A Macrosimulation Model of the Effect of Fertility on Economic Growth: Evidence from Nigeria

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Abstract

In this study, we investigate the economic effects of a decline in fertility. We use data from Nigeria and construct a macrosimulation model based on work by Ashraf et al. (2013), in which the evolution of key economic and demographic outcomes are observed under a "baseline" scenario, where fertility falls slowly over time, and are then compared to alternative scenarios in which fertility declines more rapidly. We extend the Ashraf et al. (2013) model to incorporate five channels that have been previously ignored: the effect of fertility on savings; a feedback from education to fertility; the effect of a more realistic two-sector model; the effect of fertility on health; and the effect of market imperfections. Compared to the Ashraf et al. (2013) results, adding these channels more than doubles the effect of a fertility decline on income per capita after 20 years and almost triples the effect after 50 years.

I. *Introduction*

The relationship between population change, fertility, and economic growth in Sub-Saharan Africa has received much attention in recent years due to changes in the demographic outlook in this region of the world. Over the next 50 years, many countries in Sub-Saharan Africa will position themselves to experience a demographic transition that will be characterized by an evolution in the population health state from one of high mortality, low child survival, and high fertility to one of low mortality, increased life expectancy, and low birth rates (Angeles, 2010; Caldwell, Orubuloye, & Caldwell, 1992; Thomas & Muvandi, 1994). These resulting declines in mortality and fertility in turn will contribute to a change in the population age structure and may lead to a subsequent decline in the proportion of non-working-age dependents to working age individuals – the dependency ratio – as large youth cohorts age into their productive working years (Lee, 2003).

A reduction in fertility, combined with a decline in the dependency ratio, creates the potential for a "demographic dividend" in which a larger labor force opens a window of opportunity for economic growth (Bloom, Canning, Fink, & Finlay, 2007, 2010; Bloom, Canning, & Sevilla, 2003). In particular, a decreasing fertility rate may allow for more resources to be allocated to each child, and each child can further benefit from these additional health, education, and capital investments (Becker, Duesenberry, & Okun, 1960; Becker & Lewis, 1973; Bloom et al., 2010). Having fewer children also implies that parents, particularly mothers, can continue to invest in their own human capital by increasing their educational attainment, which in turn provides them with the additional training and skills needed to participate in income-generating labor market activities (Bloom, Canning, Fink, & Finlay, 2009). At the country level, as more people in the workforce become more productive, more resources can be directed towards investments that aim to promote economic growth and development.

Many studies have documented the recent demographic trends in Sub-Saharan Africa and have explored the potential benefits of the demographic transition for economic growth in the region (Bloom, Canning, Fink, et al., 2007; Bloom et al., 2003). However, few studies have attempted to quantitatively assess the extent to which the rate of fertility decline may play a role in accelerating its economic development outcomes. The lack of evidence is not surprising, given that the interactions between changes in population, fertility, and economic growth, both at the individual and aggregate levels, are complex and are poorly observed in the empirical data. Moreover, since a country's fertility changes endogenously and dynamically with other key determinants of economic growth, for example, institution quality, education, or governance, it becomes difficult to disentangle the effects of population changes from these other factors (Becker et al., 1960; Becker, Glaeser, & Murphy, 1999; Brander & Dowrick, 1994; Galor & Weil, 2000). While previous efforts to estimate the aggregate economic effects of fertility using sophisticated econometric methods (e.g. "growth regression" models à la Barro (1991) or Mankiw, Romer, & Weil (1992) or instrumental variable approaches à la Acemoglu & Johnson (2007) or Bloom et al. (2009)), have made some progress, many of these reduced-form approaches are susceptible to the range of identification problems, criticisms, and pitfalls that are associated with endogenous responses of fertility to other determinants of growth and to economic growth itself. In contrast, studies by Joshi & Schultz (2007), Miller (2010), and others have exploited unique data sources to investigate the links between fertility and measures of individual and household well-being. While these micro level data approaches have allowed for cleaner identification strategies, they are most limited in their ability to estimate the total impact of a reduction in fertility on aggregate economic growth, given that many of the effects of such a reduction work through externalities and other general equilibrium responses (Acemoglu, 2010).

An alternative approach to estimating the effect of fertility changes on economic outcomes has been to structurally define, parameterize, and interact key economic and demographic macrorelationships of interest into a comprehensive simulation model. First developed by Coale & Hoover (1958) in their assessment of Indian fertility scenarios, the simulation approach has been widely used to effectively identify aggregate effects of changes in fertility and population growth in general equilibrium settings and over long time horizons. Through sensitivity checks and comparative statics analyses, these models have also helped to identify the relative importance of individual channels and mechanisms, such as savings (Enke, 1971), capital accumulation (Ashraf, Weil, & Wilde, 2013; Coale & Hoover, 1958), and shifting dependency ratios (Bloom & Canning, 2008), through which changes in fertility affect economic outcomes.

In this study, we employ a structural approach to investigating the economic effects of a decline in fertility and subsequent demographic change in a developing country context where initial fertility is high. We construct an aggregate demographic-economic macrosimulation model in which we characterize the evolutions of economic growth and development outcomes under a "baseline" scenario, in which fertility falls slowly, and then compare these outcomes to outcomes under alternative scenarios in which fertility exogenously declines more rapidly over time. Our model builds on the work by Coale & Hoover (1958) and on a more recent macrosimulation study by Ashraf, Weil, & Wilde (2013), the latter of which itself also heavily draws on the Coale-Hoover and Enke models. In developing our model framework, we particularly follow the example of Ashraf, Weil, & Wilde (2013), hereafter referred to as AWW 2013, and we maintain much of the core AWW 2013 model structure when considering dynamic evolutions of population growth and changes in the population age structure. We also follow AWW 2013 by employing a traditional Solovian economic growth framework to model demographic responses in physical and human capital accumulation and natural resource use (land) over time.

However, we significantly depart from AWW 2013 and previous related work in the following five key ways:

1. ***Endogenous Fertility Responses to Education:*** A significant drawback of previous simulation approaches is that few macrosimulation models, including AWW 2013, successfully endogenize the evolution of fertility in response to changes in income or other economic growth determinants over time. Consequently, these models have been criticized for being overly simplistic and for underestimating the true aggregate effect of an exogenous change in fertility (Sanderson, 1980). The rationale for endogenizing fertility is further supported by trends that show a significant decline in fertility in the developing world over the last few decades, even in Sub-Saharan Africa where access to and use of family planning and reproductive health services is poor (Bongaarts, Cleland, Townsend, Bertrand, & Das Gupta, 2012). This empirical evidence is suggestive of a potential feedback mechanism which AWW 2013 and others previous models of fertility do not capture. We address this shortcoming by imposing a channel through education, which allows us to observe the ripple effects of an initial fertility shock dynamically across both demographic and economic systems.
2. ***Endogenous and Dynamic Savings:*** Little is known about the relationship between fertility and savings in the literature, and even less is known about the mechanisms through which savings behavior would endogenously affect fertility and other outcomes of interest in a general equilibrium setting. We aim to fill these gaps in the evidence by introducing a more complex and realistic savings function and by incorporating feedback channels from fertility to savings and back. In particular, we relax the commonly used simplifying assumption of fixed and low

aggregate savings rates when depicting capital flows, and we instead model savings behavior to be dynamically dependent on: 1) past savings; 2) wages and incomes; and 3) shifts in the population age structure, specifically the old age dependency ratio.

3. ***Inclusion of Health as Human Capital:*** In contrast to previous studies, we extend our analysis of human capital by assessing the health effects, in addition to the education effects, of a change in fertility. Previous studies have shown fertility and health to be gross substitutes, and fewer health expenditures and investments in nutrition, particularly in children, are made as family size increases (Rosenzweig & Wolpin, 1986). For our analysis, we choose adult height, which is considered to be an effective indicator for early nutrition and lifetime health status, as our proxy for health as human capital (Schultz, 2002; Strauss & Thomas, 1998).
4. ***A Two Sector Model with Labor Market Inefficiencies:*** We depart from the AWW 2013 assumption of a one-sector model in which the labor market clears as workers are paid their marginal product, the efficient wage. To do so, we first add a second sector and then differ sector productivities in the tradition of Lewis (1968), which allows us to infer effects of changes in population on labor force movements across sector markets. The addition of an inefficient sector, such as agriculture, frees the model from the AWW 2013 over-reliance on market clearing and allows for the absorption of excess labor in which workers in the inefficient sector are paid a subsistence wage that is less than their marginal product. This complexity in the analysis better reflects the excess labor supply and resource constrained conditions that are often observed in less efficient sectors in developing countries.
5. ***Wage Distortions and Other Inefficiencies:*** As part of a more nuanced analysis, we add wage distortions and sectorial inefficiencies, including a tax on capital that serves as a risk premium for capital investment, and we trace the effects of fertility on market allocations under these distortions. These additional wrinkles in our model separate us even further from the more traditional assumptions employed by AWW 2013 and related work, many of which place too much faith in the functionality and efficiency of labor and capital markets in low- and middle-income settings.

