Convergence to Population stability: An Assessment of Demographic Potential of Indian States

Background:

Rapid population growth is one of the major challenges for developing countries and has been the subject of considerable debates. Some countries have experienced historically rapid declines to replacement levels of fertility. In China, Japan, and South Korea, such a fertility decline was accomplished in two decades or less whereas some countries, such as India, have begun a more gradual fertility decline which may still take next 25 years to achieve replacement fertility (Li & Tuljapurkar, 1999). India accounts for nearly 17% of the world's population and is experiencing rapid demographic changes with highest demographic heterogeneities ever experienced anywhere in the world at the regional and state levels (James, 2011). According to United Nation Population Division projection, the ultimate size of India's population when population stabilization is achieved will be about 1.72 billion around 2060. Nathan Keyfitz, Nagnur, and Sharma (1967) argued that the demographic history of a population is inscribed in its age distribution. Since age structural transition is an integral part of a demographic transition therefore the prospective vital rates along with age distribution can give us idea about the future scenario of any population. And this is main motivation for this study.

Why this study is important in Indian context

Since different states are at different level of their demographic transition therefore it would be quite informative to study the experiences of demographically leader states especially to set the pathways for laggard states. For policy perspective it would be quite informative to know the time frame of population stability and the ultimate population structure of each Indian state in advance based their current vital rates and age structure. The aggregate level pictures about demographic transition and demographic dividend mask the significant differences of Indian states.

The present study is focused to investigate some important research questions. The first objective is to investigate about the potentiality of the current age structure for further population growth in major Indian states and second but most important objective of this study to examine how fast the Indian states are approaching towards stability and what will be their ultimate stationary population. Since, different states are at different level of their demographic transition and hence aggregate level estimates mask the state heterogeneity, which motivates us to go for state-wise analysis and focus on the population stabilization and other population dynamics for major Indian state.

Conceptual Description:

The idea of stable population dates back to Euler in 1760 (Euler, 1970) but modern development was pioneered by Alfred J. Lotka (Lotka, 1907, 1939). The important proposition by Lotka about stable population is that if a population maintains fixed rates of fertility and survivorship at all ages it will gradually approach a condition in which its age composition, and hence its crude death and birth rates, remain constant (N. Keyfitz, 1965). Lotka considered a one-sex population with assumption of age-specific fertility and mortality rate constant over time in a closed population and examined the birth sequence in such population as:

$$B(t) = \int_0^t B(t-x)\, l(x)\, f(x)\ dx \quad t > 50 \tag{1}$$

Where $B(t)$ = number of births at time t; $n(x, t)$ = number of persons aged x at time t; $f(x)$ = rate of bearing female children for women aged x; $l(x)$ = probability of survival of women from birth to exact age x.

Fisher's Reproductive value of a population: The concept of reproductive value was propounded by R. A. Fisher (Fisher, 1930) which provides a quantitative measure of relative impacts on long-term population dynamics of individuals living at the same time. Fisher proposed the concept as being the present value of future births to a person discounted by the asymptotic rate of population increase (by Lotka's r). Nathan Keyfitz examined this concept and showed that reproductive value is proportional to ultimate size of posterity (N. Keyfitz, 1985). The reproductive value of a person aged x is given as:

$$v(\mathrm{x}) = \int_x^\infty \frac{l(y)}{l(x)}\, f(y)\, e^{-r(y-x)}\, dy \tag{2}$$

and, total reproductive value of population at time t can be given as:

$$V(t) = \int_0^\infty n(x;t) v(x) dx \tag{3}$$

R.A. Fisher proved that changed rate of the population's total reproductive value is equal to the Lotka's intrinsic growth rate r i.e. $\frac{dV(t)}{Vd(t)} = r$, which is very important property for aggregate population modelling.

Concept of Demographic Potential: The concept of reproductive value is for time-independent reproductive regimes only. But, for general case, D.M. Ediev introduced a new concept known as 'Demographic Potential' which allow generalization of Fisher's result (Ediev, 2001, 2003) as:

$$c(x,t) = \int_x^\infty \frac{l(y,t)}{l(x,t)}\, f(y,t)\, c(0,t+y) dy \tag{4}$$

$$C(t) = \int_0^\infty n(x,t)\, c(x,t-x) dx \tag{5}$$

$c(x,t)$ is the demographic potential of the person born at time t at age x and $C(t)$ is total demographic potential of the population at time t. Function in (4) are explicitly written as function of cohorts and is dual to Lotka's renewal equation. This duality is even clearer in expression written for newborns:

$$c(0,t) = \int_0^\infty l(y,t)\, f(y,t)\, c(0,t+y) dy \tag{6}$$

When fertility and mortality function used in (4) are time-constant, demographic potentials became proportional to Fisher's reproductive values due to ergodic property. Demographic potentials decrease as exponential function of time, i.e. $c(x,t) = c(x,0)e^{-rt}$, and are equal to Fisher's reproductive values if demographic potential of a new born is set to be one and other potentials are expressed in terms of newborn's potential

$$\frac{c(x,t-x)}{c(0,t)} = c(x) = \frac{e^{rx}}{l(x)} \int_x^\infty l(y)\, f(y) e^{-ry} dy \tag{7}$$

Demographic potential is an aggregate index proportional to future ultimate size of population posterity. It reflects the demographic power of the nation and its ability to provide future population growth.

