Estimating Neonatal Mortality*

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Abstract

In recent years, much of the focus in monitoring child mortality has been on assessing changes in the under-five mortality rate (U5MR). However, as the U5MR decreases, the share of neonatal deaths (within the first month) tends to increase, warranting increased efforts in monitoring this indicator in addition to the U5MR. A Bayesian splines regression model is presented for estimating neonatal mortality rates (NMR) for all countries. In the model, the relationship between NMR and U5MR is assessed and used to inform estimates, and spline regression models are used to capture country-specific trends. As such, the resulting NMR estimates incorporate trends in overall child mortality while also capturing data-driven trends. The model is fitted to 195 countries over the period 1990–2015.

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1 Introduction

In order to evaluate a country's progress in reducing child mortality, it is important to: obtain accurate estimates; be able to project mortality levels; and have some indication of the uncertainty in the estimates and projections. In practice, obtaining reliable mortality estimates is often most difficult in developing countries where mortality is relatively high. Many do not have well-functioning vital registration systems, data are limited, and the data that are available are often of poor quality.

The Millennium Development Goal 4 calls for a two-thirds reduction in under-five mortality between 1990 and 2015. As part of monitoring progress towards this goal, the focus in recent years has been on assessing changes in the under-five mortality rate (U5MR), which refers to the number of deaths before the age of 5 per 1000 live births. However, as the U5MR decreases, the share of neonatal deaths (within the first month) tends to increase, warranting increased efforts in monitoring this indicator in addition to the U5MR (e.g. Lawn et al. 2004; Mekonnen et al. 2013; Lozano et al. 2011; Bhutta et al. 2010). Notably, The Lancet published a series on newborn health in May 2014 (`http://www.thelancet.com/series/everynewborn`).

The United Nations Inter-agency Group for Child Mortality Estimation (IGME) publishes estimates of the neonatal mortality rate (NMR, the number of neonatal deaths per 1,000 live births) for all countries (IGME 2014). The estimation process currently used by IGME is described in Oestergaard et al. (2011). In this process, a multilevel model is used to obtain estimates for countries without high quality vital registration data, using the U5MR as a predictor. While the method has worked well to capture the main trends in the NMR, it has several disadvantages. For instance, for countries that do no have vital registration data available, trends in the NMR are driven by trends in the U5MR rather than by the data, which means that the NMR estimates do not provide new insights into progress in reducing neonatal deaths. Another disadvantage is that all data points are treated equally; information about the varying reliability of different data sources is not taken into account. Lastly, in the estimation approach, the estimated NMR may exceed the estimated U5MR.

In this paper, we propose a new model for estimating country-specific NMR that attempts to address these concerns. The dataset and model are summarized in the next sections, followed by some key results. Additional details about the model and computation are provided in the Appendix.

2 Data

Data on NMR are available for 189 countries. The source, nature and reliability of data varies from country to country. Broadly, data on NMR levels comes from vital registration (VR) systems; sample vital registration (SVR); and survey data. Data for a specific country may come from one or several of these sources, and for many time points there are overlapping NMR estimates based on the different data sources. Some countries have a relatively full data set from 1950 onwards, while other countries have only one or two historical series.

Table 1 summarizes the availability of data. Vital registration (VR) data are available for 105 countries. For most developed countries, a full time series of VR data exists. For other countries with VR data, the time series is often incomplete and is supported by other sources of data. Sample vital registration data (SVR) are available for China, India and South Africa. In terms of the survey data, the majority are from Demographic and Health Surveys (DHS), but there are also some series from other surveys such as the Multiple Indicator Cluster Survey and Family Health Surveys. Note that 'Other DHS' refers to non-standard DHS, that is, Special Interim and National DHS, Malaria Indicator Surveys, AIDS Indicator Surveys and World Fertility Surveys. Much of the data from surveys had reported sampling errors, but in some cases sampling errors were not reported.

Table 1: Summary of the NMR data series and observations by source type and whether or not sampling errors were reported.

Source	No. of Series	No. of Observations
VR	105	2915
SVR	3	49
DHS (with sampling errors)	235	1415
DHS (without sampling errors)	21	65
Other DHS (with sampling errors)	42	262
Other DHS (without sampling errors)	25	76
MICS (with sampling errors)	13	72
MICS (without sampling errors)	6	24
Others (with sampling errors)	25	147
Others (without sampling errors)	72	152

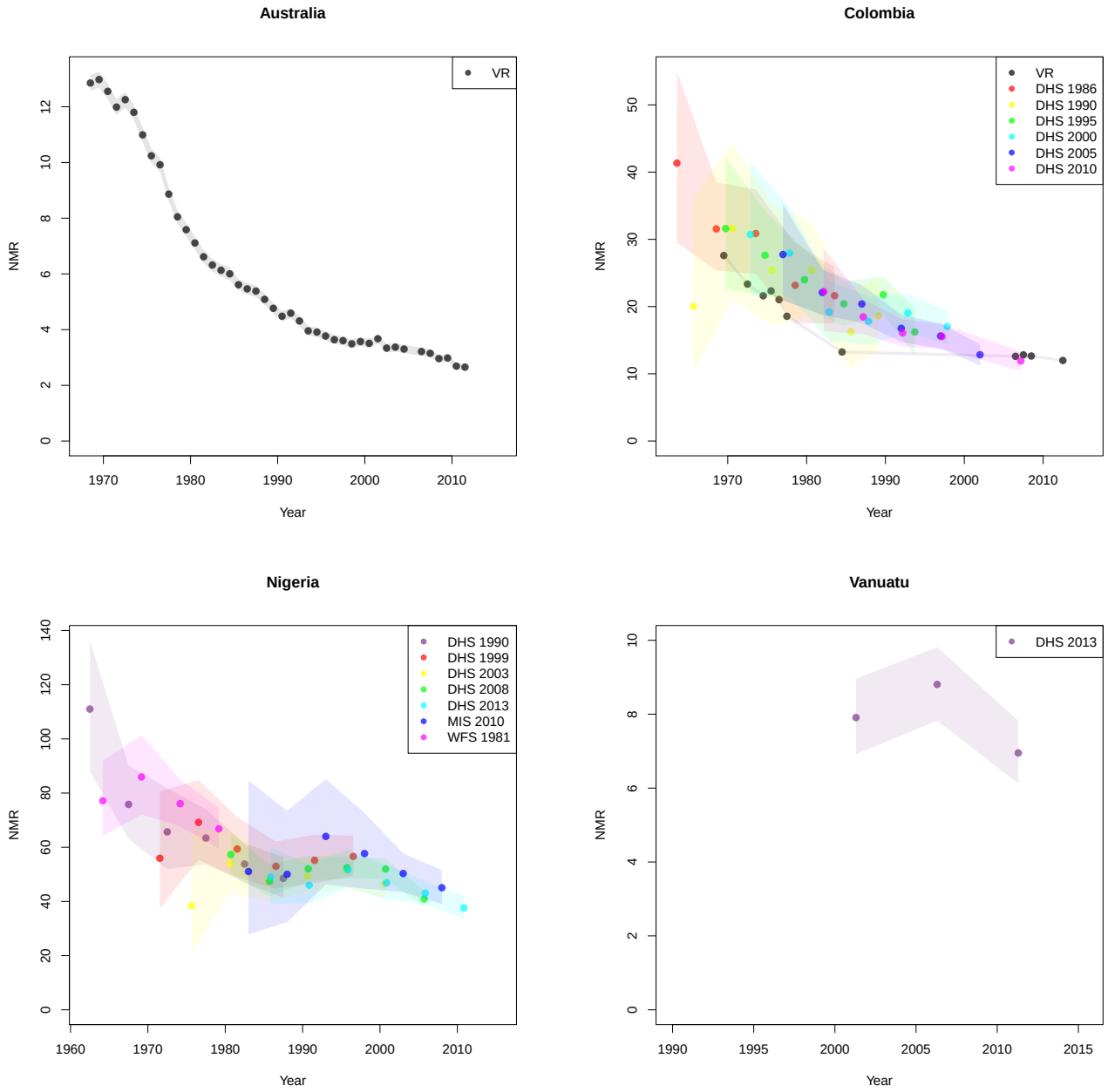
Countries can be grouped into four categories:

1. **Well-functioning VR**. Most developed countries have well-functioning VR systems, and the trend in NMR is relatively clear.
2. **Other VR**. In contrast to group 1, VR data for less-developed countries often has incomplete time series. Additionally, for smaller countries, trends in VR data can be unclear and prone to higher levels of stochastic error. Other SVR and survey data sources may exist, and there are potentially multiple estimates of the NMR for some time points.
3. **No VR, other data available**. For many less-developed countries, VR data do not exist. In these situations data is from SVR systems or estimates from surveys. Again, multiple estimates of NMR potentially exist for some time points.
4. **No VR, limited data available**. For some countries very few data points may exist (or none at all).

Figure 1 below illustrates examples of the four categories. The NMR for Australia, as estimated from a full VR data time series, has a trend which is relatively regular and uncertainty is low. Colombia has an incomplete VR data time series but has additional data from several Demographic and Health Surveys. There are multiple estimates for some years, and the uncertainty around the estimates varies by source and year. Nigeria has no VR data, and the estimates are constructed from various DHS and other surveys. Again, there are multiple estimates for the some time points, and uncertainty varies. Finally, Vanuatu has only three observation points from one DHS.

The aim is to produce estimates and projections of NMR for all countries in the world, and report the associated uncertainty around these estimates and projections. The model needs to be flexible enough to estimate NMR in a variety of situations, as illustrated above. The estimates should follow the data closely for countries with reliable data and low uncertainty. On the other hand, the model estimates need to be adequately smooth in countries with relatively large uncertainty and erratic trajectories. The model also needs to be able to estimate NMR over the period 1990-2015 (at least) for countries where there are limited or no data available.

Figure 1: Neonatal Mortality Data (deaths per 1,000 births) for selected countries.



3 Method

As the level of U5MR decreases, the proportion of deaths under 5 that are neonatal increases. Our model utilizes this relationship between the ratio of NMR to U5MR and the level of U5MR. Additionally, the model allows for country-specific effects.

To guarantee that NMR estimates are not greater than U5MR estimates, estimates of

the ratio of NMR to U5MR will be obtained, i.e. we will estimate

$$\frac{NMR_{c,t}}{U5MR_{c,t}} \leq 1$$

for each country c and year t. Estimates for $NMR_{c,t}$ are easily obtained by multiplying the estimated ratio by the relevant U5MR.

Write N_{ct} and $U_{c,t}$ as the NMR and U5MR for country c at time t, respectively. Noting that

$$\text{logit}\left(\frac{N_{c,t}}{U_{c,t}}\right) = \log\left(\frac{N_{c,t}}{U_{c,t} - N_{c,t}}\right),$$

and preferring the more intuitive ratio on the log scale rather than the logit scale, we will explain the model set-up in terms of the ratio

$$R_{c,t} = \frac{N_{c,t}}{U_{c,t} - N_{c,t}},$$

which refers to the (true) ratio of neonatal deaths compared to deaths in months 2 to 59. We define $r_{c,i}$ to be observation i of the ratio in country c. The observed ratio can be expressed as a combination of the true ratio and some error, i.e.

$$\begin{aligned} r_{c,i} &= R_{c,t[c,i]} \cdot \varepsilon_i \\ \implies \log(r_{c,i}) &= \log(R_{c,t[c,i]}) + \delta_i \end{aligned} \tag{1}$$

where $t[c,i]$ refers to the observation year for the i-th observation in country c, ε_i is the error of observation i and $\delta_i = \log(\varepsilon_i)$.

The true ratio $R_{c,t}$ is modeled as the product of the expected ratio given the current level of U5MR, and a country-specific deviation from the overall relationship:

$$R_{c,t} = f(U_{c,t}) \cdot P_{c,t},$$

where $f(U_{c,t})$ is the overall expected ratio given the current level of U5MR and $P_{c,t}$ is a country specific multiplier. This corresponds to an additive relation on the log-scale:

$$\log(R_{c,t}) = \log\left(f(U_{c,t})\right) + \log(P_{c,t}). \tag{2}$$

3.1 Global relationship with U5MR

The first step is to find an appropriate function $f(\cdot)$, which captures the expected value of the ratio given the current level of U5MR. Figure 2 suggests that the log ratio decreases linearly with decreasing log(U5MR) until a U5MR of around 20 deaths per 1,000 births. Below that level, however, there does not appear to be a relationship between the two. Given this observed relationship, $f(\cdot)$ is given by a constant up to