By specifying our model assumptions through a structural framework, our simulation approach allows us to explicitly identify and analyze potential channels through with fertility declines impact key health and economic indicators. Our simulation equations are calibrated and parameterized using estimates from well-identified microeconomic evidence, field studies, and classic economic and demographic theory, which together enable us to ground our choices of parameters in the micro-founded evidence, as opposed to solely basing them on theory. Our model can also be adapted using country-specific population data, which allows us to compare and contrast derived model predictions from several countries with varying fertility paths.

Comparing our results to AWW 2013, we find that adding these additional channels almost triples the effect of fertility decline on income per capita. For example, AWW 2013 find that moving from the medium to the low UN fertility variant would increase income per capita by 11.7 percent at a time horizon of 50 years, and increase by 5.6 percent at a horizon of 20 years. Once we add these additional channels, income is 30.9 percent higher after 50 years ($12,131 vs. $9,270) and 14.2 percent higher after 20 years ($3,921 vs. $3,434). We conclude that these previously ignored channels are not only important, but perhaps are even more important than the more traditional channels considered in the literature.

The remaining sections are divided as follows. In Section II, we present the demographic section of our model and outline the key population effects. Section III offers a description of the economic model in which the demographic simulations are included. Section IV presents simulation results of our model using 2010 country demographic and economic data from Nigeria and compares our results from those obtained by AWW 2013, who also use Nigeria data. In Section V, we discuss the key findings and insights of our model based on the simulations and sensitivity analyses. Section VI concludes.

II. *The Demographic Model*

Population

The demographic part of our model takes age-specific mortality and fertility rates as inputs to project the population over time. In practice, population is divided into 5-year age groups, and each time period t in our model corresponds to five years accordingly. Our population model follows AWW 2013, in which the population at time t in age group i[1], $Pop_{i,t}$, is given by

$$Pop_{i,t} = (1 - d_i)Pop_{i-1,t-1}$$

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[1] In this paper, the cohort index i refers to each 5-year age cohort and is defined such that the first index value $i = 15$ refers to the 15-19 age group, followed by $i = 20$, which refers to the 20-24 age group, and so on.

where d_i refers to the age-specific mortality rate for age group i. Since our measure of time is divided into coarse five-year intervals, we are less able to measure the exact number of children that are born in a specific year. We instead estimate the number of children aged 0 to 5 that are born within the five-year time interval t and who are alive at the time of data collection, $Pop_{0,t}$, which is a function of the age-specific fertility rate for cohort i over the five-year interval t, $f_{i,t}$, and the five-year age distribution of the female population at t, $Popf_{i,t}$, and we adjust our estimates of child survival with a mortality term $1 - d_0$, where d_0 is an exogenous constant that reflects the average mortality rate from birth up to age 5, accounting for higher rates of neonatal, postneonatal, and infant mortality:

$$Pop_{0,t} = (1 - d_0) \sum_{i=15}^{45} f_{i,t} \, Popf_{i,t}$$

For simplicity, our demographic projections are calculated over a closed, female-only population. Considering a population of both males and females, however, would not qualitatively alter the results of the model so long as the sex ratio at birth is constant over time.

III. The Economic Model

Inputs

Labor Supply: We assume that children enter the labor force at 15 and workers leave the labor force at 65. For each gender, we calculate the total labor supply contribution at time t as a function of the labor force participation rate at each age group i and time period t, $partm_{i,t}$ for males and $partf_{i,t}$ for females, and the size of the gender-specific population of age i at time t. Specifically, we employ gender- and age-specific labor force participation rates to construct total labor force participation rates by age, using the fraction of males and females in each age group as population weights. Total male labor supply at time t is determined by

$$LS_{male.t} = \sum_{i=15}^{60} partm_{i,t} Popm_{i,t}$$

where $Popm_{i,t}$ is the projected fraction of the male population in age group i at time t. In following the AWW 2013 model framework, we modify the age-specific female labor force participation rate at t to reflect the effect of a decrease in total female labor supply due to increases in time devoted to childrearing, namely

$$partf_{i,t} = partf_{i,0} + \pi(f_{i,0} - f_{i,t})$$

where $partf_{i,0}$ is the baseline female labor force participation rate for age group i, π (which is initially set to zero) is a measure that reflects the effect of fertility on female labor supply, and $f_{i,t}$ captures the age-specific fertility rate for cohort i at time t. Through this equation, we can predict the expected increase in female labor force participation rates as age-specific fertility rates decline (i.e. as the difference between baseline age-specific fertility rate $f_{i,0}$ and $f_{i,t}$ grows larger and becomes increasingly more positive), particularly among the younger female cohorts. Total female labor supply at time t is therefore given by

$$LS_{female.t} = \sum_{i=15}^{60} partf_{i,t} Popf_{i,t}$$

In our specification, we assume that there is no selection into labor force participation by either education or health apart from any modeled effects of fertility on female labor supply - cohort participation rates for both men and women are exogenously determined and are considered to be independent of average education and health factors.

Education by Cohort: Our treatment of schooling and educational experience follows the standard literature. In our model, we assume that a given cohort's educational attainment (quantified in years of schooling) is entirely amassed before age 20, after which the level of schooling for that cohort is held constant for the remainder of that cohort's lifetime. We also expect that lower fertility will raise the average level of schooling. Models of the fertility transition stress the movement of households along a "quality-quantity" frontier in which investment per child in health and education rises as the number of children falls (Becker, 1981; Becker & Lewis, 1973; Lam, 2003). It does not follow from this observation, however, that the change in schooling that would result from an exogenous change in fertility is the same as the change that would accompany declining fertility when both measures are evolving endogenously. To better capture these effects, we depart from the exogeneity assumption of fertility by introducing a feedback mechanism in our fertility-to-education equations.

To estimate the effects of fertility on a given cohort's educational attainment at time t, we assume that the cohort's average years of schooling amassed by age 20, denoted $E_{20,t}$, is given by:

$$E_{20,t} = E_{20,0}\left[1 + \theta_E(f_{i,-20} - f_{i,t-20})\right]$$

where $E_{20,0}$ is an exogenous measure of the average number of years of schooling acquired for the baseline cohort of 20-year olds at time $t = 0$, $f_{i,t-20}$ and $f_{i,-20}$ represent the age-specific fertility rates at time t and time 0, lagged respectively by 20 years, and θ_E is an exogenous constant that captures the direct effect of fertility on childhood education. In particular, this specification implies that for every one-birth reduction in the total fertility rate at t, education rises by θ_E percent above education at baseline where fertility does not change. Few studies have sought to directly estimate the returns to education from changes in fertility, with the best estimates coming from Schultz (2004) who finds a 2.5 percent annual return to primary education across a sample of six African countries. The average years of schooling past age 20 for a given cohort i, $E_{i,t}$, remain constant. In particular:

$$E_{i,t} = E_{i-1,t-1}, \qquad i > 20$$

Finally, we calculate the average years of schooling of the workforce at time t, E_t, by taking an average of cohort-specific schooling estimates at t, $E_{i,t}$, weighted by the fraction of the total labor force that is employed in each age cohort i, which we compute using the baseline labor force participation rates for each cohort, $part_{i,0}$ and the population in each age group i at time t, $Pop_{i,t}$ as follows

$$E_t = \sum_{i=20}^{65} \left[\frac{part_{i,0} \cdot Pop_{i,t}}{\sum_{i=20}^{65} \left(part_{i,0} \cdot Pop_{i,t} \right)} \right] E_{i,t}$$

In our simulation model, we calculate average years of schooling separately for each gender, namely

$$E_t^{male} = \left[\frac{partm_{i,0} \cdot Popm_{i,t}}{\sum_{i=20}^{65} \left(partm_{i,0} \cdot Popm_{i,t} \right)} \right] E_{i,t}^{male}$$

$$E_t^{female} = \left[\frac{partf_{i,0} \cdot Popf_{i,t}}{\sum_{i=20}^{65} \left(partf_{i,0} \cdot Popf_{i,t} \right)} \right] E_{i,t}^{female}$$

and then combine the gender-specific estimates in a weighted average to estimate the average years of schooling for the entire workforce at time t

$$E_t = \frac{E_t^{male} \cdot Popm_t + E_t^{female} \cdot Popf_t}{Pop_t}$$

Feedback from Fertility to Education: In contrast to AWW 2013, we endogenize the evolution of fertility over time by introducing a feedback mechanism from education to fertility in a log-linear form as follows

$$\ln f_{i,t} = \ln f_{i,0} + \psi\left(E_{i,0} - E_{i,t}\right)$$

In this manner, each additional year of schooling for cohort i, from a base of $E_{i,0}$, increases the base log fertility of the cohort, $\ln f_{i,0}$, by ψ, which is an exogenous parameter that captures the direct effect of education on fertility.

