \Sigma / \partial \theta_m = 0$, $\partial \Sigma / \partial \theta_{f2} = 0$, and $\partial \Sigma / \partial \theta_{f2} = 0$. These derivatives are monotonic, so the derived cutoffs θ^{**}_m, θ^{**}_{f1}, and θ^{**}_{f2} are the unique global maximizers, proving the social efficiency of these investments.

When $\overline{\mu} = \overline{\nu}$, the derivatives $\partial\Sigma/\partial\theta_m$, $\partial\Sigma/\partial\theta_{f2}$, and $\partial\Sigma/\partial\theta_{f2}$ are not necessarily zero simultaneously. However, $\lim_{\theta_m \to \theta_m^{*+}} \partial\Sigma/\partial\theta_m > 0$, $\lim_{\theta_m \to \theta_m^{*-}} \partial\Sigma/\partial\theta_m < 0$; $\lim_{\theta_{f1} \to \theta_{f1}^{*+}} \partial\Sigma/\partial\theta_{f1} > 0$, $\lim_{\theta_{f1} \to \theta_{f1}^{*-}} \partial\Sigma/\partial\theta_{f1} < 0$; $\lim_{\theta_{f2} \to \theta_{f2}^{*+}} \partial\Sigma/\partial\theta_{f2} > 0$, $\lim_{\theta_{f2} \to \theta_{f2}^{*-}} \partial\Sigma/\partial\theta_{f2} < 0$. Since the derivatives are monotonic, θ_m^*, θ_{f1}^*, and θ_{f2}^* still achieve global maximum.

A.6 Proof of Proposition 1 (Changes in College Investments)

Take any relevant variable x.

$$\frac{dG_m^{R_1}}{dx} = \frac{d[1 - G_m(\theta_m^*)]}{dx} = -g_m(\theta_m^*)\frac{d\theta_m^*}{dx},$$

and

$$\frac{dG_f^{R_1}}{dx} = \frac{d[1 - G_f(\theta_{f1}^*)]}{dx} = -g_f(\theta_{f1}^*)\frac{d\theta_{f1}^*}{dx}.$$

Since $g_m, g_f > 0$, the signs of $dG_m^{R_1}/dx$ and $dG_f^{R_1}/dx$ are the same as the signs of $-d\ln\theta_m^*/dx$ and $-d\ln\theta_f^*/dx$, respectively. Therefore, it suffices to derive $d\ln\theta_m^*/dx$ and $d\ln\theta_f^*/dx$, and $dG_m^{R_1}/dx$ and $dG_f^{R_1}/dx$ have the opposite signs. It is supposed that $\overline{\mu}^* > \overline{\nu}^*$, so

$$\ln\theta_m^* = \ln c_m - \ln(\Delta w_m + S_{hl} - S_{ll}),$$

$$\ln\theta_{f1}^* = \ln[c_f + (1-\phi_2)S_{ll}] - \ln[\Delta w_f + \phi_2(S_{hh} - S_{hl})],$$

Let me show the detailed derivations one by one.

WTS: $dG_m^{R_1}/dc_m < 0$**.**

$$\frac{d\ln\theta_m^*}{dc_m} = \frac{1}{c_m} > 0.$$

WTS: $dG_m^{R_1}/d\Delta w_m > 0$**.**

$$\frac{d\ln\theta_m^*}{d\Delta w_m} = -\frac{1}{\Delta w_m + S_{hl} - S_{ll}} < 0.$$

WTS: $dG_m^{R_1}/dS_{hl} > 0$**.**

$$\frac{d\ln\theta_m^*}{dS_{hl}} = -\frac{1}{\Delta w_m + S_{hl} - S_{ll}} < 0.$$

WTS: $dG_m^{R_1}/dS_{ll} < 0$**.**

$$\frac{d\ln\theta_m^*}{dS_{ll}} = \frac{1}{\Delta w_m + S_{hl} - S_{ll}} > 0.$$

WTS: $dG_f^{R_1}/dc_f < 0$**.**

$$\frac{d\ln\theta_{f1}^*}{dc_f} = \frac{1}{c_f + (1-\phi_2)S_{ll}} > 0.$$

WTS: $dG_f^{R_1}/d\Delta w_f > 0$**.**

$$\frac{d\ln\theta_{f1}^*}{d\Delta w_f} = -\frac{1}{\Delta w_f + \phi_2(S_{hh} - S_{hl})} < 0.$$

WTS: $dG_f^{R_1}/d\phi_2 > 0$**.**

$$\frac{d\ln\theta_f^*}{d\phi_2} = -\frac{S_{ll}}{c_f + (1-\phi_2)S_{ll}} - \frac{S_{hh} - S_{hl}}{\Delta w_f + \phi_2(S_{hh} - S_{hl})} < 0.$$

WTS: $dG_f^{R_1}/dS_{hh} > 0$**.**

$$\frac{d\ln G_f^{R_1}}{dS_{hh}} = -\frac{\phi_2}{\Delta w_f + \phi_2(S_{hh} - S_{hl})} < 0.$$

WTS: $dG_f^{R_1}/dS_{hl} < 0$**.**

$$\frac{d\ln G_f^{R_1}}{dS_{hl}} = \frac{\phi_2}{\Delta w_f + \phi_2(S_{hh} - S_{hl})} > 0.$$

WTS: $dG_f^{R_1}/dS_{ll} < 0$**.**

$$\frac{d\ln G_f^{R_1}}{dS_{ll}} = \frac{1-\phi_2}{c_f + (1-\phi_2)S_{ll}} > 0.$$

Finally,

$$\frac{dG_f^{R_1}}{dc_m} = \frac{dG_f^{R_1}}{d\Delta w_m} = \frac{dG_f^{R_1}}{dS_{lh}} = \frac{dG_f^{R_1}}{d\phi_3} = 0.$$

and

$$\frac{dG_m^{R_1}}{dc_f} = \frac{dG_m^{R_1}}{d\Delta w_f} = \frac{dG_m^{R_1}}{d\phi_2} = \frac{dG_m^{R_1}}{dS_{hh}} = \frac{dG_m^{R_1}}{dS_{lh}} = \frac{dG_m^{R_1}}{d\phi_3} = 0.$$

■

A.7 Proof of Proposition 2 (The Shrinking College Gender Gap)

Take any relevant variable x,

$$\frac{d\Delta G^{R_1}}{dx} = g_f(\theta_{f1}^*)\frac{d\theta_{f1}^*}{dx} - g_m(\theta_m^*)\frac{d\theta_m^*}{dx}.$$

WTS: $d\Delta G^{R_1}/dc_f < 0$**.**

$$\frac{d\Delta G^{R_1}}{dc_f} = g_f(\theta_{f1}^*)\frac{d\theta_{f1}^*}{dc_f} - g_m(\theta_m^*)\frac{d\theta_m^*}{dc_f} = g_f(\theta_{f1}^*)\frac{d\theta_{f1}^*}{dc_f} > 0$$

because $d\theta_m^*/dc_f = 0$ and $d\theta_{f1}^*/dc_f > 0$ as shown in the proof of Proposition 1.
WTS: $d\Delta G^{R_1}/d\Delta w_f > 0$.

$$\frac{d\Delta G^{R_1}}{d\Delta w_f} = g_f(\theta_{f1}^*)\frac{d\theta_{f1}^*}{d\Delta w_f} - g_m(\theta_m^*)\frac{d\theta_m^*}{d\Delta w_f} = g_f(\theta_{f1}^*)\frac{d\theta_{f1}^*}{d\Delta w_f} < 0$$

because $d\theta_m^*/d\Delta w_f = 0$ and $d\theta_{f1}^*/d\Delta w_f < 0$ as shown in the proof of Proposition 1.
WTS: $d\Delta G^{R_1}/d\phi_2 > 0$.

$$\frac{d\Delta G^{R_1}}{d\phi_2} = g_f(\theta_{f1}^*)\frac{d\theta_{f1}^*}{d\phi_2} - g_m(\theta_m^*)\frac{d\theta_m^*}{d\phi_2} = g_f(\theta_{f1}^*)\frac{d\theta_{f1}^*}{d\phi_2} < 0$$

because $d\theta_m^*/d\phi_2 = 0$ and $d\theta_{f1}^*/d\phi_2 < 0$ as shown in the proof of Proposition 1.
WTS: $d\Delta G^{R_1}/dS_{hh} > 0$.

$$\frac{d\Delta G^{R_1}}{dS_{hh}} = g_f(\theta_{f1}^*)\frac{d\theta_{f1}^*}{dS_{hh}} - g_m(\theta_m^*)\frac{d\theta_m^*}{dS_{hh}} = g_f(\theta_{f1}^*)\frac{d\theta_{f1}^*}{dS_{hh}} < 0$$

because $d\theta_m^*/dS_{hh} = 0$ and $d\theta_{f1}^*/dS_{ll} < 0$ as shown in the proof of Proposition 1.
WTS: $d\Delta G^{R_1}/dS_{hl} > 0$.

$$\frac{d\Delta G^{R_1}}{dS_{hh}} = g_f(\theta_{f1}^*)\frac{d\theta_{f1}^*}{dS_{hl}} - g_m(\theta_m^*)\frac{d\theta_m^*}{dS_{hl}} < 0$$

because $d\theta_m^*/dS_{hl} > 0$ and $d\theta_{f1}^*/dS_{hl} < 0$ as shown in the proof of Proposition 1. ■

A.8 Proof of Proposition 3 (Reversal of the College Gender Gap)