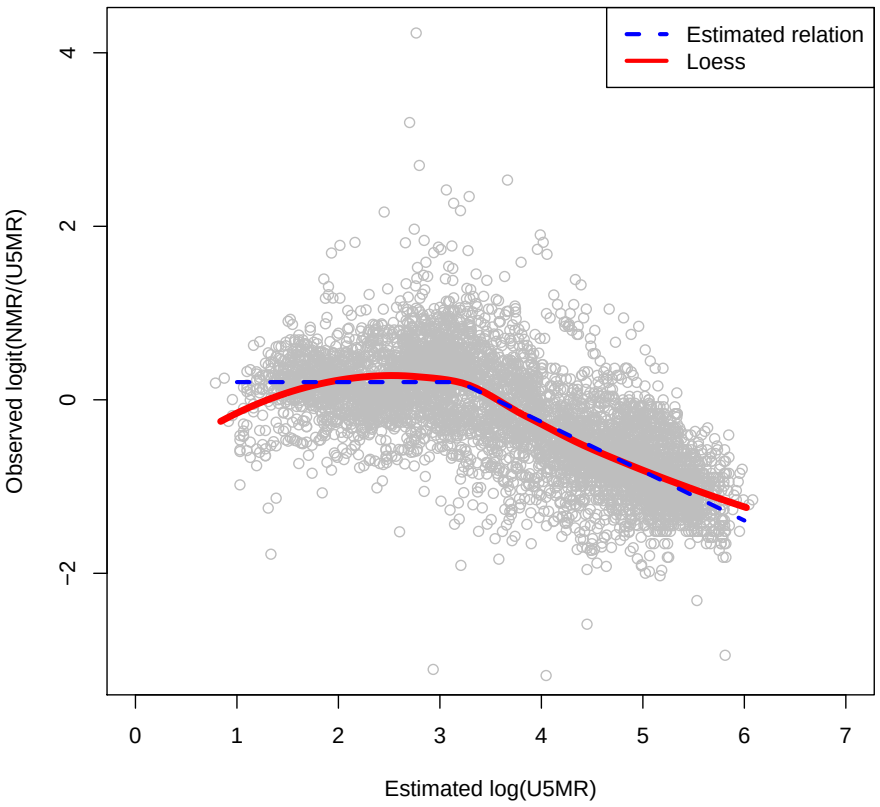
a level of a $\log(U_{c,t})$, defined as $\log(U_{cut})$. For $\log(U_{c,t}) > \log(U_{cut})$, $f(\cdot)$ is a linear function of $\log(U_{c,t})$:

$$f(U_{c,t}) = \beta_0 + \beta_1 \cdot (\log(U_{c,t}) - \log(U_{cut}))_{[\log(U_{c,t}) > \log(U_{cut})]},$$

where β_0, β_1, and U_{cut} are unknown parameters which will be estimated. When $\log(U_{c,t}) < \log(U_{cut})$, the indictor function associated with the second term is zero, and $f(U_{c,t}) = \beta_0$.

The fitted relationship between the ratio and the level of U5MR is illustrated in Figure 2. U_{cut} is estimated to be around 23 deaths per 1,000 births. At U5MR levels that are higher than U_{cut}, the β_1 coefficient suggests that a 1% increase in the U5MR leads to a 0.6% decrease in the ratio $R_{c,t}$. The fitted line is quite similar in shape to the loess curve fitted to the data.

Figure 2: $\log(R_{c,t})$ versus $\log(U_{c,t})$. Observations are displayed with grey dots. The Loess fit to the observations is shown in red, and the estimated relation (function $f(U_{c,t})$) is added in blue (dashed line).



3.2 Country-specific multiplier

Although there is a relationship between $R_{c,t}$ and $U_{c,t}$ at the aggregate level, the relationship is likely to differ from country to country. For instance, some countries may have higher levels of NMR than what we expect given the level of U5MR. The country-specific term $P_{c,t}$ in equation 2 captures data-driven differences across countries and also within countries over time.

The $P_{c,t}$ term was modeled using a penalized spline regression model. On the log-scale, it has the following form:

$$\log(P_{c,t}) = \sum_{k=1}^{K_c} B_{c,k}(t)\alpha_{c,k}$$

where $B_{c,k}(t)$ is the kth basis spline evaluated at time t and $\alpha_{c,k}$ is splines coefficient k. The coefficients to be estimated are the $\alpha_{c,k}$'s. They can be rewritten to be in the form of an intercept and fluctuations around that intercept:

$$\alpha_{c,k} = \lambda_c + [A \cdot \boldsymbol{\varepsilon}_c]_k.$$

The λ_c is similar to a country-specific estimate. If $\lambda_c > 1$, then country c's level of NMR is generally higher than expected given its U5MR level, and vice versa for $\lambda_c < 1$. The $\boldsymbol{\varepsilon}_c$ term represents fluctuations around this country-specific intercept (A is some known constant, see Appendix for more details). These fluctuation terms allow for the $P_{c,t}$ term to be influenced by the changes in the level of the underlying data. However, the size of these fluctuations is restricted to ensure relatively smooth trajectories.

3.3 Data model

Equation 1 indicates the observed ratio $r_{c,i}$ is modeled on the log-scale as the true ratio $R_{c,t}$ plus some error term δ_i. This error term δ_i is modeled differently based on the source of the data.

For the VR data, it is assumed that the error term is due to the stochastic standard error of the estimates. This type of error will be larger for countries with smaller populations, because there is a greater chance that these countries have abnormally high or low deaths purely by chance. The stochastic standard errors for VR data was estimated through a Poisson model of deaths (see section A.2).

For the non-VR data, δ_i is assumed to be a combination of sampling and non-sampling error. The sampling errors were reported for the majority of the non-VR observations (see Table 1). For those observations where sampling error was not reported, it was set to be 10%. The non-sampling error was estimated through the modeling process.