Adult Height by Cohort: Our treatment of health, as proxied by adult height, parallels our model assumptions on educational attainment and schooling in the previous section. We assume that a given cohort's average height is attained by age 20, after which the average height for that cohort is held constant for the remainder of that cohort's lifetime. In following from our previous argument for a quality-quantity tradeoff in both education and health, we expect that lower fertility contribute positively to average adult height in the cohort. This effect is reflected in the additional investments that households with fewer children are able to make to improve child health and nutrition, which in turn reduce stunting and positively contribute to growth and development into adulthood.

To estimate the effects of fertility on a given cohort's height at time t, we assume that the cohort's average height amassed by age 20, denoted $H_{20,t}$, is given by:

$$H_{20,t} = H_{20,0}\left[1 + \theta_H\left(f_{i,-20} - f_{i,t-20}\right)\right]$$

where $H_{20,0}$ is an exogenous measure of the average height of the baseline cohort of 20-year olds at time $t = 0$, $f_{i,t-20}$ and $f_{i,-20}$ again represent the age-specific fertility rates at time t and time 0, lagged respectively by 20 years, and θ_H is an exogenous constant that captures the direct effect of fertility on adult height. In a similar fashion to our education specification, this specification implies that for every one-birth reduction in the total fertility rate at t, adult height increases by θ_H percent above height at baseline where fertility does not change. The average height past age 20 for a given cohort i, $H_{i,t}$, remain constant. In particular:

$$H_{i,t} = H_{i-1,t-1}, \qquad i > 20$$

Finally, we calculate, as a proxy for average workforce health, the average adult height of the workforce at time t, H_t, by taking an average of cohort-specific height estimates at t, $H_{i,t}$, weighted by the fraction of the total labor force that is employed in each age cohort i, which we compute using the baseline labor force participation rates for each cohort, $part_{i,0}$ and the population in each age group i at time t, $Pop_{i,t}$ as follows

$$H_t = \sum_{i=20}^{65} \left[\frac{part_{i,0} \cdot Pop_{i,t}}{\sum_{i=20}^{65} \left(part_{i,0} \cdot Pop_{i,t} \right)} \right] H_{i,t}$$

As was the case with our education estimates, we simulate average years of schooling separately for each gender, namely

$$H_t^{male} = \left[\frac{partm_{i,0} \cdot Popm_{i,t}}{\sum_{i=20}^{65} \left(partm_{i,0} \cdot Popm_{i,t} \right)} \right] H_{i,t}^{male}$$

$$H_t^{female} = \left[\frac{partf_{i,0} \cdot Popf_{i,t}}{\sum_{i=20}^{65} \left(partf_{i,0} \cdot Popf_{i,t} \right)} \right] H_{i,t}^{female}$$

and then combine the gender-specific estimates in a weighted average to estimate the average height for the entire workforce at time t

$$H_t = \frac{H_t^{male} \cdot Popm_t + H_t^{female} \cdot Popf_t}{Pop_t}$$

Production

We extend the base model of AWW 2013 by considering a Lewis development economy with two sectors, manufacturing and agriculture, which share the total labor supply across sectors to produce distinct commodities. Aggregate production in manufacturing at time t is given by a standard Cobb-Douglas production function, with physical capital K_t, labor allocated to manufacturing LM_t, average years of schooling in the workforce (as a proxy for education) E_t, and average height of the workforce (as a proxy for health) H_t as factor inputs such that aggregate manufacturing and services output at t, YM_t, is given by

$$YM_t = AM_t K_t^{\alpha} LM_t^{1-\alpha} e^{\gamma E_t + \lambda H_t}$$

where AM_t is the total factor productivity of manufacturing at t. Estimates for schooling E_t and health H_t are fed into the economic model from our demographic simulations as described in the previous section.

In a similar fashion, aggregate production in agriculture at t is also modeled by a Cobb-Douglas production function, with available agricultural land X (which is used as a placeholder variable for all fixed factors of production) and labor allocated to agriculture LA_t as factor inputs such that aggregate agricultural output at t, YA_t, is given by

$$YA_t = AA_t X^{\beta} LA_t^{1-\beta}$$

where AA_t is the total factor productivity of agriculture at t.

Capital Accumulation and Savings

In our base case model, we extend the standard Solovian framework for capital accumulation by assuming that capital stock in the period $t+1$, K_{t+1}, evolves over time according to the equation

$$K_{t+1} = s_t Y_t + (1-\delta) K_t$$

where s_t is the savings rate at time t and δ is the rate of depreciation of capital that is assigned a standard value of 7 percent (Schmitt-Grohe & Uribe, 2006). In contrast to AWW 2013, we depart from the simplifying assumption of a constant savings rate and follow the example of Bloom, Canning, Mansfield, & Moore (2007), in which the evolution of the savings rate is defined by

$$s_t = \frac{S_t}{Y_t} = \phi_0 + \phi_1 s_{t-1} + \phi_2 w_t + \phi_3 w_t^2 + \phi_4 \frac{Old_t}{WA_t}$$

Here, $s_{t-1} = \frac{S_{t-1}}{Y_{t-1}}$ is the savings rate in the previous time period $t-1$, w_t is the annual aggregate wage at time t, which is defined as a fixed proportion of per-capita income in the same period (i.e. $w_t = (1-a) y_t$ for some fixed a), and $\frac{Old_t}{WA_t}$ captures the old-age dependency ratio, the ratio of old-age dependents to the working age population, at t. We assume that savings begins in a steady state equilibrium at time $t=0$, and we calibrate the constant term ϕ_0 to fit the baseline steady state savings, wage, and dependency ratio conditions. Further details on the derivation and interpretation of the savings equation can be found in Appendix 1.

We derive the standard equation for capital flows from our Cobb-Douglas production function by setting the marginal product of capital equal at time t to its real user cost plus an endogenously determined tax, which serves as a risk premium for investment in capital. Rearranging terms and solving for capital yields

$$K_t = \frac{\alpha YM_{t-1}}{r + tax}$$

or in log terms

$$\log K_t = \log YM_{t-1} + \log \alpha - \log(r + tax)$$

Wages and Worker Allocation across Sectors

Our model specification requires that manufacturing and agricultural wages, which endogenously adjust within their respective labor markets, will in turn determine equilibrium labor supply allocations across the two sectors. Total labor supply L_t is shared across manufacturing and agriculture such that

$$L_t = LM_t + LA_t$$

Wages in the manufacturing sector at time t, wM_t, are set to be equal to the marginal product of labor in manufacturing for an additional worker with average levels of education and health, or in log terms

$$\log wM_t = \log\left[(1-\alpha)\frac{YM_t}{LM_t}\right]$$

In following Lewis (1954)'s dual-sector model of surplus labor, we assume that the agricultural sector is less developed and more labor intensive with little to no capital endowment, thereby resulting in subsistence agriculture in which wages at t are determined by the average product, or in log terms:

$$\log wA_t = \log \frac{YA_t}{LA_t}$$

This wage condition captures a common observation in low-income countries in which family members share incomes and communities pool and divide resources as a means of insuring against risk. Given that the stock of land for agriculture is fixed and agriculture is labor intensive, there may come a point when the fixed stock of land is fully utilized by a threshold level of workers, after which any additional labor will not yield an increase in output, yet there may exist an excess supply of workers employed in the agricultural sector that contribute very little, if anything, to total agricultural output but who still receive a wage equal to the average product. These excess workers, whose marginal productivity is zero, reflect the inherent inefficiency in the agricultural labor market.