The proof is by contradiction. Suppose $\Delta U^* \geq \Delta V^*$ in equilibrium. When $\phi_2 = 1$, by the optimal cutoff functions,

$$\theta_m^* = \frac{c}{\Delta w + \Delta U^*} \leq \theta_{f1}^* = \frac{c}{\Delta w + \Delta V^*} < \theta_{f2}^* = \frac{c + (1-\phi_3)\underline{V}^*}{\Delta w + \phi_3 \Delta V^*}.$$

Since the distributions of abilities are the same for men and women, more men become high wage men than women become high wage fit women through college investments,

$$\int_{\theta_m^*}^1 \theta dG(\theta) > \int_{\theta_{f1}^*}^1 \theta dG(\theta),$$

and more men become high wage men than women become high wage fit women through career reinvestments,

$$\int_{\theta_m^*}^1 (1-\theta)\theta dG(\theta) > \phi_3 \int_{\theta_{f2}^*}^1 (1-\theta)\theta dG(\theta).$$

Therefore, there is a bigger mass of high wage men than that of high wage fit women in equilibrium,

$$\overline{\mu}^* = \int_{\theta_m^*}^1 \theta dG(\theta) + \int_{\theta_m^*}^1 (1-\theta)\theta dG(\theta) > \overline{v}^* = \int_{\theta_{f1}^*}^1 \theta dG(\theta) + \phi_3 \int_{\theta_{f2}^*}^1 (1-\theta)\theta dG(\theta).$$

Since there is positive assortative matching of men and fit women, a positive mass of $\overline{w}$ men and $(\overline{w}, \overline{r})$ women and a positive mass of $\overline{w}$ men and $(\underline{w}, \overline{r})$ women are matched. The equilibrium payoffs then satisfy $\overline{U}^* + \overline{V}^* = S_{hh}$ and $\overline{U}^* + \underline{V}^* = S_{hl}$. In addition, by the stability condition, $\underline{U}^* + \underline{V}^* \geq S_{ll}$. The conditions imply that $\Delta U^* \leq S_{hl} - S_{ll}$ and $\Delta V^* = S_{hh} - S_{hl}$. By symmetry, $\Delta V^* = S_{hh} - S_{lh}$. By the complementarity of the surplus function, $\Delta U^* \leq S_{hl} - S_{ll} < S_{hh} - S_{lh} = \Delta V^*$ which contradicts the initial assumption $\Delta U^* \geq \Delta V^*$. Therefore, $\Delta U^* < \Delta V^*$ always holds in equilibrium. When $\phi_2 = 1$, $\theta_m^* = c/(\Delta w + \Delta U^*) > \theta_{f1}^* = c/(\Delta w + \Delta V^*)$ and $G_m^{R_1} = 1 - G(\theta_m^*) < G_f^{R_1} = 1 - G(\theta_{f1}^*)$. ■

A.9 Proof of Proposition 3' (Reversal of the College Gender Gap, Generalized)

Directly by the cutoff functions, $\theta_{f1}^* < \theta_m^*$ if and only if

$$\frac{c + (1-\phi_2)\underline{V}^*}{\Delta w + \phi_2 \Delta V^*} < \frac{c}{\Delta w + \Delta U^*},$$

which rearranges to

$$\phi_2 > \frac{c\Delta U^* + \underline{V}^*(\Delta w + \Delta U^*)}{c\Delta V^* + \underline{V}^*(\Delta w + \Delta U^*)}.$$

By the same logic as in the previous proof, $\Delta U^* < \Delta V^*$, so the right hand side is smaller than 1. When $\overline{v}^* < \overline{\mu}^* < \underline{v}^* + \overline{v}^*$, $\Delta U^* = S_{hl} - S_{ll}$, $\Delta V^* = S_{hh} - S_{hl}$, and $\underline{V}^* = S_{ll}$. Therefore, when

$$\phi_2 > \frac{c(S_{hl} - S_{ll}) + S_{ll}(\Delta w + S_{hl} - S_{ll})}{c(S_{hh} - S_{hl}) + S_{ll}(\Delta w + S_{hl} - S_{ll})},$$

θ_{f1}^* is guaranteed to be smaller than θ_m^* and more women go to college than men in equilibrium. ■

A.10 The College Gender Gap Under Submodular Surplus

Suppose S is strictly submodular, i.e. $S_{hh} + S_{ll} < S_{hl} + S_{lh}$. I show that there cannot be strictly more women making college investments than men doing it. There is a positive mass of matches between high wage men and low wage fit women: $\pi^*(\overline{w}_m, \underline{w}_f, \overline{r}) > 0$. Therefore, they divide up their marriage surplus: $\overline{U}^* + \underline{V}^* = S_{hl}$. There is a positive mass of matches between low wage men and high wage fit women: $\pi(\underline{w}_m, \overline{w}_f, \overline{r}) > 0$. They divide up their marriage surplus as well: $\underline{U}^* + \overline{V}^* = S_{lh}$. Subtracting the two equations, $\Delta U^* - \Delta V^* = S_{hl} - S_{lh} = 0$. When $\phi_2 = 1$, $\theta_m^* = \theta_{f1}^*$. When $\phi_2 < 1$, $\theta_m^* < \theta_{f1}^*$. ■

A.11 Proof of Proposition 4 (Changes in Career Investments)

The proportion of males who make a career investment (action R_2) is $G_m^{R_2} = \int_{\theta_m^*}^1 (1-\theta) dG_m(\theta)$. Take any variable x,

$$\frac{dG_m^{R_2}}{dx} \equiv -(1-\theta_m^*) g_m(\theta_m^*) \frac{d\theta_m^*}{dx}.$$

Therefore, the sign is the same as that of $dG_m^{R_1}/dx$ and that of $-d\ln\theta_m^*/dx$. The sign of $d\ln\theta_m^*/dx$ has been shown in the proof of Proposition 1. All the changes that increase men's college investments also increase men's career investments.

The proportion of females who make a career reinvestment (action R_2) is $G_f^{R_2} \equiv \int_{\theta_{f2}^*}^1 (1 - \theta) G_f(\theta)$. Take any variable x,

$$\frac{dG_f^{R_2}}{dx} = -(1-\theta_{f2}^*) g_f(\theta_{f2}^*) \frac{d\theta_{f2}^*}{dx}.$$

Therefore, the sign is the same as that of $-d\ln\theta_{f2}^*/dx$.

$$\ln\theta_{f2}^* = \ln[c_f + (\phi_2 - \phi_3)S_{ll}] - \ln[\Delta w_f + \phi_3(S_{hh} - S_{hl})].$$

WTS: $dG_f^{R_2}/dc_f < 0$**.**

$$\frac{d\ln\theta_{f2}^*}{dc_f} = \frac{1}{c_f + (\phi_2 - \phi_3)S_{ll}} > 0.$$

WTS: $dG_f^{R_2}/d\Delta w_f > 0$**.**

$$\frac{d\ln\theta_{f2}^*}{d\Delta w_f} = -\frac{1}{\Delta w_f + \phi_3(S_{hh} - S_{hl})} < 0.$$

WTS: $dG_f^{R_2}/d\phi_2 < 0$.

$$\frac{d\ln\theta_{f2}^*}{d\phi_2} = \frac{S_{ll}}{c_f + (\phi_2 - \phi_3)S_{ll}} > 0.$$

WTS: $dG_f^{R_2}/d\phi_3 > 0$.

$$\frac{d\ln\theta_{f2}^*}{d\phi_3} = -\frac{S_{ll}}{c_f + (\phi_2 - \phi_3)S_{ll}} - \frac{S_{hh} - S_{hl}}{\Delta w_f + \phi_3(S_{hh} - S_{hl})} < 0.$$

WTS: $dG_f^{R_2}/dS_{hh} > 0$.

$$\frac{d\ln\theta_{f2}^*}{dS_{hh}} = -\frac{\phi_3}{\Delta w_f + \phi_3(S_{hh} - S_{hl})} < 0.$$

WTS: $dG_f^{R_2}/dS_{hl} < 0$.

$$\frac{d\ln\theta_{f2}^*}{dS_{hh}} = \frac{\phi_3}{\Delta w_f + \phi_3(S_{hh} - S_{hl})} > 0.$$

WTS: $dG_f^{R_2}/dS_{ll} < 0$.

$$\frac{d\ln\theta_{f2}^*}{dS_{ll}} = \frac{\phi_2 - \phi_3}{c_f + (\phi_2 - \phi_3)S_{ll}} > 0.$$

Finally,

$$\frac{dG_f^{R_2}}{dc_m} = \frac{dG_f^{R_2}}{d\Delta w_m} = \frac{dG_f^{R_2}}{dS_{lh}} = 0.$$

■

A.12 Proof of Proposition 5 (The Shrinking Gender Gap in Career Investments)

The gender difference between the proportions of career investments is

$$\Delta G^{R_2} \equiv G_m^{R_2} - G_f^{R_2} = \int_{\theta_m^*}^{1} (1-\theta)dG_m(\theta) - \int_{\theta_{f\,2}^*}^{1} (1-\theta)dG_f(\theta),$$

and the change with respect to any variable x is

$$\frac{d\Delta G^{R_2}}{dx} = \frac{dG_m^{R_2}}{dx} + (1-\theta_{f2}^*)g_f(\theta_{f2}^*)\frac{d\theta_{f2}^*}{dx}.$$