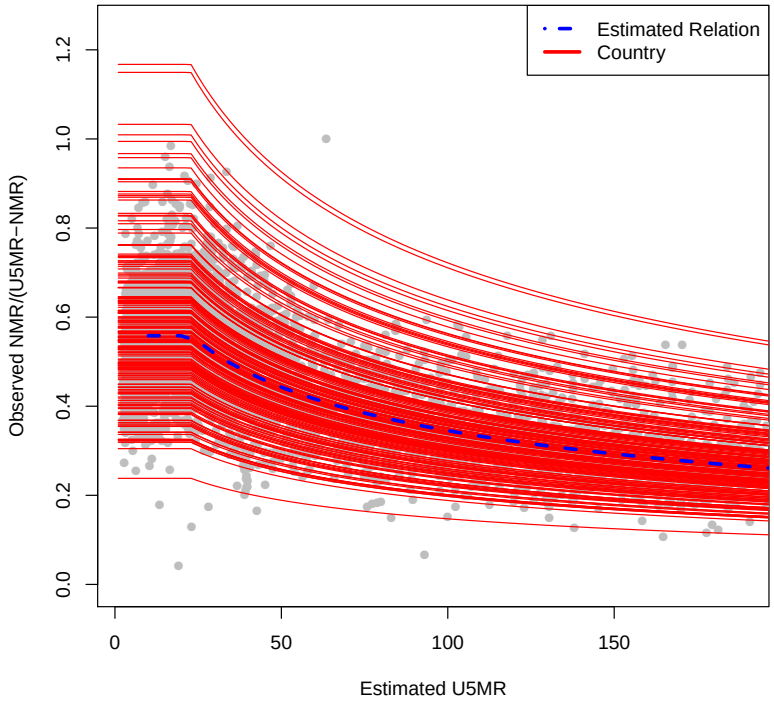
The model was fitted in a Bayesian framework using the statistical software R. Details of the computational setup are outlined in the Appendix, section A.5.

4 Results

Estimates of NMR were produced for 195 countries spanning at least the period 1990 – 2015. In this section we highlight some key results and compare results to those produced by the current IGME method.

Figure 3 visualizes the estimated country-specific intercepts (λ_c's) and the global relationship with U5MR. The blue-dashed line represents the $f(U_{c,t})$ as before in Figure 2. The λ_c's were modeled as normally distributed around a mean of zero and as such many of the country-specific lines are close to the overall line. Those countries which have particularly high or low levels compared to the global relationship will be discussed in the next section.

Figure 3: $R_{c,t}$ versus $U_{c,t}$. Observations are displayed with grey dots. The estimated relation (function $f(U_{c,t})$) is added in blue (dashed line). The red lines represent the country-specific effects.



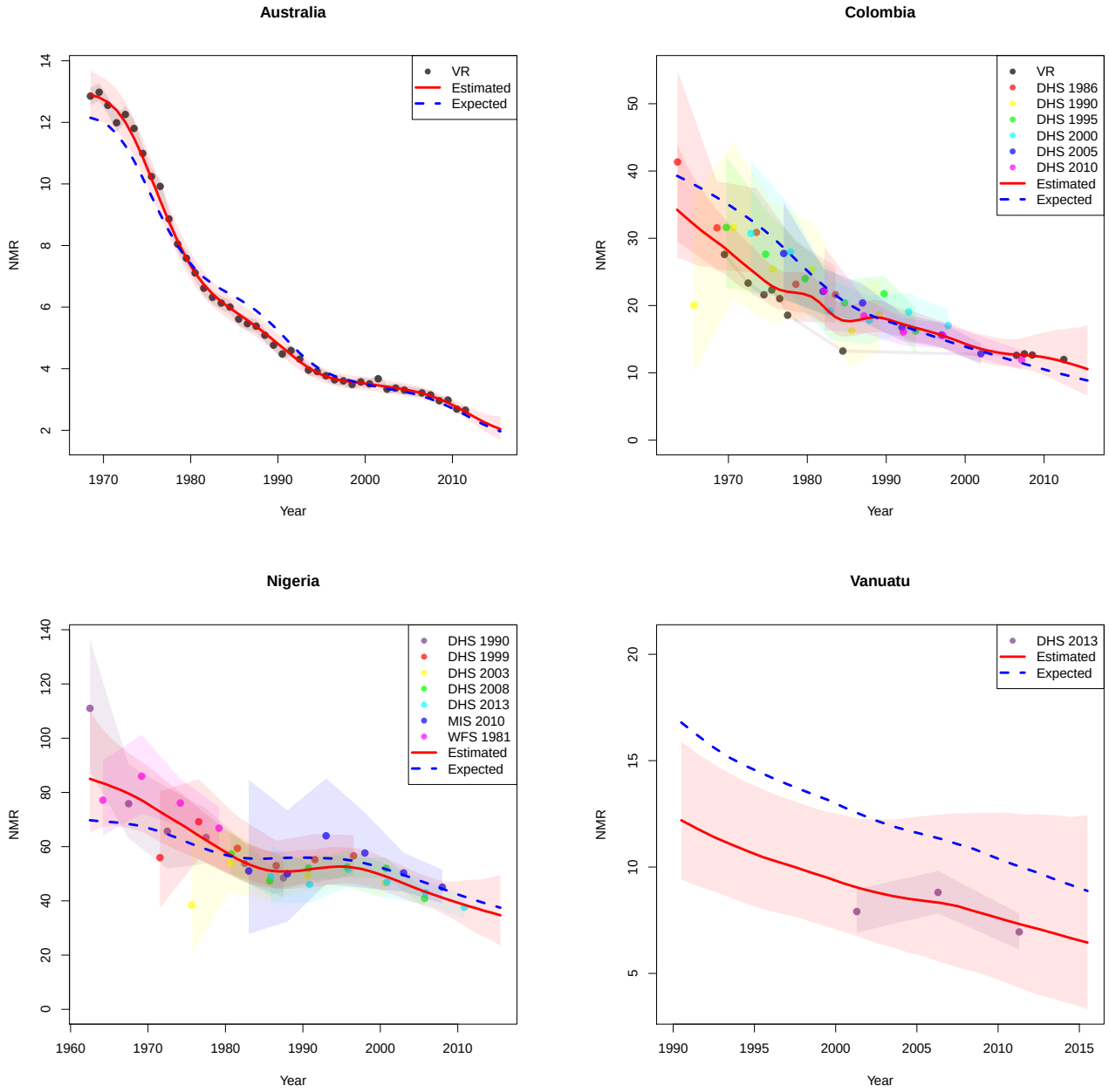
The charts in Figures 4 illustrate the performance of the model in the variety of different data-availability situations as discussed in Section 2. In each of the figures, the red line represents the estimated fitted line (with 95% credible intervals). The blue line represents the estimation with the $f(U_{c,t})$ only (without the country-specific effect, $P_{c,t}$). The blue line can be interpreted as the expected level of NMR in a particular year given the level of U5MR.

