Forecasting mortality by using statistical moments

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Abstract

Forecasting mortality by predicting the moments of the distribution of deaths leads to coherent results that can flexibly incorporate exogenous information such as the mortality experience of neighboring (Li-Lee 2005) or world record countries (Torri-Vaupel, 2012). Forecasting the moments of the distribution of deaths yields the further advantage of reducing the forecast dimension by requiring the projection of a lower number of moments than the number of age classes used by other methods (e.g., Lee-Carter 1992, Renshaw-Haberman 2003). In the present paper, a method allowing to determining the age-schedule of death rates is presented, by forecasting a number of statistical moments and reconstruction of the density function, starting from the obtained results. The method is back tested using US female data.

Keywords: mortality forecasting; statistical moments; density function, VAR

Introduction

A surprisingly few mortality forecast methods acknowledge with the exception of projections made by compositional data analysis (Oeppen 2008) that the mortality experience of a country is described not only by a hazard rate and the number of deaths but by a probability density function as well. The distribution of deaths, or life table d_x is constrained by $\sum_x d_x = 1$, time-series extrapolations of its trends are likely to violate this assumption. Oeppen (2008) used compositional data analysis to constrain the distribution forecast to sum up to unity. While this approach solves the problem of respecting the unit sum constraint, it also necessitates changing the coordinate system from Descartes to Aitchison geometry which might hinder the interpretation of the results. However, reconstructing the density function from a set of moments automatically avoids this problem.

The k[th] sample moment, ν_k, of d_x is given by

$$\nu_k = \sum_{x=0}^{\omega} d_x x^k \quad ,$$

where x denotes age and ω the highest age attained in the population, respectively. Note that ν_1 is the mean age at death or life expectancy at birth, e_0 (Canudas-Romo 2010).

Out method can be broken down into three parts:

1. Assess the evolution of observed moments of the distribution of deaths.
2. Forecast moments by multivariate time series analysis.
3. Reconstruct the forecast d_x distribution.

Evolution of the observed sample moments

In order to further simplify the problem of forecasting moments and increase the possibility of involving exogenous information, we calculated the sample central moments by

$$\nu_k' = \sum_{x=0}^{\omega} d_x (x - e_0)^k \quad .$$

Forecast moments by multivariate time series analysis

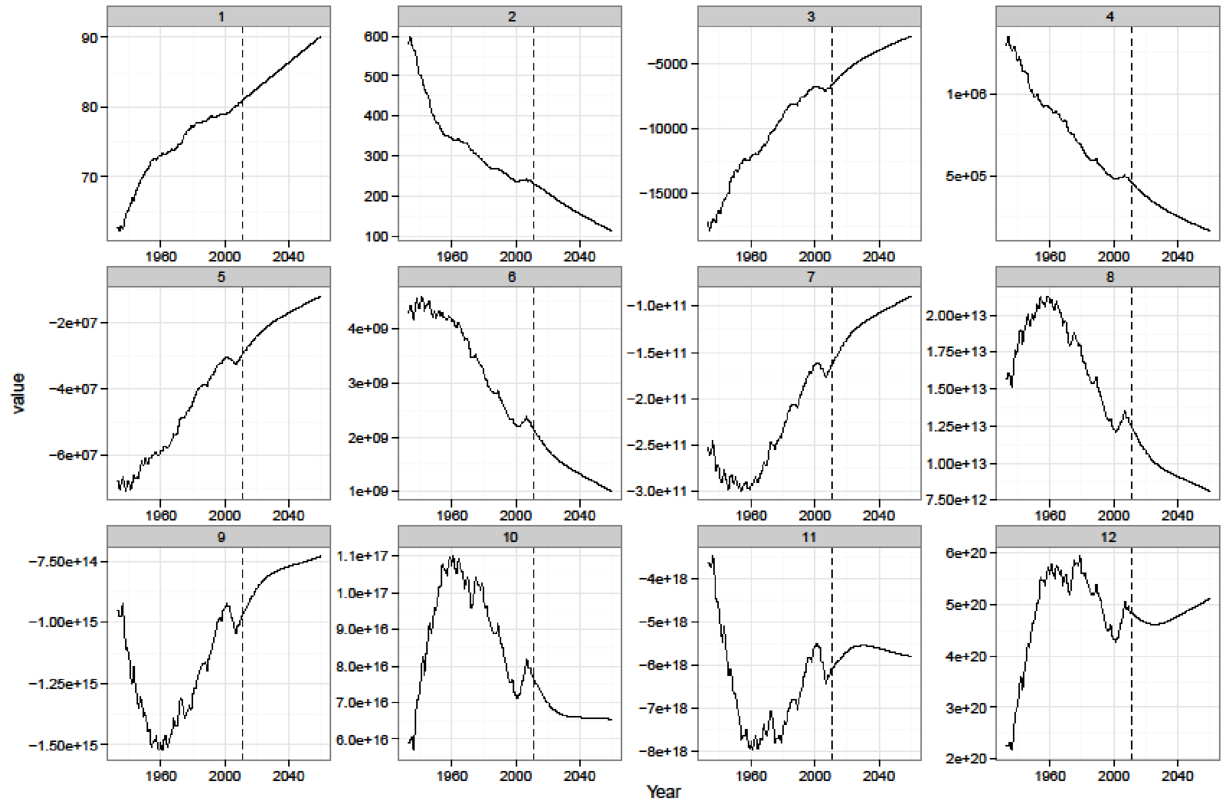
Figure 1 shows the observed (left of the dashed line) and forecast moments (right of the dashed line). Currently, we forecast the second-twelfths central moments by stationary vector-autoregressive time series models, where the p-lag vector autoregressive ($VAR(p)$) model has the form

$$Y_t = c + \Pi_1 Y_{t-1} + \Pi_2 Y_{t-2} + \cdots + \Pi_p Y_{t-p} + \varepsilon_t \quad , \quad t = 1, \ldots, T$$

where Π_i are $(n \times n)$ coefficient matrices and ε_t is an $(n \times 1)$ unobservable zero mean white noise vector process with time invariant covariance matrix Σ.