Since

$$\frac{dG_m^{R_2}}{dc_f} = \frac{dG_m^{R_2}}{d\Delta w_f} = \frac{dG_m^{R_2}}{d\phi_2} = \frac{dG_m^{R_2}}{dS_{hh}} = \frac{dG_m^{R_2}}{dS_{lh}} = \frac{dG_m^{R_2}}{d\phi_3} = 0,$$

the change in the gender gap has the same sign as $d\ln\theta_{f2}^*/dx$. Therefore, following the results in the previous proof,

$$\frac{d\Delta G^{R_2}}{d(-c_f)}, \frac{d\Delta G^{R_2}}{d\Delta w_f}, \frac{d\Delta G^{R_2}}{d\phi_3}, \frac{d\Delta G^{R_2}}{d(-\phi_2)}, \frac{d\Delta G^{R_2}}{dS_{hh}}, \frac{d\Delta G^{R_2}}{d(-S_{hl})}, \frac{d\Delta G^{R_2}}{d(-S_{ll})} < 0.$$

■

A.13 Proof of Proposition 6 (The Shrinking Gender Gap in Wages)

Males' average wage is

$$\begin{aligned} \mathbb{E}w_m^* &= \underline{w}_m \int_{\theta_m^*}^1 (1-\theta)^2 dG_m(\theta) + \overline{w}_m \int_{\theta_m^*}^1 (2-\theta)\theta dG_m(\theta) \\ &= \underline{w}_m + \Delta w_m \int_{\theta_m^*}^1 (2-\theta)\theta dG_m(\theta), \end{aligned}$$

and females' average wage is

$$\mathbb{E}w_f^* = \underline{w}_f + \Delta w_f [\int_{\theta_{f\,1}^*}^{\theta_{f\,2}^*} \theta dG_f(\theta) + \int_{\theta_{f\,2}^*}^1 (2-\theta)\theta dG_f(\theta)].$$

Take a relevant variable x (not $\underline{w}_f$ or $\overline{w}_f$),

$$\frac{d\mathbb{E}w_m^*}{dx} = -\Delta w_m (2-\theta_m^*)\theta_m^* g_m(\theta_m^*)\frac{d\theta_m^*}{dx}$$

and

$$\frac{d\mathbb{E}w_f^*}{dx} = -\Delta w_f [\theta_{f1}^* g_f(\theta_{f1}^*)\frac{d\theta_{f1}^*}{dx} + (1-\theta_{f2}^*)\theta_{f2}^* g_f(\theta_{f2}^*)\frac{d\theta_{f2}^*}{dx}].$$

The change in the gender gap in wages is

$$\frac{d\Delta w^*}{dx} = \Delta w_f [\theta_{f1}^* g_f(\theta_{f1}^*)\frac{d\theta_{f1}^*}{dx} + (1-\theta_{f2}^*)\theta_{f2}^* g_f(\theta_{f2}^*)\frac{d\theta_{f2}^*}{dx}] - \Delta w_m (2-\theta_m^*)\theta_m^* g_m(\theta_m^*)\frac{d\theta_m^*}{dx}.$$

$d\theta_{f1}^*/dx$, $d\theta_{f2}^*/dx$, and $d\theta_m^*/dx$ have been derived in the previous proofs. ■

A.14 Proof of Proposition 7 (Grand Gender Convergence)

Suppose $\phi_2 = \phi_3 = 1$, $G_m = G_f = G$, $\Delta w_m = \Delta w_f = \Delta w$, $c_m = c_f = c$, and $S_{hl} = S_{lh}$. Then

$$\begin{aligned}
\overline{\delta}(k) &= \int_{\theta_m^*(k)}^1 \theta(2-\theta)dG(\theta) - \int_{\theta_{f\,1}^*(k)}^1 \theta dG(\theta) - \int_{\theta_{f\,2}^*(k)}^1 (1-\theta)\theta dG(\theta) \\
&= \int_{\frac{c}{\Delta w + k(S_{lh}-S_{ll})+(1-k)(S_{hh}-S_{hl})}}^1 \theta(2-\theta)dG(\theta) - \int_{\frac{c}{\Delta w+(1-k)(S_{lh}-S_{ll})+k(S_{hh}-S_{hl})}}^1 \theta(2-\theta)dG(\theta).
\end{aligned}$$

Since $\overline{\delta}(0.5) = 0$, there is a unique equilibrium associated with $k^* = 0.5$. In the equilibrium,

$$\theta_m^* = \theta_{f1}^* = \theta_{f2}^* = \frac{c}{\Delta w + (S_{hh} - S_{ll})/2}.$$

Since the ability distributions and investment strategies are gender symmetric, men's and women's investments, marriage market distributions, and wages are all the same in this equilibrium. ■

A.15 Proof of Proposition 8 (Investment Uncertainty Delays Marriage)

A man who decides whether or not to entering the marriage market before realizing the investment compares the expected utility he or she receives in the marriage market as a type $\theta_m \circ \overline{w}_m + (1-\theta_m) \circ \underline{w}_m$ and the expected utility that he receives by entering the market as a $\overline{w}_m$ with probability θ_m and as a type $\underline{w}_m$ with probability $1-\theta_m$. The only difference is in expected marriage payoffs. Suppose a type $\theta_m \circ \overline{w}_m + (1-\theta_m) \circ \underline{w}_m$ male is stably assigned to a type (w_f, r) partner, so that

$$U(\theta_m \circ \overline{w}_m + (1-\theta_m) \circ \underline{w}_m) = \theta_m S(\overline{w}_m, w_f, r) + (1-\theta_m) S(\underline{w}_m, w_f, r) - V(w_f, r).$$

By stability, $U(\overline{w}_m) \geq S(\overline{w}_m, w_f, r) - V(w_f, r)$ and $U(\underline{w}_m) \geq S(\underline{w}_m, w_f, r) - V(w_f, r)$. Combining the two inequalities,

$$\theta_m U(\overline{w}_m) + (1-\theta_m) U(\underline{w}_m) \geq \theta_m S(\overline{w}_m, w_f, r) + (1-\theta_m) S(\underline{w}_m, w_f, r) - V(w_f, r)$$

and the right hand side exactly equals $U(\theta_m \circ \overline{w}_m + (1-\theta_m) \circ \underline{w}_m)$. Therefore,

$$\theta_m U(\overline{w}_m) + (1-\theta_m) U(\underline{w}_m) \geq U(\theta_m \circ \overline{w}_m + (1-\theta_m) \circ \underline{w}_m).$$

■

A.16 Proof of Proposition 9 (Changes in Marriage Age)

Take any variable x,

$$\frac{d\mathbb{E}a_m^*}{dx} = -(2-\theta_m^*)g_m(\theta_m^*)\frac{d\theta_m^*}{dx},$$

and

$$\frac{d\mathbb{E}a_f^*}{dx} = -g_f(\theta_{f1}^*)\frac{d\theta_{f1}^*}{dx} - (1-\theta_{f2}^*)g(\theta_{f2}^*)\frac{d\theta_{f2}^*}{dx}.$$

The comparative statics results of $d\theta_m^*/dx$, $d\theta_{f1}^*/dx$ are shown in the Proof of Proposition 1, and those of $d\theta_{f2}^*/dx$ follow from proof of Proposition 4. The comparative statics results follow. ■

A.17 Proof of Proposition 10 (The Shrinking Gender Gap in Marriage Age)

Take any relevant variable x,

$$\frac{d\Delta a^*}{dx} = -(2-\theta_m^*)g(\theta_m^*)\frac{d\theta_m^*}{dx} + g_f(\theta_{f1}^*)\frac{d\theta_{f1}^*}{dx} + (1-\theta_{f2}^*)g(\theta_{f2}^*)\frac{d\theta_{f2}^*}{dx}.$$

$d\theta_m^*/dx = 0$ for the related variables, so

$$\frac{d\Delta a^*}{dx} = g_f(\theta_{f1}^*)\frac{d\theta_{f1}^*}{dx} + (1-\theta_{f2}^*)g(\theta_{f2}^*)\frac{d\theta_{f2}^*}{dx}.$$

$d\theta_{f1}^*/dx$ and $d\theta_{f2}^*/dx$ are derived in the previous proofs. ■

A.18 Proof of Proposition 11 (Males' Marriage Age-Personal Income Relationship)

All the ability $\theta_m \le \theta_m^*$ men marry at age 1. They all earn $\underline{w}_m$ and make up mass $G_m(\theta_m^*)$ of the population. Mass $\int_{\theta_m^*}^1 \theta dG_m(\theta)$ of males marry at age 2 and everyone of them earns high wage $\overline{w}_m$. The rest of the males marry at age 3. Mass $\int_{\theta_m^*}^1 (1-\theta)\theta dG_m(\theta)$ earn a high wage $\overline{w}_m$ and mass $\int_{\theta_m^*}^1 (1-\theta)^2 dG_m(\theta)$ earn a low wage $\underline{w}_m$. The average wage of those who marry at age 3 is

$$\mathbb{E}w_{m3} = \frac{\overline{w}_m \int_{\theta_m^*}^1 (1-\theta)\theta dG_m(\theta) + \underline{w}_m \int_{\theta_m^*}^1 (1-\theta)^2 dG_m(\theta)}{\int_{\theta_m^*}^1 (1-\theta) dG_m(\theta)}$$

strictly between $\mathbb{E}w_{m1} = \underline{w}_m$ and $\mathbb{E}w_{m2} = \overline{w}_m$. ■

A.19 Proof of Proposition 12 (Females' Marriage Age-Personal Income Relationship)