Given the wage differential between the two sectors, the surplus laborers whose marginal productivities are zero will have an incentive to move from the agricultural sector to the urban sector. This labor migration will neither result in decrease in agricultural output nor change the relative scarcity of agricultural and manufacturing goods. However, since wage is determined at the average and not on the margin, there will be fewer workers remaining in the agricultural sector as surplus laborers migrate to the manufacturing sector, resulting in an increase of the rural wage. In equilibrium, workers will migrate between sectors and wages will adjust such that

$$\log wM_t - \log b = \log wA_t$$

where b is an endogenous constant that is set to the baseline differential in sector wages and that captures additional non-wage costs of labor in the formal sector. If we replace manufacturing and agricultural wages with their respective wage-output equilibrium conditions, substitute manufacturing and agricultural output with their respective production functions, and set health $H_t = 0$ for the time being, we obtain:

$$Z_t LM_t^{-\alpha} = (L_t - LM_t)^{-\beta}$$

where

$$Z_t = \frac{(1-\alpha) \cdot AM_t K_t^{\alpha} e^{\gamma E_t}}{b \cdot AA_t X^{\beta}}$$

For $\alpha = \frac{1}{3}$ and $\beta = \frac{1}{6}$, we can explicitly solve for LM_t as

$$LM_t = \frac{1}{2}\left(Z_t^3 \sqrt{Z_t^6 + 4L_t} - Z_t^6\right)$$

We can verify that $0 \leq LM_t \leq L_t$, and we calibrate the value of b so that the initial manufacturing labor LM_t matches the data, and then we fix b to that value in all subsequent simulations.

Calibration

Table 1 describes each parameter that was used in the model, the parameter value that was used to calibrate the model, and the source from which these values were obtained.

Table 1: Parameter Descriptions

Variable Name	Value	Description	Source(s)
π	0.02	Effect of fertility on female labor supply	Ashraf et al. (2013)
θ_E	0.2	Effect of fertility on childhood education	Joshi & Schultz (2007); Rosenzweig &

			Wolpin (1980)
ψ	-0.15	Effect of education on fertility	Osili & Long (2008)
θ_H	XX	Effect of fertility on adult height	Finlay and Canning (XXXX)
α	0.33	Capital share of output	Hall & Jones (1999)
β	0.167	Land share of output	Kawagoe et al. (1985); Williamson (1998, 2002)
γ	0.1	Education effect on output	Banerjee & Duflo (2005); Oyelere (2010); Psacharopoulos (1994); Psacharopoulos & Patrinos (2004)
λ	0.08	Effect of health on output	Schultz (2002, 2005)
δ	0.07	Depreciation rate of capital	Schmitt-Grohe & Uribe (2006)
ϕ_1	0.758	Effect of lagged savings on current savings	Bloom et al. (2007)
ϕ_2	0.133	Effect of aggregate wages on savings	Bloom et al. (2007)
ϕ_3	-0.006	Effect of squared aggregate wages on savings	Bloom et al. (2007)
ϕ_4	-0.209	Ratio of old to working age population on current savings	Bloom et al. (2007)
r	0.03 – 0.09	Real user cost of capital	Banerjee & Duflo (2005)

Estimates of key parameters that illustrate the direct relationships between fertility and other factors are drawn from several sources. To identify the direct labor market cost of an additional child, π, we follow the parameterization approach described in Ashraf et al. (2013), who interpolate Filipino data from Tiefenthaler (1997) and find that lifetime female labor supply declines by an estimated 2 percent for each additional birth. Parameter estimates for the direct effect of fertility on educational attainment, θ_E, are derived from Rosenzweig & Wolpin (1980) and Joshi & Schultz (2007), who draw upon quasi-experimental evidence from a family planning intervention in Matlab, Bangladesh and find that a 15 percent reduction in total fertility, which is equivalent to having one fewer birth, increases the number of years of schooling in children by 20 percent. When considering the endogenous response of fertility to changes in education, we parameterize our coefficient ψ, the direct effect of education on fertility, using results from Osili & Long (2008), who examined the causal impact of a universal primary education program in Nigeria using OLS and instrumental variable approaches and found that each additional year of female schooling reduced fertility by 0.26 to 0.48 births, which constitutes a 11 to 19 percent reduction. We obtain our parameter value of 15 percent for ψ by averaging across the various Osili-Long estimates. Finally, we parameterize our estimate for θ_H, which captures the impact of fertility on health (as proxied by adult height), by drawing upon recent work by Finlay and Canning (XXXX), who find that each additional birth reduces height by XX percent.

Standard estimated values for production factor shares are extracted from the classic economic growth literature, including the capital share of output estimate of $\alpha = \frac{1}{3} = 0.33$ (Hall & Jones, 1999), the land share of output[2] estimate of $\beta = \frac{1}{6} = 0.167$ (Kawagoe, Hayami, & Ruttan, 1985; Williamson, 1998, 2002), parameter $\gamma = 0.1$, which is an approximate average of the estimated returns to education (Banerjee & Duflo, 2005; Oyelere, 2010; Psacharopoulos, 1994; Psacharopoulos & Patrinos, 2004), and the health parameter $\lambda = 0.08$, which is proxied through estimated wage returns to adult height (Schultz, 2002, 2005). In modeling agricultural output as a function of land and labor, we recognize that our agricultural production equation is a simplification of the Kawagoe model since we do not consider the significant contributions of other reproducible factors to output, including livestock, fertilizer, and machinery.

Data Sources

Our simulation analysis is focused on considering interventions that alter the path of fertility from what would otherwise occur along a given baseline. In following AWW 2013, we use the current fertility and mortality schedules to construct a stable population, and in the baseline scenario we assume that fertility and mortality will be constant going forward. Our model may be tailored to consider different baseline and intervention scenarios. For most of this study, we examine baseline and intervention scenarios constructed using demographic data from Nigeria. This approach allows us to better understand the timing by which different demographic-economic channels operate. Our baseline (high-variant) and alternative (medium-variant and low-variant) scenarios are constructed using current vital rates from Nigeria, although we may easily adapt the model by feeding in data from other countries. Baseline data on male and female age-specific fertility rates and projected populations are gathered from 2010 United Nations World Population Prospects estimates.

For our economic model, we collect baseline data for manufacturing and agricultural outputs, manufacturing and agricultural labor inputs, and available land from World Development Index estimates, and we use capital stock estimates from the Penn World Table repository. Data on

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[2] In our parameterization of the land factor share, β, we refer to Kawagoe et al. (1985)'s examination of the agricultural production function, in which the authors estimate an agricultural factor share between 0.1 and 0.2. Given that the parameter is small relative to the manufacturing factor share, we set β to be 0.167, which yields a tractable solution for manufacturing labor LM_t.