Convergence to Stability as fundamental property of any general population:

A large part of mathematical demography is built upon one fundamental theorem, the "Strong Ergodic Theorem" of Demography which is first proven by (Sharpe & Lotka, 1911). According to this theorem, the age distribution of a closed population with unchanging fertility and mortality schedules must converge to a fixed and stable form. Later, López (1961) proved that this property of ergodicity applies even if the fertility and mortality schedules are changing. This is regarded as "Weak Ergodic Theorem" of demography.

Data Sources and methodology:

The present study will be based on multiple sources of data. It will use the secondary source of data from Indian census data since 1961, SRS (Sample Registration System, 1971-2012) annual reports and Abridged Life-Table by SRS to assess the demographic trends. Since the early 1970s, India's Sample Registration System has been the reliable source of fertility and mortality estimates for the country.

Monotonic measures of convergence to stability: Tuljapurkar (1982) discussed how the speed of convergence is determined by the stable net maternity function and introduced 'Kullback distance', the informational distance, as a monotonic measure of convergence of age structure to the stable equivalent. In information theory, the Kullback distance is a measure of the distance between two probability distributions. But here it concerns with the distance between an observed population and its stable equivalent.

The objective is to focus on how an observed population will converge towards its stable equivalent. Therefore, first, stable equivalent population structure of each state will be estimated based on their current vital rates and age-structure. Then the distance between observed and its stable equivalent age structure along with the speed of convergence to stability will be estimated.

Following N. Keyfitz (1985), we can estimate the stable equivalent population once we have the estimates of stable equivalent number of births at any time *t* given by $Q(t)$ as

$$Q(t) = \int_0^\infty N(x,t)V'(x)dx \tag{8}$$

Here, $V'(x)$ is the $(1/\mu_0)$ times $v(x)$, the reproductive value at age *x*.

So we can define $S(x,t)$ as the number of persons aged *x* in the stable equivalent population at time *t* and could be estimated as

$$S(x,t) = Q(t)e^{-rx}l(x). \tag{9}$$

We will use the concept of Kullback distance developed by Tuljapurkar (1982) and extensively discussed with demographic interpretation in Schoen and Kim (1991) for assessing the distance between actual age structure and stable age structure and how fast any population is approaching towards its stability.

At time t, the Kullback distance K(t)is defined as

$$K(t) = \int_0^\infty q(x,t)\, ln[q(x,t)/s(x)]\, dx\ ; \tag{10}$$

where, $q(x,t) = N(x,t)V'(x)/\int_0^\infty N(a,t)V'(a)da$ (11)

$= N(x,t)V'(x)/Q(t)$

is the probability distribution for the observed population at time *t*, and

$$s(x) = e^{-rx}\, l(x)\, V'(x) \tag{12}$$

is the limiting probability distribution for the stable population.

The Kullback distance is the minus the weighted sum of the logarithms of the age-specific momentum values; each age is weighted by the fraction it contributes to the stable equivalent number of births (Schoen & Kim, 1991). This is followed from Equation (10) which can also be written as

$$K(t) = -\int_0^\infty q(x,t) \ln \Omega(x,t)\, dx \tag{13}$$

Here $\Omega(x,t)$ is the age specific population momentum defined as $\Omega(x,t) = S(x,t)/N(x,t)$.

Kullback distance decreases monotonically to 0, which follows that every population, at every point in time, moves to reduce the distance that separates it from its stable equivalent.

Rate of convergence to stability: Rate of convergence to stability given by $h(t)$, is the instantaneous rate by which Kullback distance $K(t)$ decrease and can be defined as:

$$h(t) = -\frac{d[\ln K(t)]}{dt} = r - r_k(t) \tag{14}$$

here, $r = \int_0^\infty r(x,t)\, q(x,t) dx$; and $r_k(t) = \int_0^\infty r(x,t)\, q(x,t) \ln \Omega(x,t)\, dx \,/\, K(t)$ and $r(x,t)$ is the population's age specific growth rates. Since $h(t)$ is independent of level of $K(t)$, it can be used to compare the rates of convergence to stability in different populations.

Results and Discussion:

Since this study is still on-going and probably it will be finished in couple of weeks therefore the findings will be discussed in the revised version of the paper. But hopefully findings of this study will give more insightful and prospective pictures of the Indian states.

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