Mass $G_f(\theta^*_{f1})$ marry at age 1 and earn low wage: $\mathbb{E}w_{f1} = \underline{w}_{f1}$. Those who marry at age 2 achieve either a high wage or a low wage. Among them, mass $\int_{\theta^*_{f1}}^{1} \theta dG_f(\theta)$ earn high wage and mass $\int_{\theta^*_{f1}}^{\theta^*_{f2}} (1-\theta) dG_f(\theta)$ earn low wage, for an average wage of

$$\mathbb{E}w_{f2} = \frac{\overline{w}_f \int_{\theta^*_{f1}}^{1} \theta dG_f(\theta) + \underline{w}_f \int_{\theta^*_{f1}}^{\theta^*_{f2}} (1-\theta) dG_f(\theta)}{\int_{\theta^*_{f1}}^{1} \theta dG_f(\theta) + \int_{\theta^*_{f1}}^{\theta^*_{f2}} (1-\theta) dG_f(\theta)}.$$

Those who marry at age 3 have abilities above θ^*_{f2}, and they realize either high or low wages, for an average of

$$\mathbb{E}w_{f3} = \frac{\overline{w}_f \int_{\theta^*_{f2}}^{1} (1-\theta)\theta dG_f(\theta) + \underline{w}_f \int_{\theta^*_{f2}}^{1} (1-\theta)^2 dG_f(\theta)}{\int_{\theta^*_{f2}}^{1} (1-\theta) dG_f(\theta)}.$$

$\mathbb{E}w_{f2}$ is monotonically decreasing in θ^*_{f2} which increases in ϕ_3. $\mathbb{E}w_{f3}$ is monotonically increasing in θ^*_{f2} which increases in ϕ_3.

When $\phi_3 \to 0$, $\theta^*_{f2} \to 1$,

$$\mathbb{E}w_{f2} \to \frac{\int_{\theta^*_{f1}}^{1} [\theta \overline{w}_f + (1-\theta)\underline{w}_f] dG_f(\theta)}{1 - G_f(\theta^*_{f1})}$$

and $\mathbb{E}w_{f3} \to \overline{w}_f$. Therefore, when ϕ_3 is small $\mathbb{E}w_{f1} < \mathbb{E}w_{f2} < \mathbb{E}w_{f3}$. The equilibrium marriage age-personal income relationship is positive.

When $\phi_3 \to 1$, $\theta^*_{f2} \to \theta^*_{f1}$, $\mathbb{E}w_{f2} \to \overline{w}_f$, and,

$$\mathbb{E}w_{f3} = \frac{\overline{w}_f \int_{\theta^*_{f2}}^{1} (1-\theta)\theta dG_f(\theta) + \underline{w}_f \int_{\theta^*_{f2}}^{1} (1-\theta)^2 dG_f(\theta)}{\int_{\theta^*_{f2}}^{1} (1-\theta) dG_f(\theta)}.$$

Therefore, when ϕ_3 is large enough, the equilibrium marriage age-personal income tends to be inverse-U shaped. ■

B General Framework and Results

I first describe the general theoretical framework and then reprove some of the main results from the text under the generalized framework. The generalizations from the basic theoretical framework are that 1) agents discount, 2) sets of wage offers are continuous, 3) the distribution of wages an agent draws from in the career reinvestment is not necessarily the same as the distribution after the college investment, and 4) surplus function is continuously differentiable and can be positive for an unfit woman.

B.1 Generalized Framework

There is an infinite number of discrete periods. At the beginning of each period, unit masses of males and females are born with heterogeneous abilities. In each period, the heterogeneous abilities $\theta_m \in \Theta_m = [0, 1]$ and $\theta_f \in \Theta_f = [0, 1]$ are distributed according to continuous and strictly increasing gender-specific mass distributions G_m and G_f.

Each agent lives for three periods, referred to as ages 1, 2, and 3. Each agent derives utility from wage (w_m, w_f) in the labor market plus payoff (U, V) in the marriage market and net any investment cost. All the agents are risk-neutral and discount by a common discount factor $\delta \leq 1$. For example, if a man pays investment cost c_{m1} at age 1 and accepts a w_m job and earns marriage surplus U at age 2, his lifetime utility discounted to age 1 is $\delta(w_m + U) - c_{m1}$.

Each newborn learns his or her own ability θ and chooses whether to go to college (R_1) or not (A_1). An agent who does not go to college earns a low wage $\underline{w}_m$, $\underline{w}_f$ from the labor market and enters the marriage market immediately. On the other hand, an agent who goes to college delays entry to the marriage market and pays a positive investment cost (c_{m1}, c_{f1}).

At the beginning of age 2, each ability θ agent who has made the college investment receives a wage offer according to distribution $P_{m2}(\cdot|\theta)$ and $P_{f2}(\cdot|\theta)$. One can either accept (A_2) or reject (R_2) the offer. An agent who accepts the job offer starts to earn the lifetime wage and enters the marriage market in that period. An agent who rejects the job offer delays entrance to the marriage market and pays another investment cost (c_{m2}, c_{f2}) to receive another draw next period.

At the beginning of age 3, each ability θ agent who has rejected the job offer at age 2 gets a wage offer drawn from $P_{m3}(\cdot|\theta)$, $P_{f3}(\cdot|\theta)$. The agent at this point has no choice but to accept the offer and to enter the marriage market.

Assume that $P_{m2}(\cdot|\theta)$, $P_{m3}(\cdot|\theta)$, $P_{f2}(\cdot|\theta)$, and $P_{f3}(\cdot|\theta)$ are continuous in wages and strictly increasing in the supports $\mathcal{W}_m \equiv [\underline{w}_m, \overline{w}_m]$ and $\mathcal{W}_f \equiv [\underline{w}_f, \overline{w}_f]$. Assume higher ability agents draw from better distributions of offers: $P_{m2}(\cdot|\theta)/P_{m3}(\cdot|\theta)/P_{f2}(\cdot|\theta)/P_{f3}(\cdot|\theta)$ first order stochastically dominates $P_{m2}(\cdot|\theta')/P_{m3}(\cdot|\theta')/P_{f2}(\cdot|\theta')/P_{f3}(\cdot|\theta')$ if $\theta > \theta'$. Assume that $P_{m2}(\cdot|\theta)$ first order stochastically dominates $P_{m3}(\cdot|\theta)$ and $P_{f2}(\cdot|\theta)$ first order stochastically dominates $P_{f3}(\cdot|\theta)$ for all $\theta \in [0,1]$. Finally assume that $c_{m1} \leq c_{m2}$ and $c_{f1} \leq c_{f2}$.

In summary, men and women face the same investment strategies. They make two investment decisions respectively at age 1 and at age 2, and the age 2 decision is contingent on their wage offer; $\sigma_m^1, \sigma_f^1 : [0,1] \to \{A_1, R_1\}$ and $\sigma_m^2, \sigma_f^2 : [0,1] \times \mathcal{W} \to \{A_2, R_2\}$.

A couple's marriage surplus is $S(w_m, w_f, r)$ where r is a woman's reproductive fitness. Assume that the surplus is continuously differentiable, strictly increasing in all three arguments and strictly supermodular in wages. A woman's fitness level is realized after she enters the marriage market. A woman who enters the marriage market at age a is fit ($\bar{r}$) with probability ϕ_a and unfit ($\underline{r}$) with probability $1-\phi_a$. Assume that a woman maintains her reproductive fitness with increasingly declining probabilities, $\phi_1 = 1 \geq \phi_2 \geq \phi_3$ and $\phi_1 - \phi_2 \leq \phi_2 - \phi_3$.

Measures μ on males' wage set $\mathcal{W}_m$ and ν on $\mathcal{W}_f \times \{\underline{r}, \bar{r}\}$ describe the marriage market. Men and women frictionlessly match and bargain over division of their marriage surplus. An outcome of the marriage market specifies a feasible matching and stable marital payoffs. The matching function $\pi : \mathcal{W}_m \times \mathcal{W}_f \times \{\underline{r}, \bar{r}\} \to \mathbb{R}_+$ describes the masses of matches between different types of men and women and is feasible if π has marginals μ and ν. The marital payoff functions $U : \text{supp}(\mu) \to \mathbb{R}_+$ and $V : \text{supp}(\nu) \to \mathbb{R}_+$ specify payoffs of men and women, respectively.

An outcome (π, U, V) is stable if π solves the primal problem $\sup\left\{\int S d\tilde{\pi} | \tilde{\pi} \text{ is feasible}\right\}$, and U and V solve the dual problem $\inf\left\{\int \tilde{U} d\mu + \int \tilde{V} d\nu | \tilde{U} \geq 0, \tilde{V} \geq 0\right\}$. The primal problem and the dual problem have solutions and there is no gap between the solutions (Gretsky et al. 1992, Theorem 1), so a stable outcome exists. The solutions satisfy the stability conditions, $U(w_m) + V(w_f, r) \geq S(w_m, w_f, r)$ for any $(w_m, w_f, r) \in \text{supp}(\mu) \times \text{supp}(\nu)$ and $U(w_m) + V(w_f, r) = S(w_m, w_f, r)$ if $\pi(w_m, w_f, r) > 0$ (Gretsky et al. 1992, Lemma 3).

Assume that those who produce zero surplus are matched. Finally, if there are unequal masses of men and women, assume that everyone with the same matching characteristic has the same probability of being unmatched, and that an agent unmatched in the current period enters the marriage market in the subsequent period if alive.