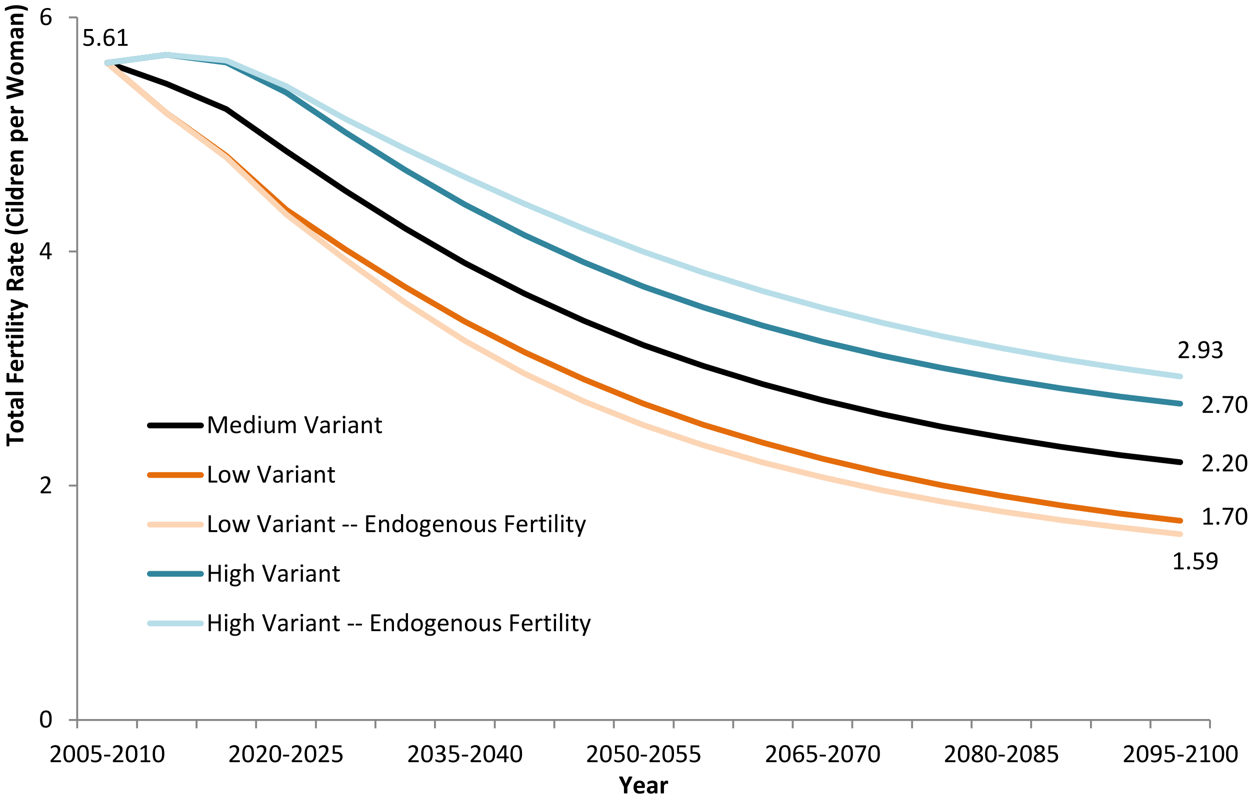
average schooling and average height are obtained from our demographic model simulations, while estimates of age-specific savings rates are gathered from Bloom, Canning, Mansfield, & Moore, (2007). Baseline labor force participation rates are obtained from the ILO.

IV. Simulation Results: The Case of Nigeria

Demographic Scenario

Figure 1 presents the changing pathways of fertility under the three main scenarios. Under the medium-variant scenario (the black line), total fertility declines from an initial 5.61 children per woman in 2005-2010 to the replacement level of 2.20 children per woman in 2095-2100. The total fertility rate under the high-variant scenario (the dark blue line) progresses on a slower trajectory than the medium variant such that the fertility rates between these scenarios differ by 0.25 births per woman in 2010-2015, 0.40 births per woman in 2015-2020, and by a fixed 0.50 births per woman from 2020 onwards. Similarly, the low-variant total fertility (the dark orange line) is projected such that the difference in fertility between the low-variant and medium-variant scenarios is the same at each time period as the difference in fertility between the medium-variant and the high variant scenarios.

Figure 1: Fertility under high-, medium-, and low-variant scenarios, Nigeria 2010



When accounting for the endogenous responses of fertility from the education channel, we see that both the low-variant and high-variant projections diverge further away from the medium-variant projection. By adjusting for these systemic effects, we see that fertility under the low-variant scenario is projected to fall by an additional 0.11 births, declining to a rate of 1.59 children per woman by 2100. This new pathway is indicated by the light blue line in Figure 1. In contrast, the total fertility rate under the high variant scenario would be 0.23 births per woman higher once the education feedback is incorporated, which is shown by the light orange line in Figure 1.

Figure 2: Population under high-, medium-, and low-variant scenarios, Nigeria 2010

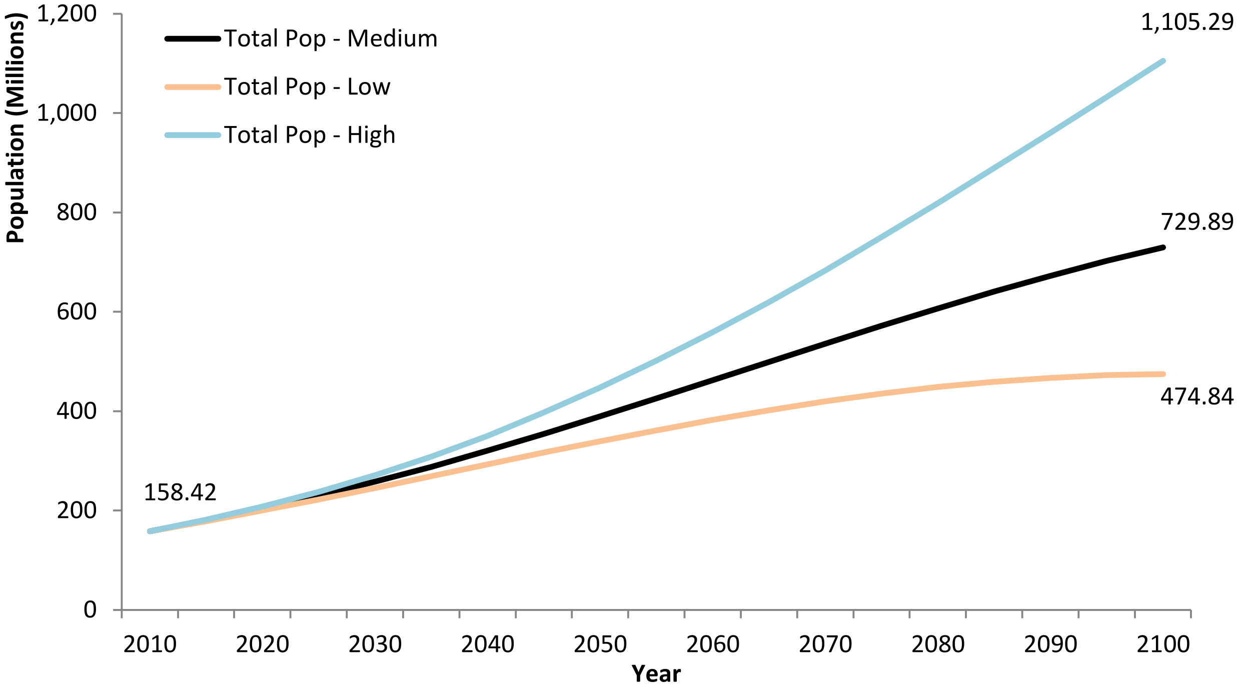
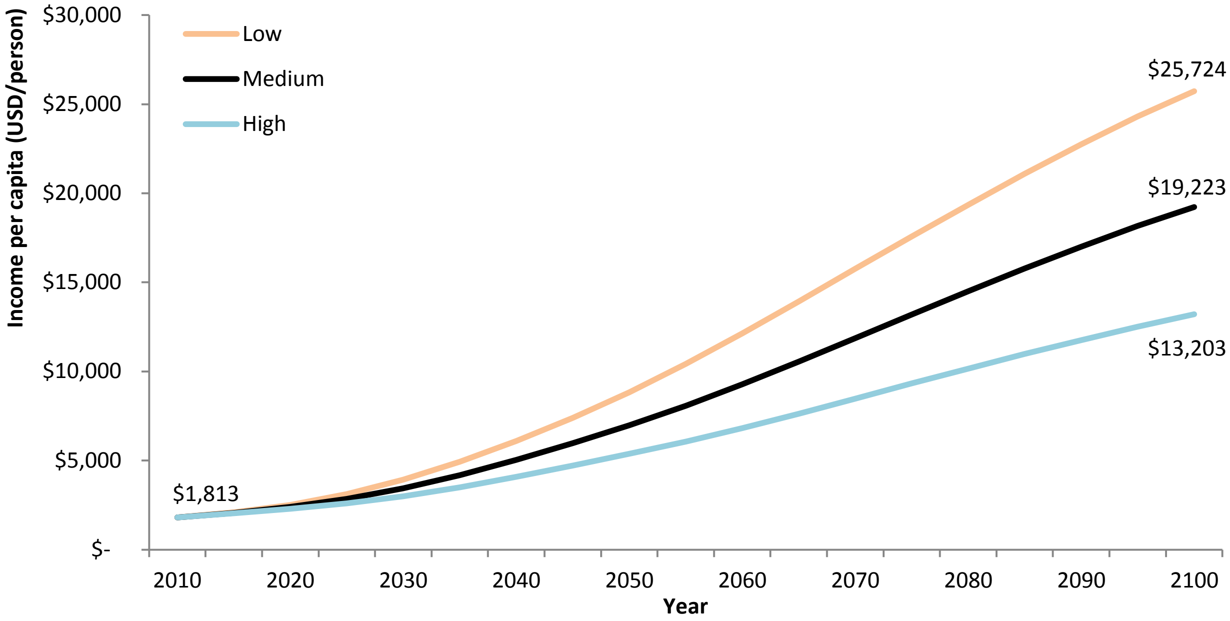


Figure 2 presents the evolution of total population under each of the three endogenous fertility scenarios. By these estimates, population under the low-variant scenario will be 13.1 percent lower than population in the medium-variant scenario and 24.3 percent lower than the population in the high-variant scenario in 2050.