Presently, life expectancy is separately forecast by a univariate time series model, ARMA(1,1) and included as exogenous information in the multivariate model.

Figure 1: Multivariate forecast of the first 12 moments of US female probability of death



Reconstruction of the forecast distribution

The problem of reconstructing a function from a given number of moments can be regarded as a finite dimensional version of the Hausdorff moment problem (Shohat and Tamarkin, 1943). It is known in the mathematical literature as the finite moment problem (Chebyshev, 1961) and has been extensively studied from a theoretical perspective. A practical approach consisting on spline-based reconstruction algorithm has been proposed by John et al. (2007) validated for chemical engineering applications. The spline-based reconstruction will be adopted in the present paper as well, having the advantage of allowing us to maintain the process more restriction free.

In order to obtain a perfect reconstruction of the required density function one needs to have information about all the moments up to infinity. But taking advantage of the regularity of human mortality in demography the reconstruction of a density function can be obtained by imposing an a priori restriction of the class of function where the solution is sought. In this way only a small number (3 to 12) of moments are needed to determine the best fit within the assumed law of mortality. As a priori functions employed for age-schedule of death rates reconstruction we can use the Gompertz (1825)-Makeham (1860), Gamma distributed frailty with Gompertz mortality schedule model (Vaupel 1979), the Siler competing hazard model (1983) that capture the mortality during 'immaturity', adulthood and senescence. Alternatively, we can also choose an observed probability distribution of deaths and use them as an a priori shape that approximates the target distribution whose moments are known (Tekel-Cohen 2012).

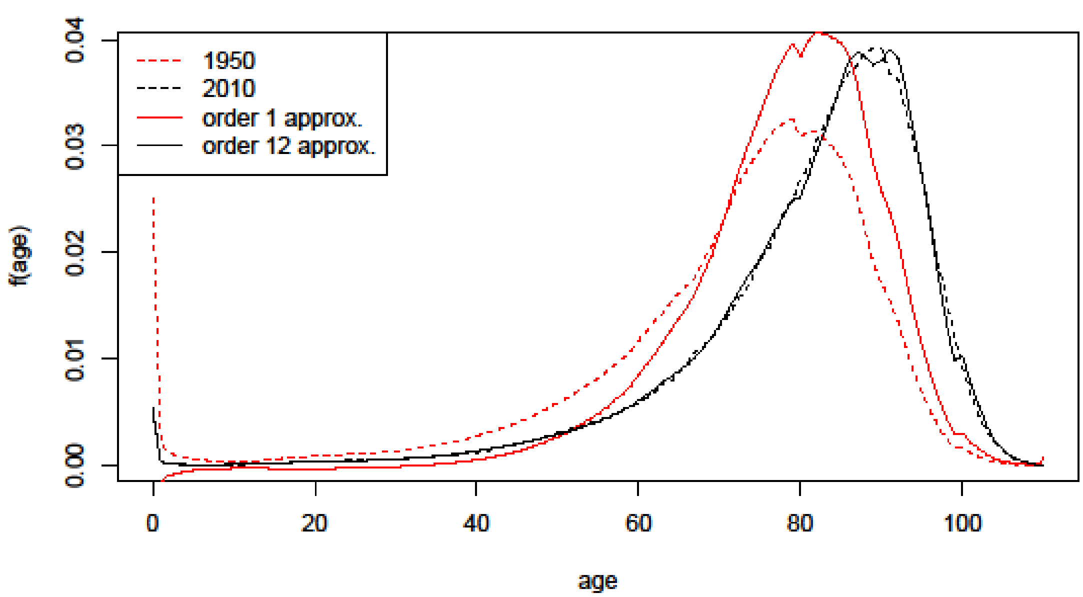
Future steps

Our most important future step is to find the optimal way of reconstructing forecast moments. Our current results show that the probability distribution of deaths can be reconstructed using observed sample moments and an initial distribution of deaths that gives an a priori knowledge of the shape of the probability density function. Figure 2 shows the reconstructed probability density function of deaths of US females in 2010 based on the initial distribution of deaths in 1950 and the first 12 sample moments of 2010.

The multivariate time series analysis employed in this abstract is currently based on the observed sample moments of all years between 1933 and 2010. As Tuljapurkar et al. (2000) noted, the periods preceding and following 1950 are structurally different and forecasts could probably start rather from 1950 than 1933.

Information on smoking can also be included as an exogenous variable to improve the coherence of the forecast in the multivariate time series projection (Janssen et al. 2013).

Figure 2 – Reconstruction of the density of deaths, US females.



Data source: Human Mortality Database

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