For any stable marriage market outcome (π, U, V) on $\text{supp}(\mu) \times \text{supp}(\nu)$, extend it to be defined on $\mathcal{W}_m \times \mathcal{W}_f \times \{\underline{r}, \bar{r}\}$, $U(w_m) = \sup_{w_m \in \text{supp}(\mu)}[S(w_m, w_f, r) - V(w_f, r)]$ and $V(w_f, r) = \sup_{(w_f, r) \in \text{supp}(\nu)}[S(w_m, w_f, r) - U(w_m)]$.

Definition 2. $(\sigma_m^*, \sigma_f^*, \pi^*, U^*, V^*)$ is a stationary equilibrium if the strategies (σ_m^*, σ_f^*) are optimal with respect to (π^*, U^*, V^*) and (π^*, U^*, V^*) is an extended stable outcome of the marriage market (μ^*, ν^*) induced by (σ_m^*, σ_f^*).

B.2 Generalized Results

Lemma (Stable Marriage Market Outcome)**.** *A stable matching π exists. Stable marriage payoff functions U and V are continuous in wages, and are strictly increasing in the three arguments if there is gender balance.*

Proof. The existence of a stable outcome follows from Gretsky et al. (1992, Theorems 1 and 2). The continuity of stable payoff functions follows from Gretsky et al. (1992, Theorem 6). The stable payoff functions U and V defined on $\mathcal{W}_m \times \mathcal{W}_f \times \{\underline{r}, \overline{r}\}$ satisfy

$$U(w_m) = \sup_{w_m \in \text{supp}(\mu)} [S(w_m, w_f, r) - V(w_f, r)] \geq 0,$$

and

$$V(w_f, r) = \sup_{(w_f, r) \in \text{supp}(\nu)} [S(w_m, w_f, r) - U(w_m)] \geq 0.$$

Since S is strictly increasing in each of the three arguments, $S(w_m, w_f, r) - V(w_f, r)$ is strictly increasing in w_m for all (w_f, r) and $S(w_m, w_f, r) - U(w_m)$ is strictly increasing in w_f and r for all w_m. Then the supremums of the increasing functions, U and V, are strictly increasing. ■

Lemma (Males' Optimal Strategies)**.** *Suppose (π, U, V) is stable. Let θ_m^* be the unique solution to*

$$\delta \int_{\underline{w}_m}^{\overline{w}_m} [U(w_{m2}) + w_{m2}] dP_{m2}(w_{m2}|\theta_m^*) - U(\underline{w}_m) - \underline{w}_m - c_{m1} = 0,$$

and let $w_{m2}^*(\theta_m)$ be

$$\min\{w_{m2} \in [\underline{w}_m, \overline{w}_m] | \delta \int_{\underline{w}_m}^{\overline{w}_m} [U(w_{m3}) + w_{m3}] dP_{m3}(w_{m3}|\theta_m) - U(w_{m2}) - w_{m2} - c_{m2} \leq 0.\}$$

Males' optimal strategies are

$$\sigma_m^{1*}(\theta) = \begin{cases} R_1 & \theta \geq \theta_m^* \\ A_1 & \theta < \theta_m^* \end{cases}, \quad \sigma_m^{2*}(\theta, w_2) = \begin{cases} R_2 & w_2 \leq w_{m2}^*(\theta) \\ A_2 & w_2 > w_{m2}^*(\theta) \end{cases}.$$

Proof. An ability θ_m male's maximal utility is

$$\begin{aligned} H_m(\theta_m) \quad = \quad & \max_{A_1, R_1}\{U(\underline{w}_m) + \underline{w}_m, -c_{m1} + \delta \int_{\underline{w}_m}^{\overline{w}_m} \max_{A_2, R_2}\{U(w_{m2}) + w_{m2}, \\ & -c_{m2} + \delta \int_{\underline{w}_m}^{\overline{w}_m} [U(w_{m3}) + w_{m3}] dP_{m3}(w_{m3}|\theta_m)\} dP_{m2}(w_{m2}|\theta_m)\} \end{aligned}$$

which can be rearranged to

$$U(\underline{w}_m)+\underline{w}_m+\max_{A_1,R_1}\Bigg\{0,-c_{m1}+\delta\int_{\underline{w}_m}^{\overline{w}_m}[U(w_{m2})+w_{m2}]dP_{m2}(w_{m2}|\theta_m)-U(\underline{w}_m)-\underline{w}_m+$$
$$\int_{\underline{w}_m}^{\overline{w}_m}\max_{A_2,R_2}\Bigg\{0,-c_{m2}+\delta\int_{\underline{w}_m}^{\overline{w}_m}[U(w_{m3})+w_{m3}]dP_{m3}(w_m|\theta_m)-U(w_{m2})-w_{m2}\Bigg\}dP_{m2}(w_{m2}|\theta_m)\Bigg\}.$$

When an ability θ_m male receives w_{m2} offer, he takes action R_2 if

$$-c_{m2}+\delta\int_{\underline{w}_m}^{\overline{w}_m}[U(w_{m3})+w_{m3}]dP_{m3}(w_{m3}|\theta_m)-U(w_{m2})-w_{m2}\geq 0,$$

and action A_2 if

$$-c_{m2}+\delta\int_{\underline{w}_m}^{\overline{w}_m}[U(w_{m3})+w_{m3}]dP_{m3}(w_{m3}|\theta_m)-U(w_{m2})-w_{m2}<0,$$

Let $w^*_{m2}(\theta_m)$ denote the reservation wage of θ_m. Namely if

$$\delta\int_{\underline{w}_m}^{\overline{w}_m}[U(w_{m3})+w_{m3}]dP_{m3}(w_{m3}|\theta_m)-U(\underline{w}_m)-\underline{w}_m-c_{m2}<0$$

then $w^*_{m2}(\theta_m)=\underline{w}_m$, and otherwise $w^*_{m2}(\theta_m)$ is the unique solution to

$$\delta\int_{\underline{w}_m}^{\overline{w}_m}[U(w_{m3})+w_{m3}]dP_{m3}(w_{m3}|\theta_m)-U(w_{m2}(\theta^*_m))-w_{m2}(\theta^*_m)-c_{m2}=0.$$

Since U is continuous and strictly increasing, and $P_{m3}(\cdot|\theta_m)$ FOSDs $P_{m3}(\cdot|\theta'_m)$ for all $\theta_m>\theta'_m$, $w^*_{m2}(\theta_m)$ is continuous and increasing. Since $c_{m2}\geq c_{m1}$, $P_{m3}(\cdot|\theta_m)$ FOSDs $P_{m2}(\cdot|\theta_m)$, and $w_{m2}\geq \underline{w}_m$, for any θ_m,

$$\begin{aligned}&-c_{m1}+\delta\int_{\underline{w}_m}^{\overline{w}_m}[U(w_{m2})+w_{m2}]dP_{m2}(w_{m2}|\theta_m)-U(\underline{w}_m)-\underline{w}_m\\ \geq\ &-c_{m2}+\delta\int_{\underline{w}_m}^{\overline{w}_m}[U(w_{m3})+w_{m3}]dP_{m3}(w_m|\theta_m)-U(w_{m2})-w_{m2}.\end{aligned}$$

That is, any male who does not benefit from a college investment will not benefit from a reinvestment. Let θ^*_m be the unique solution to

$$\delta\int_{\underline{w}_m}^{\overline{w}_m}[U(w_{m2})+w_{m2}]dP_{m2}(w_{m2}|\theta^*_m)-U(\underline{w}_m)-\underline{w}_m-c_{m1}=0.$$

Any ability $\theta_m \geq \theta_m^*$ male makes a college investment and any ability $\theta_m < \theta_m^*$ male does not. ■

Lemma (Females' Optimal Strategies)**.** *Suppose (π, U, V) is stable. Let θ_f^* be the unique solution to*

$$\delta \int_{\underline{w}_f}^{\overline{w}_f} [\phi_2 V(w_{f2}, \overline{r}) + (1-\phi_2) V(w_{f2}, \underline{r}) + w_{f2}] dP_{f2}(w_{f2}|\theta_f^*) - V(\underline{w}_f, \overline{r}) - \underline{w}_f - c_{f1} = 0,$$

and

$$w_{f2}^*(\theta_f) \equiv \min\{w_{f2} \in [\underline{w}_f, \overline{w}_f] | H_{f2}(\theta_f, w_{f2}) \leq 0\}$$

where $H_{f2}(\theta_f, w_{f2})$ is

$$-c_{f2} + \delta \int_{\underline{w}_f}^{\overline{w}_f} [\phi_3 V(w_{f3}, \overline{r}) + (1-\phi_3) V(w_{f3}, \underline{r}) + w_{f3}] dP_{f3}(w_{f3}|\theta_f) - \phi_2 V(w_{f2}, \overline{r}) - (1-\phi_2) V(w_{f2}, \underline{r}) - w_{f2},$$

Females' optimal strategies are

$$\sigma_f^{1*}(\theta) = \begin{cases} R_1 & \theta \geq \theta_f^* \\ A_1 & \theta < \theta_f^* \end{cases}, \quad \sigma_f^{2*}(\theta, w_2) = \begin{cases} R_2 & w_2 \leq w_{f2}^*(\theta) \\ A_2 & w_2 > w_{f2}^*(\theta) \end{cases}.$$