Figure 3: Per-capita income under high-, medium-, and low-variant scenarios, Nigeria 2010



Two-Sector Economic Model Results

Figures 3, 4, and 5 present the path of income per capita, the share of workers in the manufacturing sector as a percentage of total labor supply, and the evolution of manufacturing capital per worker (the capital-labor ratio), respectively. Each of these paths is presented under the three endogenous fertility scenarios. In accordance with AWW 2013, we refer to the year 2010, which is the last year before total fertility rates in each of the three scenarios start to diverge, as the starting year for our simulation.

Figure 3 indicates that the reduction of fertility from our high-variant to the medium-variant and low-variant levels of fertility results in an increase in the per-capita income by almost one and a half times (45.6 percent) and two times (94.8 percent), respectively, over a 90-year time horizon. Additionally, we can assume that per-capita income across the three scenarios will continue to diverge because fertility rates in the low- and medium-variant scenarios are consistently lower than in the high-variant scenario over the entire period.

Figure 4: Share of workers in manufacturing (% of labor force) under high-, medium-, and low-variant scenarios, Nigeria 2010

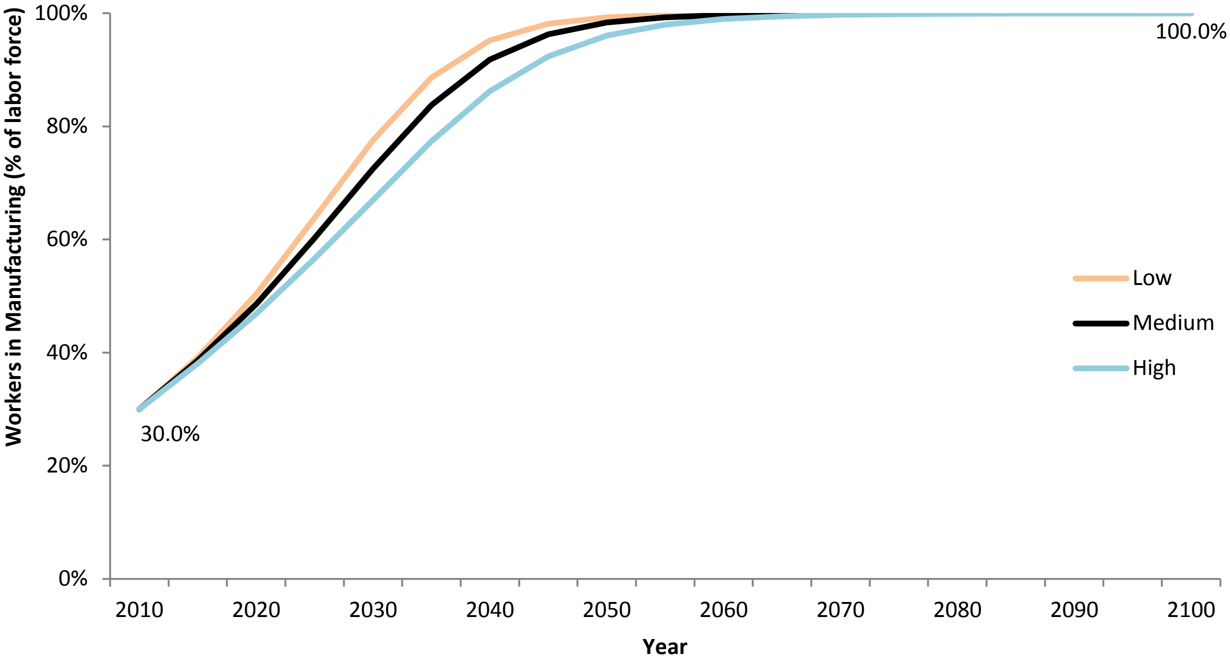


Figure 4 further illustrates the increase in the share of workers in manufacturing as a percentage of the total labor supply. Across all three fertility scenarios, we note that share of workers starts out to be smaller in the manufacturing sector than in the agricultural sector at only 30 percent of the total labor force; however, beginning around 2025, the share of workers in manufacturing exceeds the share of workers in the agricultural sector, reflecting the consequent shift in labor away from agriculture. While all three fertility scenarios depict this labor transition away from agriculture and into manufacturing, the rate at which this labor transition occurs varies considerably by fertility scenario. In particular, the share of workers in manufacturing increases the fastest and remains the highest in low-variant fertility scenario compared to medium and baseline high-variant fertility scenarios over the time horizon.

Figure 5: Manufacturing capital per worker under high-, medium-, and low-variant scenarios, Nigeria 2010

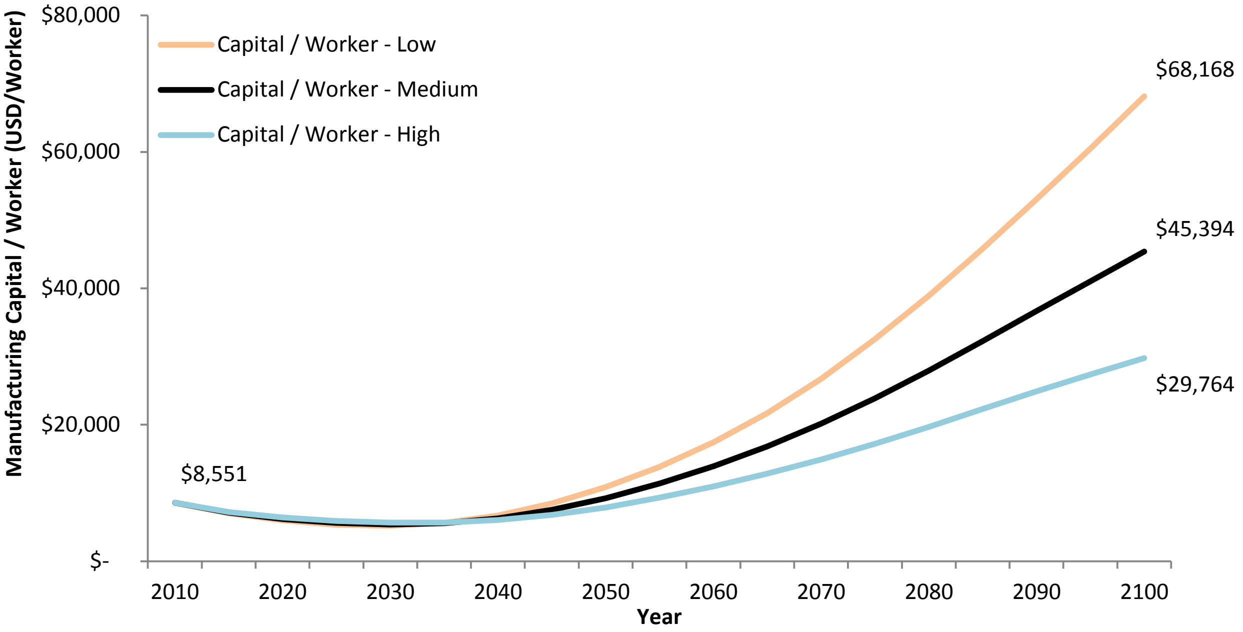


Figure 5 highlights the increasing productivity in manufacturing, as captured by the evolution in manufacturing capital per worker, over the 90 year time period. We observe that manufacturing capital per worker is approximately equal across all three fertility scenarios until around 2040, after which manufacturing capital per worker under the medium- and low-fertility scenarios is projected to grow at faster rates than under the high fertility scenario such that the level of manufacturing capital per worker in the low fertility scenario, at an estimated $68,168 per worker, is more than double that of the high fertility scenario, at an estimated $29,764 per worker, by the year 2100.

Mechanisms and their Long Run Paths

Figures 6 to 9 illustrate the evolution paths of four key mechanisms through which changes in fertility affect income per capita and other indicators of economic growth in our model. As was the case for the previous graphs, each of these figures present projected paths under the three fertility scenarios. These mechanisms include:

1. *The working age population ratio*, which is defined as the ratio between total number of workers in both sectors and the total population at each time period. This measure is a reflection of the potential for a demographic dividend in that it captures the additional productivity that

can be generated through mechanical shifts in the population age structure, which in turn is a consequence of declining fertility.