For Australia, the estimated red line follows the data closely, given the small uncertainty levels around the data. There has been a steady decrease in NMR since 1970. In earlier time periods, the level of NMR was higher than what was expected given the level of U5MR (that is, the red line is higher than the blue). This switched in the 1980s and 1990s, and more recently, the estimated and expected level are very close.

For Colombia, the estimates of NMR are being influenced by the combination of VR and survey data. The VR has a greater influence on the trajectory because of the smaller associated standard errors. While historically the level of NMR was lower than expected, since 1990 the NMR has been about the same level as the expected level. The uncertainty intervals around the estimate are much larger than for Australia due to the increased uncertainty of the data.

The level of NMR in Nigeria has decreased in recent times after a slight increase during the 1990s. The estimated level is not significantly different to the expected level. For Vanuatu, the estimated level of NMR is lower than the expected level. However, the relative absence of data for this country means that the uncertainty around the estimates is high.

Figure 4: Observed and estimated neonatal mortality (deaths per 1,000 births) for selected countries.



4.1 Outlying countries

The set-up of the model allows for the intuitive interpretation of results in comparing the estimated level of NMR to the expected level given the U5MR. Figure 5 illustrates those countries that are 'outlying' in the sense that the estimated NMR in 2010 was significantly higher or lower than the expected level by at least 10%.[1]

Countries with lower-than-expected NMR included Japan, Singapore and Republic of Korea. Countries with higher-than-expected NMR included several Southern Asian countries (such as India and Pakistan) and former Yugoslavian countries. Figure 6 shows estimates through time for two contrasting countries, Japan and India. While the gap between expected and estimated is being sustained through time for Japan, the discrepancy for India seems to be decreasing through time. Future work will investigate what is potentially causing NMR to be higher- or lower-than expected in outlying countries and whether these are real effects or artifacts of data issues.

[1] Ratio of estimated-to-expected level that are at least 1.1 or less than 0.9 in 2010.

Figure 5: Ratio of estimated-to-expected NMR in 2000 and 2010.

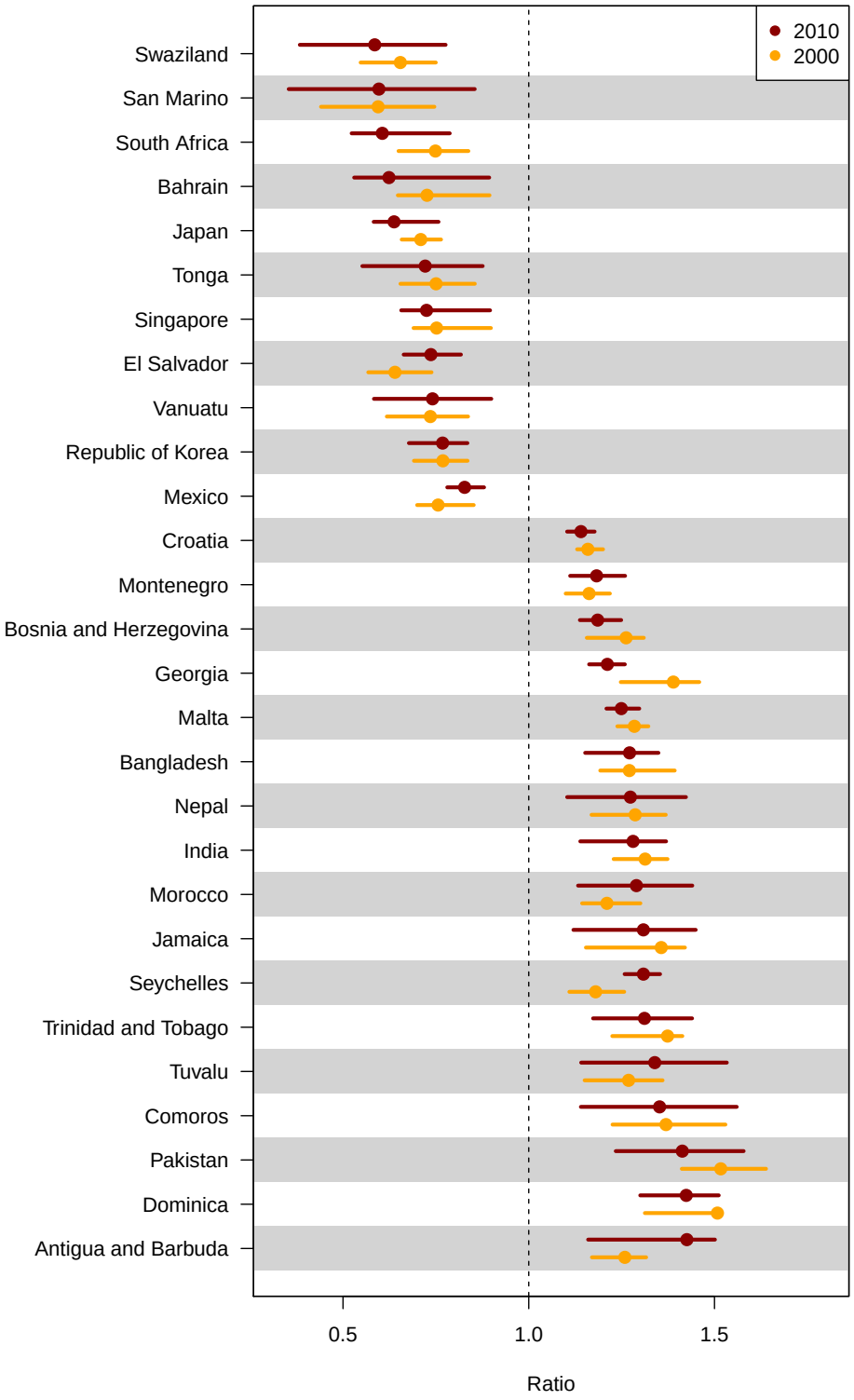
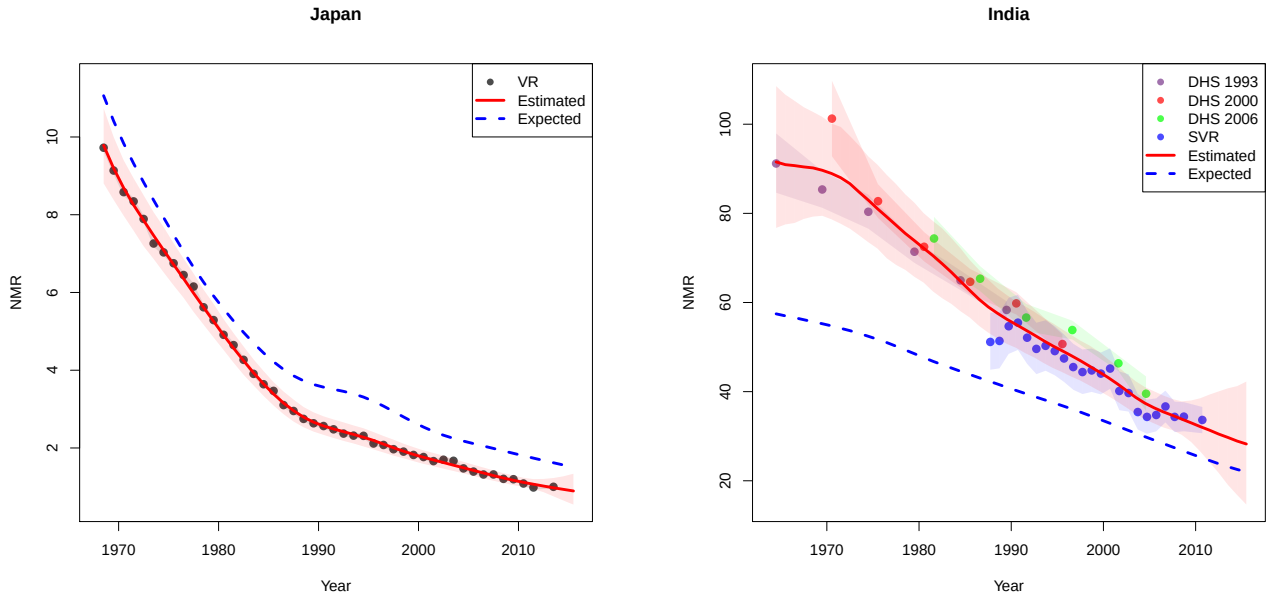