Proof. An ability θ_f female's maximal utility is

$$\begin{aligned} H_f(\theta_f) \quad = \quad & \max_{A_1, R_1} \{V(\underline{w}_f, \overline{r}) + \underline{w}_f, \\ & -c_{f1} + \delta \int_{\underline{w}_f}^{\overline{w}_f} \max_{A_2, R_2} \{\phi_2 V(w_{f2}, \overline{r}) + (1-\phi_2) V(w_{f2}, \underline{r}) + w_{f2}, \\ & -c_{f2} + \delta \int_{\underline{w}_f}^{\overline{w}_f} [\phi_3 V(w_{f3}, \overline{r}) + (1-\phi_3) V(w_f, \underline{r}) + w_{f3}] dP_{f3}(w_{f3}|\theta_f)\} dP_{f2}(w_{f2}|\theta_{f2})\} \end{aligned}$$

which can be rearranged to

$$\begin{aligned} & V(\underline{w}_f, \overline{r}) + \underline{w}_f + \max\Bigg\{0, \\ & -c_{f1} + \delta \int_{\underline{w}_f}^{\overline{w}_f} [\phi_2 V(w_{f2}, \overline{r}) + (1-\phi_2) V(w_{f2}, \underline{r}) + w_{f2}] dP_{f2}(w_{f2}|\theta_f) - V(\underline{w}_f, \overline{r}) - \underline{w}_f \\ & +\delta \int_{\underline{w}_f}^{\overline{w}_f} \max\Bigg\{0, -c_{f2} + \delta \int_{\underline{w}_f}^{\overline{w}_f} [\phi_3 V(w_{f3}, \overline{r}) + (1-\phi_3) V(w_{f3}, \underline{r}) + w_{f3}] dP_{f3}(w_{f3}|\theta_f) \\ & -\phi_2 V(w_{f2}, \overline{r}) - (1-\phi_2) V(w_{f2}, \underline{r}) - w_{f2}\Bigg\} dP_{f2}(w_{f2}|\theta_f)\Bigg\} \end{aligned}$$

An ability θ_f females rejects a w_{f2} wage offer at age 2 if and only if

$$H_{f2}(\theta_f, w_{f2}) \equiv -c_{f2} + \delta \int_{\underline{w}_f}^{\overline{w}_f} [\phi_3 V(w_{f3}, \overline{r}) + (1-\phi_3) V(w_{f3}, \underline{r}) + w_{f3}] dP_{f3}(w_{f3}|\theta_f) \\ -\phi_2 V(w_{f2}, \overline{r}) - (1-\phi_2) V(w_{f2}, \underline{r}) - w_{f2} \quad \geq \quad 0.$$

Let $w^*_{f2}(\theta_f)$ denote the reservation wage of θ_f,

$$w^*_{f2}(\theta_f) = \min\{w_{f2} \in [\underline{w}_f, \overline{w}_f] | H_{f2}(\theta_f, w_{f2}) \leq 0\}.$$

Since

$$-c_{f1} + \delta \int_{\underline{w}_f}^{\overline{w}_f} [\phi_2 V(w_{f2}, \overline{r}) + (1-\phi_2) V(w_{f2}, \underline{r}) + w_{f2}] dP_{f2}(w_{f2}|\theta_f) - V(\underline{w}_f, \overline{r}) - \underline{w}_f \geq H_{f2}(\theta_f, w_{f2}),$$

an ability θ_f female takes a college investment if and only if

$$-c_{f1} + \delta \int_{\underline{w}_f}^{\overline{w}_f} [\phi_2 V(w_{f2}, \overline{r}) + (1-\phi_2) V(w_{f2}, \underline{r}) + w_{f2}] dP_{f2}(w_{f2}|\theta_f) - V(\underline{w}_f, \overline{r}) - \underline{w}_f \geq 0.$$

An ability θ^*_f female will be indifferent, where θ^*_f is the unique solution to

$$-c_{f1} + \delta \int_{\underline{w}_f}^{\overline{w}_f} [\phi_2 V(w_{f2}, \overline{r}) + (1-\phi_2) V(w_{f2}, \underline{r}) + w_{f2}] dP_{f2}(w_{f2}|\theta^*_f) - V(\underline{w}_f, \overline{r}) - \underline{w}_f \geq 0.$$

■

Proposition (Reversal of the College Gender Gap). *Suppose the setting is gender-symmetric ($G_m = G_f \equiv G$, $P_{m2} = P_{f2} \equiv P_2$, $P_{m3} = P_{f3} \equiv P_3$, $c_{m1} = c_{f1} \equiv c_1$, and $c_{m2} = c_{f2} \equiv c_2$). When $\phi_2 = 1$ and $\phi_3 < 1$, as $\delta \to 1$, more women than men go to college in equilibrium.*

Proof. In this setting, the proportion of males making college investments is $1 - G(\theta^*_m)$ where θ^*_m solves

$$\delta \int_{\underline{w}}^{\overline{w}} [U(w_2) + w_2] dP_2(w_2|\theta^*_m) - U(\underline{w}) - \underline{w} - c_1 = 0,$$

and proportion $1 - G(\theta^*_f)$ females make a college investments where θ^*_f solves

$$\delta \int_{\underline{w}}^{\overline{w}} [V(w_2, \overline{r}) + w_2] dP_2(w_2|\theta^*_f) - V(\underline{w}, \overline{r}) - \underline{w} - c_1 = 0.$$

Define $\Delta U(w) \equiv U(w) - U(\underline{w})$ and $\Delta V(w) \equiv V(w,\bar{r}) - V(\underline{w},\bar{r})$. Rewrite the two conditions as

$$H_{m1}(\theta_m^*) \equiv \delta \int_{\underline{w}}^{\overline{w}} \Delta U(w_2) dP_2(w_2|\theta_m^*) - (1-\delta)U(\underline{w}) - \int_{\underline{w}}^{\overline{w}} w_2 dP_2(w_2|\theta_m^*) - \underline{w} = 0,$$

and

$$H_{f1}(\theta_f^*) \equiv \delta \int_{\underline{w}}^{\overline{w}} \Delta V(w_2) dP_2(w_2|\theta_f^*) - (1-\delta)V(\underline{w}) - \int_{\underline{w}}^{\overline{w}} w_2 dP_2(w_2|\theta_m^*) - \underline{w} = 0.$$

To show that $\theta_m^* > \theta_f^*$ as $\delta \to 1$, it suffices to show that $H_{m1}(\theta) < H_{f1}(\theta)$, equivalently

$$\int_{\underline{w}}^{\overline{w}} \Delta U(w_2) dP_2(w_2|\theta) < \int_{\underline{w}}^{\overline{w}} \Delta V(w_2) dP_2(w_2|\theta).$$

(to be completed) ∎

Proposition (Investment Uncertainty Delays Marriage)**.** *As $\delta \to 1$, a man prefers to marry after the investment return is realized over to marry before the investment return is realized.*

Proof. Let $x \in \mathcal{X}$ and $y \in \mathcal{Y}$ denote men's and women's possibly multi-dimensional realized marriage types. Let $P \in \mathcal{P}$ and $Q \in \mathcal{Q}$ denote distributions of the types. Take a male of type P, i.e. a male who becomes type x with probability $dP(x)$. Suppose he is matched with a female of type Q, then

$$U(P) + V(Q) = \int\int S(x,y)\, dP(x) dQ(y)$$

By the stability condition, for any $x \in \mathcal{X}$ and $Q \in \mathcal{Q}$,

$$U(x) + V(Q) \geq \int S(x,y) dQ(y) \equiv S(x,Q).$$

Therefore,

$$\int [U(x) + V(Q)] dP(x) \geq \int S(x,Q)\, dP(x) = \int\int S(x,y) dP(x) dQ(y),$$

which equals $U(P) + V(Q)$. Therefore, $\int U(x)dP(x) + V(Q) \geq U(P) + V(Q)$, so $\int U(x)dP(x) \geq U(P)$. When not all x are matched with the same female type, the inequality holds strictly. When δ is sufficiently large,

$$\delta \int U(x) dP(x) \geq U(P).$$

∎

Proposition (Males' Marriage Age - Personal Income Relationship)**.** *Males' marriage age - personal income relationship is inverse-U shaped.*

Proof. Total mass $G_m(\theta_m^*)$ of ability $\theta_m < \theta_m^*$ men marries at age 1 and earns $\mathbb{E}w_{m1}^* = \underline{w}_m$. Ability $\theta_m \geq \theta_m^*$ men accept wage offer and marry at age 2 when $w_{m2} > w_{m2}^*(\theta_m)$. The total mass is

$$\int_{\theta_m^*}^1 [1 - P(w_{m2}^*(\theta)|\theta)]dG_m(\theta)$$

and they earn on average

$$\mathbb{E}w_{m2}^* = \int_{\theta_m^*}^1 \left[\int_{w_{m2}^*(\theta)}^{\overline{w}_m} w_{m2}dP_{m2}(w_{m2}|\theta)\right] dG_m(\theta).$$

Ability $\theta_m < \theta_m^*$ men reject age 2 wage offer and marry at age 3 when $w_{m2} < w_{m2}^*(\theta_m)$. The total mass is

$$\int_{\theta_m^*}^1 P_{m2}(w_{m2}^*(\theta)|\theta)dG_m(\theta),$$

and the average wage is

$$\mathbb{E}w_{m3}^* = \int_{\theta_m^*}^1 \left[\int_{\underline{w}_m}^{\overline{w}_m} w_{m3}dP_{m3}(w_{m3}|\theta)\right] P_{m2}(w_{m2}^*(\theta)|\theta)dG_m(\theta).$$