2. *The average years of schooling attained*, which accounts for the education-as-human-capital pathway through which declining fertility contributes to economic growth and productivity.
3. *Average adult height*, which proxies for health, the other defined human capital pathway in the model.
4. *Female labor force participation*, which reflects the direct labor market opportunity cost of childbearing.

Figure 6: Working age population ratio under high-, medium-, and low-variant scenarios, Nigeria 2010

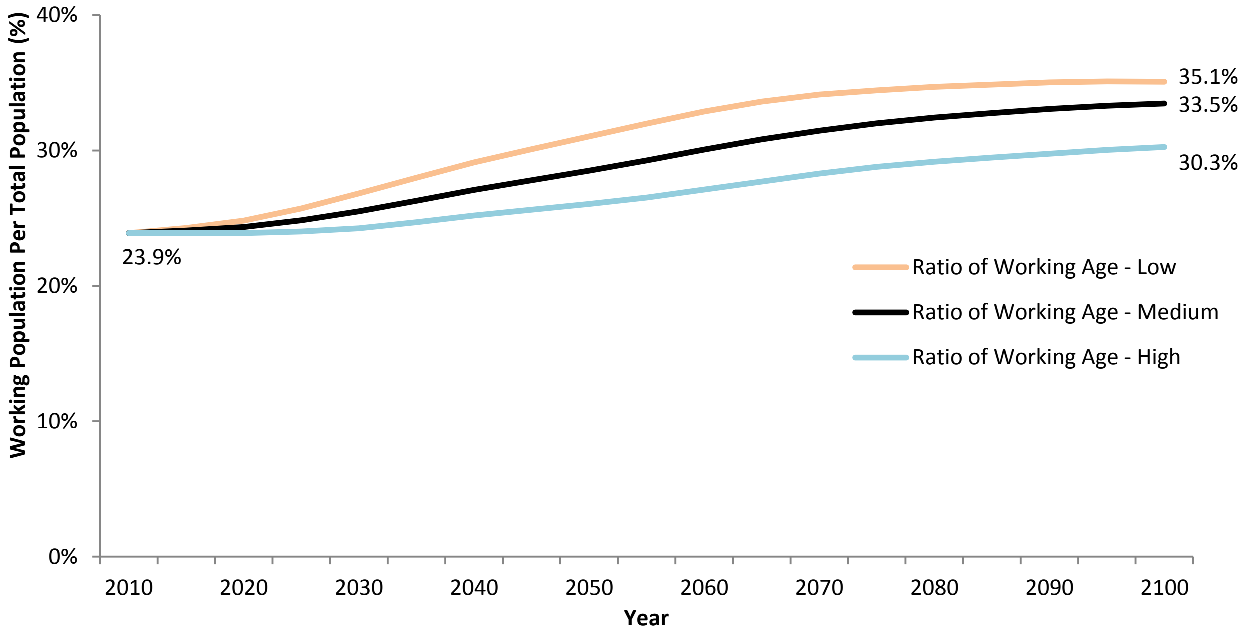


Figure 6 presents the long run effects of declining fertility on the ratio of the working age population to the total population. Reductions in the fertility rate over time contribute to a higher working age population ratio as the base of the population pyramid shrinks relative to the productive working ages. Moreover, the working age population ratio increases faster with larger declines in fertility. In comparing the high-fertility scenario with the low-fertility scenario at the end of the projection period, we see that a 1.34 birth per woman difference between these scenarios translates to a 4.8 percentage point difference in their working age population ratios.

Figure 7: Average years of education under high-, medium-, and low-variant scenarios, Nigeria 2010

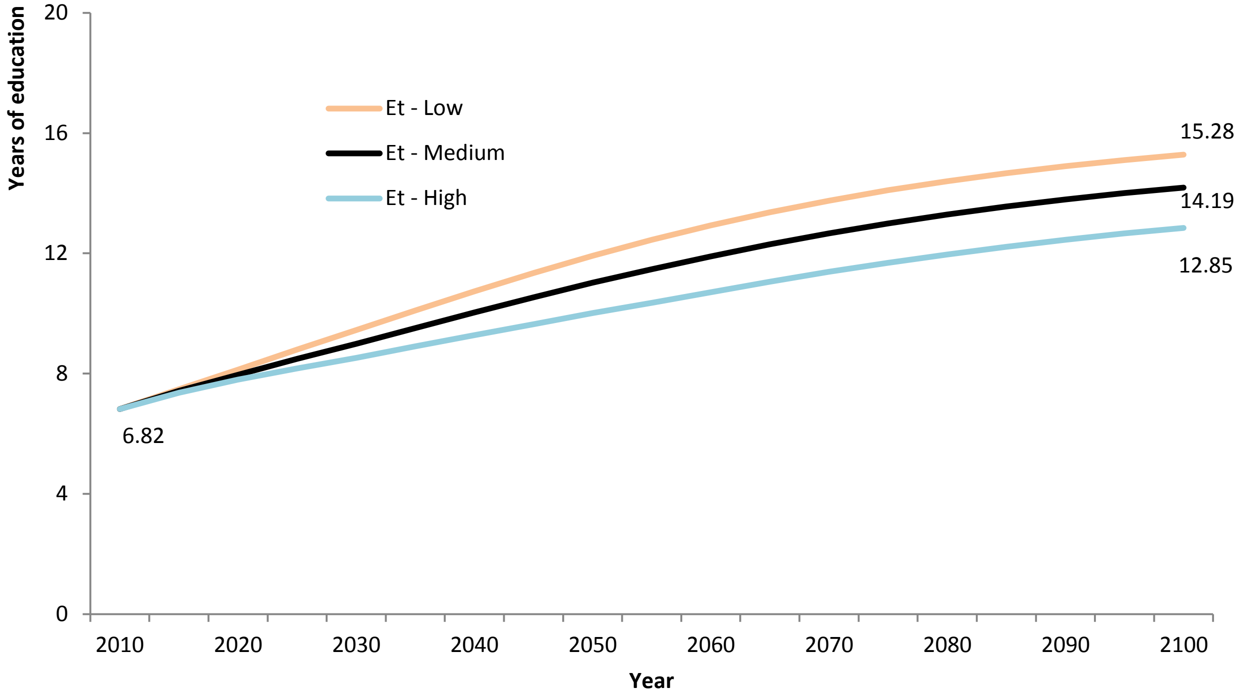
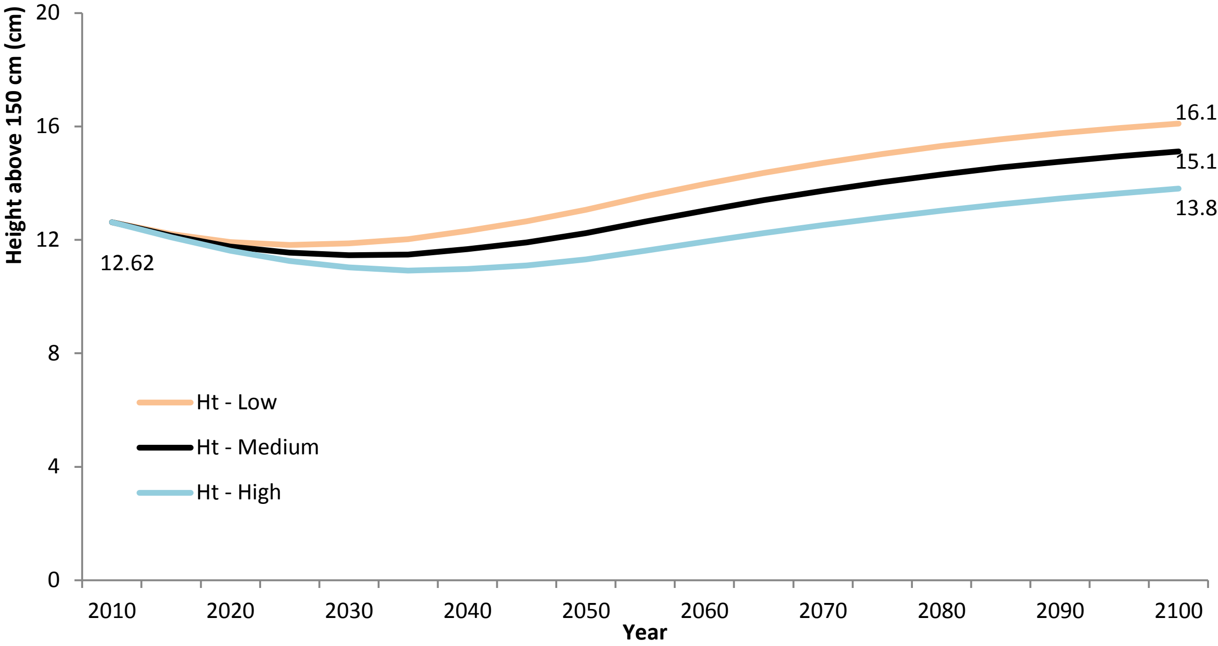