Figure 6: Observed and estimated neonatal mortality (deaths per 1,000 births), Japan and India.



4.2 Comparison with existing IGME model

It is useful to compare the results of this new model to the NMR estimates currently published by the IGME. As mentioned previously, the current NMR estimates are produced using the model described in Oestergaard et al. (2011). In this method, NMR estimates for countries with complete VR series are taken directly from the data. For countries without a complete VR series, a multilevel model is fit using U5MR as a predictor. A quadratic relationship with U5MR is specified. In addition, the model allows for country-level and region-level random effects.

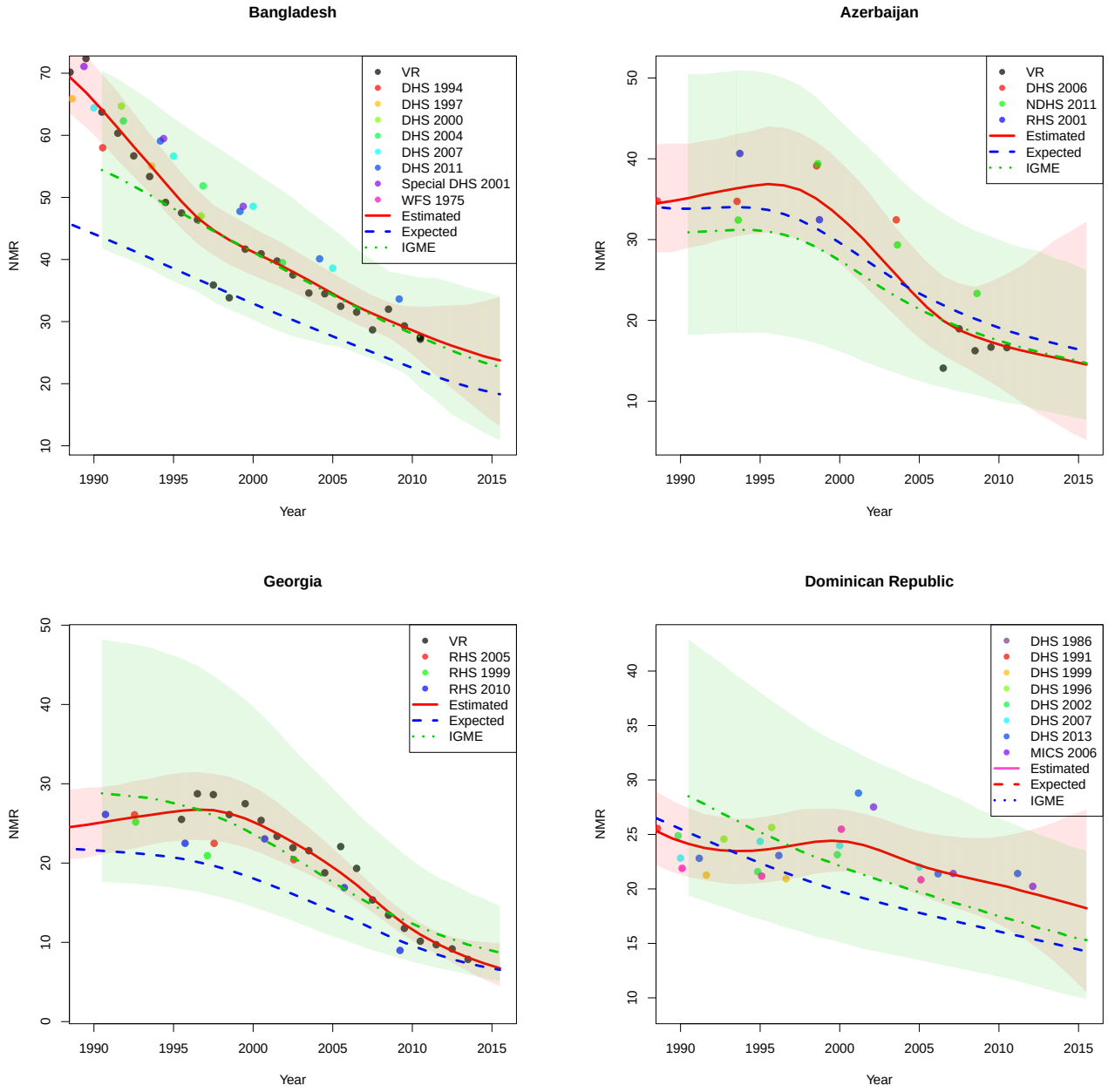
The existing model is similar in that it estimates NMR as a function of U5MR, plus some additional country-specific effect. However, one of the main differences between the two models is that for countries with non-VR data, estimates from the new model can be driven by the data, while the previous model is restricted to follow the trajectory of the U5MR in a particular country, plus or minus some country-specific intercept.