■

Proposition (Females' Marriage Age - Personal Income Relationship)**.** *Females' marriage age - personal income relationship is positive when the fitness loss is significant ($1 - \phi_2 << \phi_2 - \phi_3$), and is inverse-U shaped when the fitness loss is not significant ($1 - \phi_2 \approx \phi_2 - \phi_3$).*

Proof. Total mass $G_f(\theta_f^*)$ of ability $\theta_f < \theta_f^*$ women marries at age 1 and earns $\mathbb{E}w_{f1}^* = \underline{w}_f$. Ability $\theta_f \geq \theta_f^*$ women accept wage offer and marry at age 2 when $w_{f2} > w_{f2}^*(\theta_f)$. The total mass is

$$\int_{\theta_f^*}^1 [1 - P(w_{f2}^*(\theta)|\theta)]dG_f(\theta)$$

and they earn on average

$$\mathbb{E}w_{f2}^* = \int_{\theta_f^*}^1 \left[\int_{w_{f2}^*(\theta)}^{\overline{w}_f} w_{f2}dP_{f2}(w_{f2}|\theta)\right] dG_f(\theta).$$

Ability $\theta_f < \theta_f^*$ women reject age 2 wage offer and marry at age 3 when $w_{f2} < w_{f2}^*(\theta_f)$. The total mass is

$$\int_{\theta_f^*}^{1} P_{f2}(w_{f2}^*(\theta)|\theta)dG_f(\theta),$$

and the average wage is

$$\mathbb{E}w_{f3}^* = \int_{\theta_f^*}^{1}\left[\int_{\underline{w}_f}^{\overline{w}_f} w_{f3}dP_{f3}(w_{f3}|\theta)\right]P_{f2}(w_{f2}^*(\theta)|\theta)dG_m(\theta).$$

■

C Empirical Analyses

The data for the United States are obtained and imputed from IPUMS-USA (Ruggles et al., 2010). The details for the four years are as follows. 1960: 1% sample of 1960 US Census. Age at first marriage (*agemarr*) and total personal income (*inctot*) are directly asked on the form. Those with total income N/A are dropped. *inctot* is bottom-coded and top-coded. 1980: 1% sample of 1980 US Census. Age at first marriage (*agemarr*) and total personal income (*inctot*) are directly asked on the form. Those with total income N/A are dropped. *inctot* is bottom-coded and top-coded. 2008 and 2012: 1% sample of 2008 and 2012 American Community Surveys. Age at first marriage is calculated for those who marry once by age and year in the current marital status (*yrmarr*). Those who marry twice or thrice (no one in the sample has married more than thrice) constitute about 16% of the sample and are dropped because age at first marriage cannot be computed. Those who have total income labeled N/A are dropped. The patterns hold for white and black subgroups (Figures 18-25).

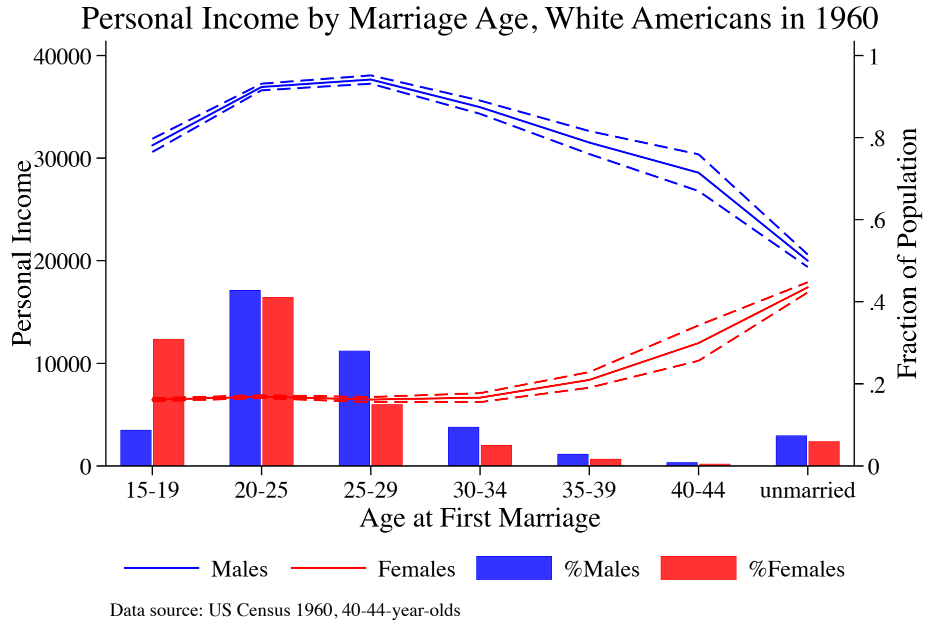

Figure 18: Marriage age distributions and marriage age-personal income relationships, 40-44-year-old white Americans in 1960.

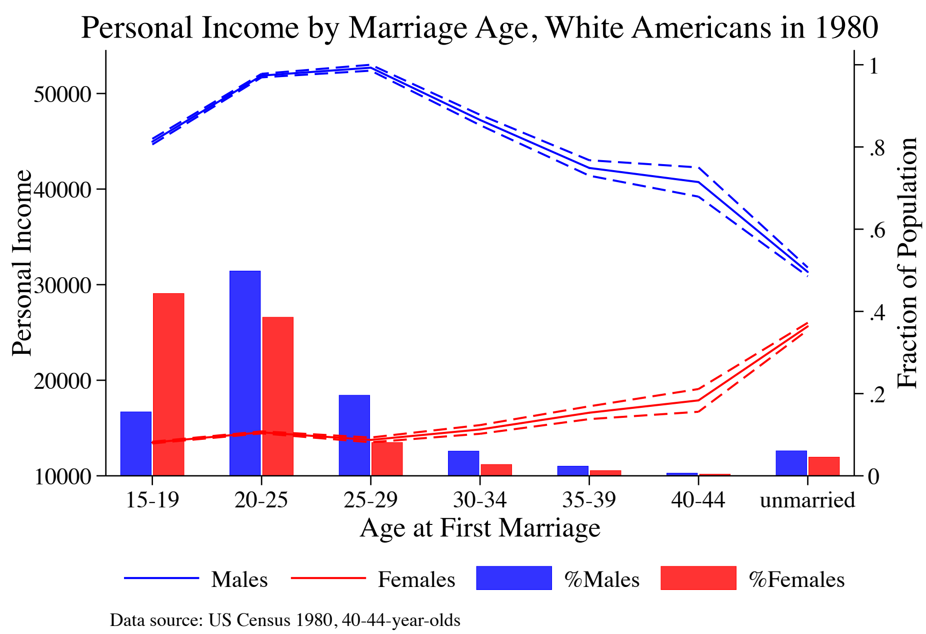

Figure 19: Marriage age distributions and marriage age-personal income relationships, 40-44-year-old white Americans in 1980.

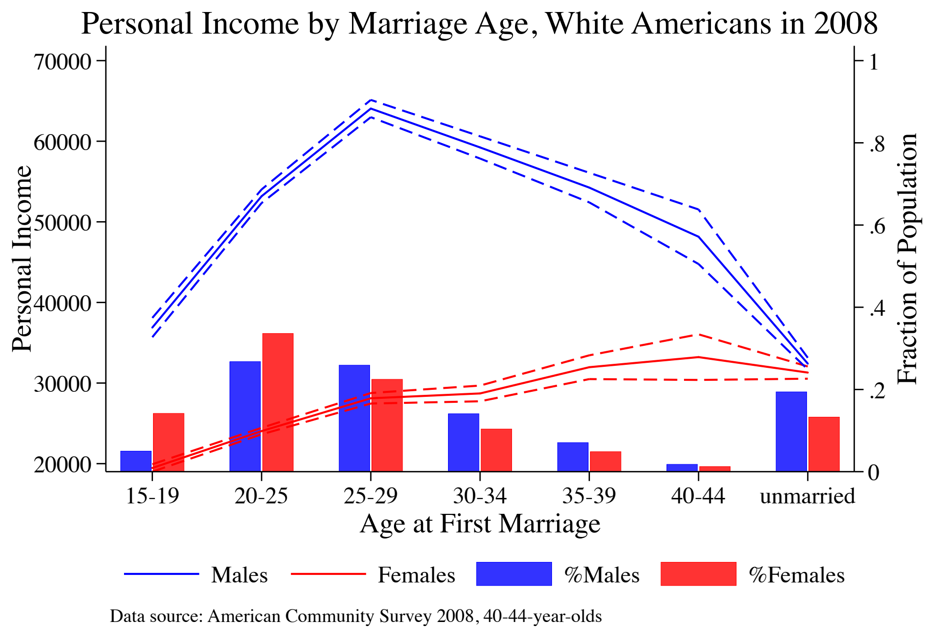

Figure 20: Marriage age distributions and marriage age-personal income relationships, 40-44-year-old white Americans in 2008.