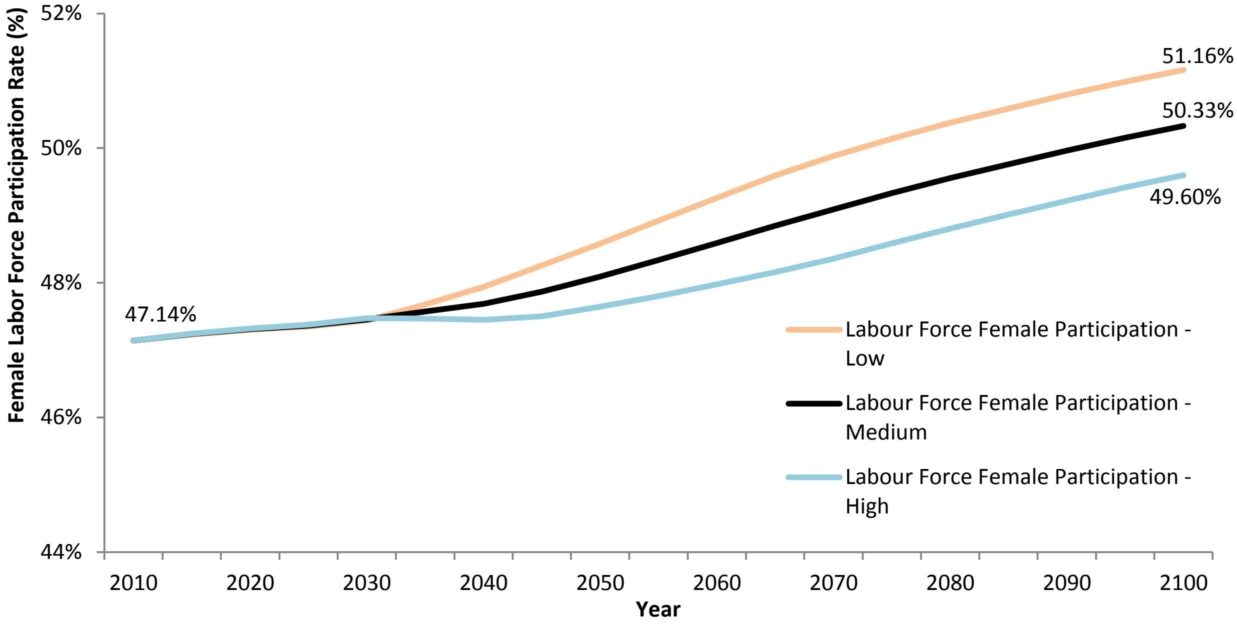
Figure 7 outlines the paths of education, measured by average years of schooling attained, under the three diverging fertility scenarios over the 90 year projection period. While educational attainment is expected to increase in the population as a whole, it will increase at faster rates under lower fertility scenarios. In particular, the average number of schooling obtained by the population under the low fertility scenario is projected to be 2.43 years more than the average amount of schooling obtained under the high fertility scenario.

Figure 8: Average height over 150 cm under high-, medium-, and low-variant scenarios, Nigeria 2010



In a similar fashion to education, we predict that health, as measured by adult stature, will also improve over time, as is shown in Figure 8; that said, adults under the low fertility scenario are predicted to gain almost 3.5 cm (1.37 in) over the projection period, while adults under the high fertility scenario will gain only 1.2 cm (0.47 in), a difference of almost one inch.

Figure 9: Female labor force participation rate under high-, medium-, and low-variant scenarios, Nigeria 2010



In assessing the female labor supply response to alternative declines in fertility over time, we observe a modest difference over time in labor force participation rates associated with these different scenarios, as is depicted in Figure 9. Most notably, we observe a 1.6 percentage point difference in the female labor participation rate when comparing the low fertility projection to the high fertility projection over the projection period.

V. Conclusions

In this paper, we created a demographic and economic model of the effect of fertility decline on economic growth. We extended the model of Ashraf et al. (2013) to incorporate five previously ignored channels: 1) the effect of fertility on savings; 2) a feedback from education back to fertility; 3) the effect of a more realistic two-sector model; 4) the effect of fertility on health; and 5) the effect of market imperfections, which are prevalent in the developing world.

Since the purpose of our paper is to provide a more comprehensive model of the relationship between fertility decline and income growth than has previously been done, a natural question is whether the additional channels we add (endogenous fertility, two sectors, market imperfections, savings, and health) change the results found previously in the literature. Comparing our results to Ashraf et al. (2013), we find that adding these additional channels almost triples the effect of fertility decline on income per capita. For example, Ashraf et al. (2013) find that moving from the medium to the low UN fertility variant would increase income per capita by 11.7 percent at a time horizon of 50 years, and increase by 5.6 percent at a horizon of 20 years. Once we add these additional channels, income is 30.9 percent higher after 50 years ($12,131 vs. $9,270) and 14.2 percent higher after 20 years ($3,921 vs. $3,434). We conclude that these previously ignored channels are not only important, but perhaps are even more important than the more traditional channels considered in the literature.

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Appendix 1: The Savings Equation

In modeling the evolution of savings, we follow the example of Bloom, Canning, Mansfield, et al. (2007) in which we consider cohort-specific savings decisions over time and aggregate across cohorts to find national savings. In the Bloom et al. (2007) savings model, the authors allow for both retirement decisions and savings decisions to depend on life expectancy, in which they argue that longer life spans lead to longer periods of retirement and increased pre-retirement savings. To derive the savings relationship, the authors first jointly solve for individuals' optimal lifetime labor supply, consumption, and savings, which are functions of life expectancy, using a lifetime utility maximization problem and derive the aggregate savings relationship (Equation 30) as follows:

$$\frac{S_t}{Y_t} = h(z, \sigma, w_t, R^*) + \frac{\sigma}{BR} - \frac{Old_t}{WA_t} + \eta \frac{Young_t}{WA_t} + \log\left(\frac{LF_t}{WA_t}\right) + \log(1-\alpha)$$

where z is life expectancy, σ is the growth rate of wages, w_t is the wage rate at time t, R^* is a mandatory retirement age constraint (usually 65), BR is the birth rate, $\frac{Old_t}{WA_t}$ captures the old-age dependency rate at t, $\frac{LF_t}{WA_t}$ captures the labor force participation rate at t, and α is the capital share of output.

To estimate the equation above, the authors test for potential non-linear effects of life expectancy, wages, and wage growth rate on savings behavior by performing a second-order Taylor series expansion on the h function around these three variables and including first-level interaction terms in h. They also include a lagged savings rate term to adjust for dynamic dependency in the time path of savings. The parameters of this saturated equation are then estimated in a dynamic fixed effects panel model using data for a panel of countries from 1960 to 2000 and a specification that is robust to country fixed effects and that allows for a dynamic evolution of aggregate savings as it adjusts towards its steady state (Table 4, Column 3). After removing insignificant variables sequentially, the authors arrive at the final regression specification below (Table 4, Column 4), which we use as our main savings equation:

$$s_t = \frac{S_t}{Y_t} = \phi_0 + \phi_1 \frac{s_{t-1}}{Y_{t-1}} + \phi_2 w_t + \phi_3 w_t^2 + \phi_4 \frac{Old_t}{WA_t}$$

To parameterize the coefficients ϕ_1 to ϕ_4 in this specification, we use estimates from the full model in Table 4, Column 3, and we then calibrate the estimate for ϕ_0 to be the value that achieves a steady state rate of savings under the baseline conditions for savings, wages, and the age dependency ratio, i.e. ϕ_0 is fit under $s_t = s_{t-1} = s^*$, the steady state savings rate, for the given s_0, w_0, and $\frac{Old_0}{WA_0}$.