Figure 7 compares the results of four countries to the estimates from the current IGME model. The charts are zoomed in to just show the period 1990–2015, which is the period for which estimates are published.[2] The estimates from the existing IGME model generally follow the same trajectory as the expected line, as determined by U5MR patterns, and is shifted up or down depending on the estimate of the country-specific effect. In contrast, the estimates from the new model follow the data more closely. The fluctuation part of the country-specific multiplier $P_{c,t}$ allows the estimated line to

[2]IGME estimates are from the May 2014 estimation round.

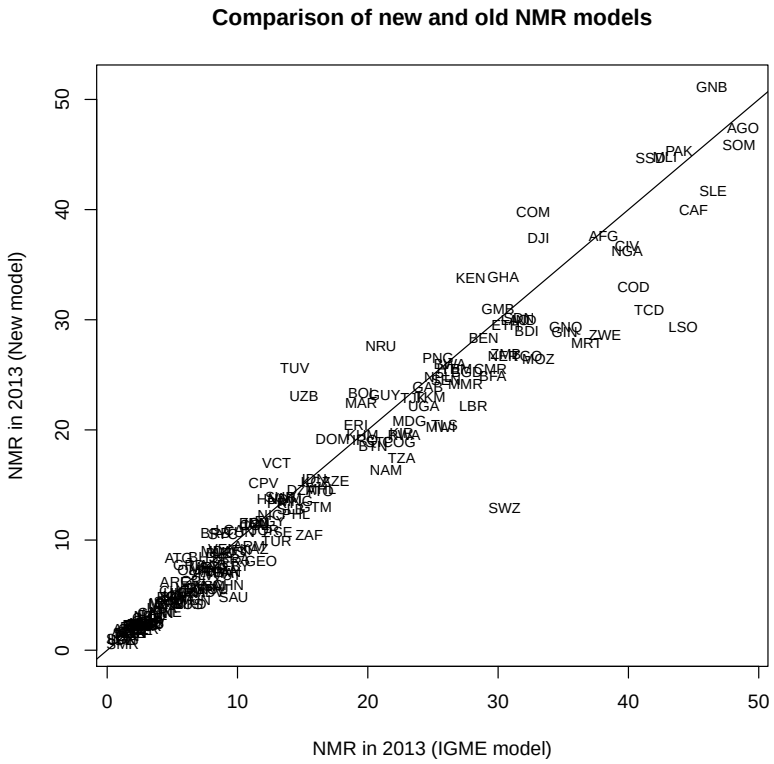
move above or below the expected line, as is the case with the Dominican Republic and Azerbaijan. In addition there is generally less uncertainty around the estimates, especially in periods where there are data.

Figure 7: Estimated Neonatal Mortality (deaths per 1,000 births); proposed versus existing model.



Despite some differences between the two models, NMR estimates in recent period are generally similar. Figure 8 compares estimates from the two models in 2013. For countries with low levels of NMR, estimates are very similar. Variability increases as NMR increases, although it is important to note that this is also often associated with greater uncertainty intervals.

Figure 8: Scatter plot of new versus old NMR estimates in 2013.



5 Summary

In this paper we introduced a new model for estimating neonatal mortality rates. The model can be expressed as the product of an overall relationship with U5MR and a country-specific effect. The overall relationship with U5MR is a simple linear function, while the country-specific effect is modeled through B-spline regression as a country-specific intercept plus fluctuations around that intercept. Estimates of the NMR were produced for 195 countries, spanning at least the period 1990–2015.

The model appears to perform well in a wide variety of situations where extent and type of data available varies. In many developed countries, where VR data series are complete and uncertainty around the data is low, NMR estimates follow the data closely. On the other hand, where there is limited data available or if uncertainty around the data is high, estimates are more influenced by the trends in U5MR.

Model estimates were compared to estimates from the existing IGME model. The notable advantage of this model is that trends in NMR for countries without VR data are driven by the data itself, rather than just reflecting trends in U5MR, as is the case with the existing model. Another advantage of this model is that it is along the same

methodological lines as the current model used by IGME to estimate U5MR (Alkema and New, 2014). Regardless, for most of the countries, estimates of NMR produced by the two models are reasonably similar, especially for more recent periods. A switch in the model used is unlikely to cause any drastic changes.

Future work will focus on the interpretation of results. In particular, for outlying countries, it will interesting to try and distinguish real country effects from probable data issues.

A Appendix

A.1 More details on modeling $P_{c,t}$

Splines regression offers a flexible method to estimate $P_{c,t}$. On the log scale, define

$$log(P_{c,t}) = \sum_{k=1}^{K_c} B_{c,k}(t)\alpha_{c,k}$$

where $B_{c,k}(t)$ is the kth basis spline evaluated at time t and $\alpha_{c,k}$ is splines coefficient k. The knot points k were chosen to be equally spaced and such that the resulting splines are non-zero for a total of 10 years. The coefficients to be estimated, $\alpha_{c,k}$, can be rewritten in the form of an intercept and fluctuations around the main intercept:

$$\alpha_{c,k} = \lambda_c + [\boldsymbol{D}'_{K_c}(\boldsymbol{D}_{K_c}\boldsymbol{D}'_{K_c})^{-1}\boldsymbol{\varepsilon}_c]_k$$

where

- λ_c is the splines intercept for country c;
- $\boldsymbol{D}_{K_c}$ is a first-order difference matrix, where $D_{K_c i,i} = -1$, $D_{K_c i,i+1} = 1$ and $D_{K_c i,j} = 0$ otherwise;
- $\boldsymbol{\varepsilon}_c$ represents fluctuations around the main intercept; $\boldsymbol{\varepsilon}_c = (\varepsilon_{c,1}...\varepsilon_{c,Q_c})'$ where $Q_c = K_c - 1$ and $\varepsilon_{c,q} = \Delta\alpha_{c,q}$ for $q = 1...Q_c$.