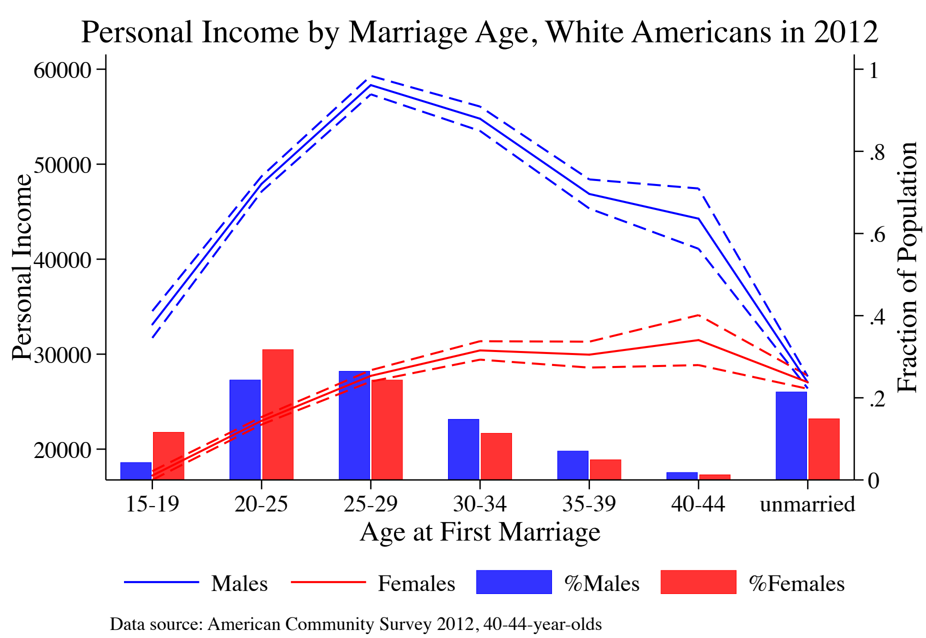

Figure 21: Marriage age distributions and marriage age-personal income relationships, 40-44-year-old white Americans in 2012.

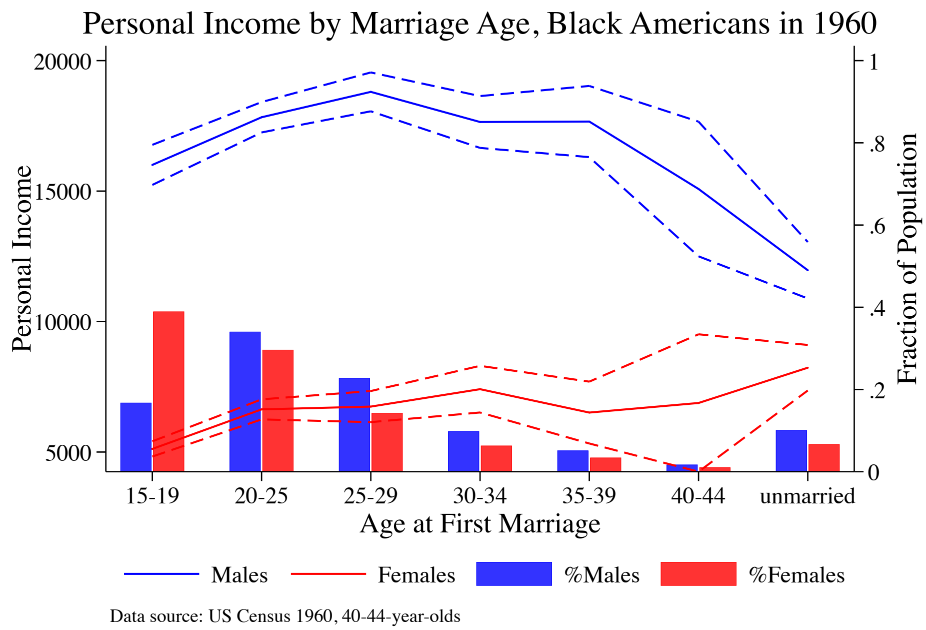

Figure 22: Marriage age distributions and marriage age-personal income relationships, 40-44-year-old black Americans in 1960.

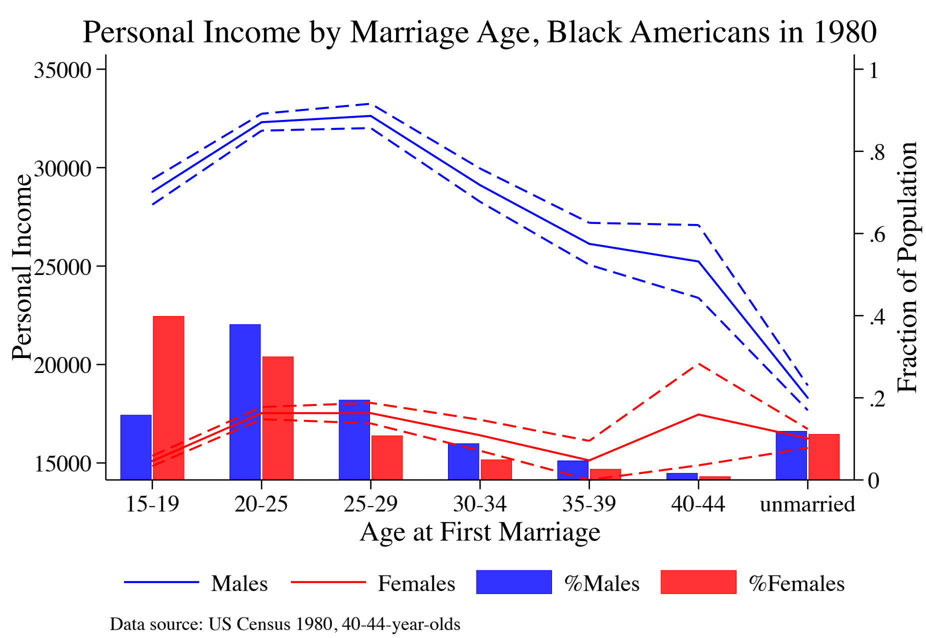

Figure 23: Marriage age distributions and marriage age-personal income relationships, 40-44-year-old black Americans in 1980.

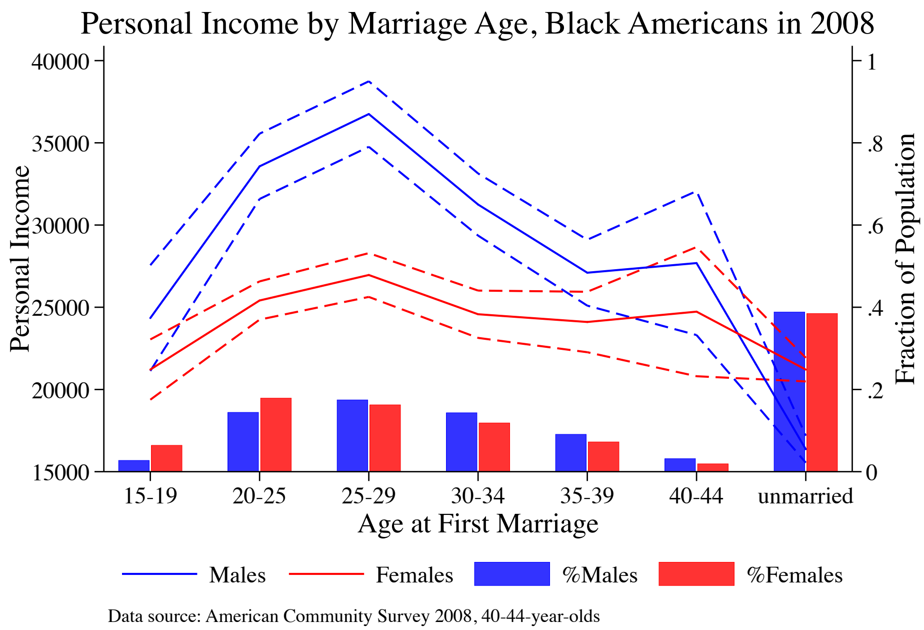

Figure 24: Marriage age distributions and marriage age-personal income relationships, 40-44-year-old black Americans in 2008.

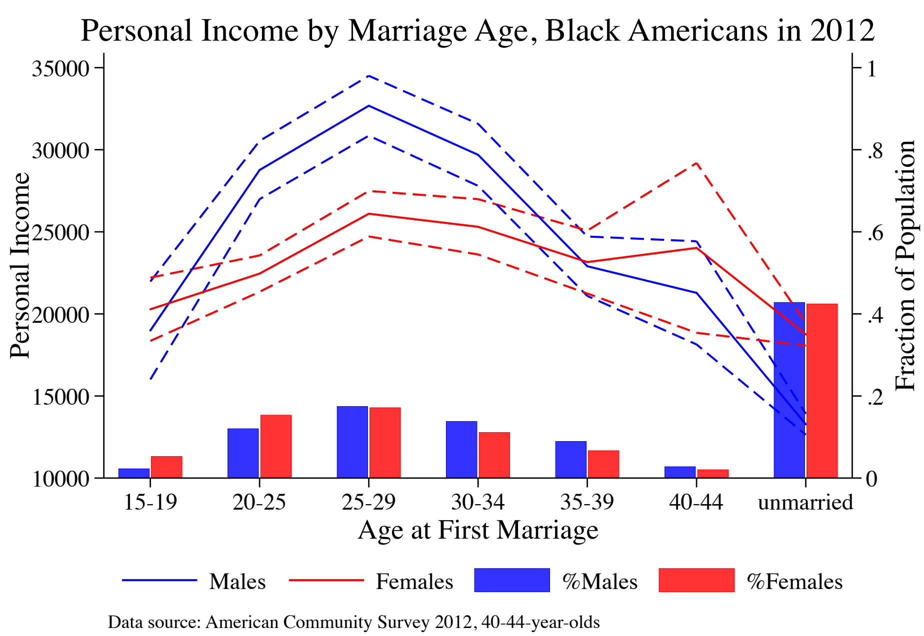

Figure 25: Marriage age distributions and marriage age-personal income relationships, 40-44-year-old black Americans in 2012.