The intercepts λ_c represent country-specific deviations from the overall intercept. As such the λ_c's were modeled hierarchically and centered at zero so that

$$\lambda_c \sim N(0, \sigma^2_\lambda)$$

with a diffuse prior on σ^2_λ.

First-order differences in adjacent spline coefficients, $\Delta\alpha_k = \alpha_k - \alpha_{k-1}$, are penalized to guarantee the smoothness of the resulting trajectory. This penalty is achieved by imposing

$$\varepsilon_{c,q} \sim N(0, \sigma^2_\varepsilon).$$

The σ^2_ε essentially acts as a global smoothing parameter, ensuring a relative level of smoothness for all trajectories worldwide. The σ^2_ε is modeled hierarchically such that

$$\log(\sigma^2_\varepsilon) \sim N(\chi, \psi^2_\sigma).$$

As σ^2_ε decreases, the fluctuations go to zero, and the $\alpha_{c,k}$'s become a country-specific intercept.

A.2 Data model

For VR data series, the error term $\delta_{c,i}$ is modeled as

$$\delta_{c,i} \sim N(0, \tau^2_{c,i}),$$

where $\tau^2_{c,i}$ is the stochastic standard error. These was obtained as follows. For each year corresponding to observation i in country c,

- A total of 3,000 simulations of under-five deaths d_5 were drawn from a Poisson distribution $d_5^{(s)} \sim Pois(\lambda_5)$ with $\lambda_5 = B \cdot_5 q_0$ where B = live births and ${}_5q_0$ = the probability of death between ages 0 and 5;
- A total of 3,000 simulations of neonatal deaths d_n were drawn from a Binomial distribution $d_n^{(s)} \sim Bin(d_5^{(s)}, p)$ where $p =_n q_0/{}_5q_0$ and ${}_nq_0$ is the probability of death in the first month of life.
- The ratio $y^{(s)} = logit\left(\frac{d_n^{(s)}}{d_5^{(s)}}\right)$ was calculated for each of the simulated samples and the standard error $\tau_{c,i}$ was calculated as $\sigma(\boldsymbol{Y})$ where $\boldsymbol{Y} = (y^{(1)}, y^{(2)}, ...y^{(s)})$, $s = 3,000$.

For non-VR data series, the error term δ_i is modeled as

$$\delta_{c,i} \sim N(0, \nu^2_{c,i} + \omega^2),$$

where $\nu_{c,i}$ is the sampling error and ω^2 is non-sampling error. The sampling error was set to 10% for those observations where sampling error was not reported (including SVR data). Diffuse priors are chosen for ω^2.

A.3 Model summary

The full model is summarized below.

$$
\begin{aligned}
r_{c,i} &\sim N(R_{c,t}, \delta_i^2) \\
\delta_i^2 &= \begin{cases} \tau_{c,i}^2 & \text{for VR data,} \\ \nu_{c,i}^2 + \omega^2 & \text{for non-VR data} \end{cases} \\
R_{c,t} &= f(U_{c,t}) \cdot P_{c,t} \\
\log(f(U_{c,t})) &= \beta_0 + \beta_1 \cdot (\log(U_{c,t}) - \log(U_{cut}))_{[\log(U_{c,t}) > \log(U_{cut})]} \\
\log(P_{c,t}) &= \sum_{k=1}^{K_c} B_{c,k}(t)\alpha_{c,k} \\
\alpha_{c,k} &= \lambda_c + [\boldsymbol{D}'_{K_c}(\boldsymbol{D}_{K_c}\boldsymbol{D}'_{K_c})^{-1}\boldsymbol{\varepsilon}_c]_k \\
\lambda_c &\sim N(0, \sigma_\lambda^2) \\
\varepsilon_{c,q} &\sim N(0, \sigma_\varepsilon^2) \\
\log(\sigma_\varepsilon^2) &\sim N(\chi, \psi^2)
\end{aligned}
$$

where

- $R_{c,t}$ is the true ratio in country c at time t, $R_{c,t} = \frac{N_{c,t}}{U_{c,t} - N_{c,t}}$;
- $r_{c,i}$ is observation i of the ratio in country c;
- $\tau_{c,i}$ is the stochastic standard error, $\nu_{c,i}$ is the sampling error and ω^2 is non-sampling error;
- β_0 is global intercept; β_1 is global slope with respect to U5MR; U_{cut} is the level of U5MR at which β_1 begins to act;
- $P_{c,t}$ is country-specific multiplier for country c at time t;
- $B_{c,k}(t)$ is the kth basis spline evaluated at time t and $\alpha_{c,k}$ is splines coefficient k;
- λ_c is the splines intercept for country c;
- $\varepsilon_{c,q}$ are fluctuations around the country-specific intercept;
- σ_ε^2 is a global smoothing parameter.

The model was fit in a Bayesian framework. Priors are given by

$$\begin{aligned}
\omega &\sim U(0, 40) \\
\beta_0 &\sim N(0, 100) \\
\beta_1 &\sim N(0, 100) \\
U_{cut} &\sim U(0, 500) \\
\sigma_{\lambda c} &\sim U(0, 40) \\
\chi &\sim N(0, 100) \\
\psi &\sim U(0, 40)
\end{aligned}$$

A.4 Projection

NMR estimates were produced up until 2015. Estimates were projected forward using the following method. Start at $k =$ the first $\alpha_{c,k}$ year that is past last year of observed data. For each time period to be projected:

- draw $u_c \sim N(0, \sigma_\varepsilon^2)$
- $\alpha_{c,k}$ is then $u_c + \alpha_{c,k-1}$

This method essentially propagates the level of the country-specific estimate λ_c with the slope of the expected trajectory.

A.5 Computation

Samples were taken from the posterior distributions of the parameters via a Markov Chain Monte Carlo (MCMC) algorithm. This was performed through the use of JAGS software (Plummer 2003). In terms of computation, six chains with different starting points were run with a total of 50,000 iterations in each chain. Of these, the first 10,000 iterations in each chain were discarded as burn-in and every 20th iteration after was retained. Thus 2,000 samples were retained from each chain, meaning there were 12,000 samples retained for each estimated parameter.

Trace plots were checked to ensure adequate mixing and that the chains were past the burn-in phase. Gelman's $\hat{R}$ (Gelman and Rubin 1992) and the effective sample size were checked to ensure a large enough and representative sample from the posterior distribution. The value of $\hat{R}$ for all parameters estimated was less than 1.1.

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