How Does Assortative Mating Impact Intergenerational Education Mobility? A Two-Sided Matching Model

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Abtract

Intergenerational education mobility estimation can be inconsistent due to potential endogeneity. In this paper, I investigate the question of whether endogeneity is due to effects of parents' assortative mating on unobserved mutually determined factors that affect their children's educational attainment. This study develops a structural model based on a two-sided matching model to identify the underlying assortative mating pattern and to filter out any bias caused by such endogeneity. I show that 1) In Chinese marriage market, the assortative matching is asymmetric and does cause an endogeneity problem；a woman considers a man's education level, hukou status and other unobserved qualities that are correlated with child's education, while a man considers unobserved qualities of women that are not correlated with child's education; 2) causal relationship of parents' education, leadership and hukou on child's education do exist, and the coefficients of father's characteristics are greater than mother's.

KEYWORDS: Assortative Mating, Integenerational Education Mobility, Endogenous Matching

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1. Introduction

Assortative mating, the nonrandomness of people matching in romantic relationships, plays a crucial role in the formation of families, and affects the characteristics of reproduction. The potential effects of assortative mating on social inequality are major reasons why sociologists are interested in the topic; to what degree families are formed by inter-marriages could be used to examine the openness of salient social boundaries, such as education, race, religion, and social status. In addition, to the extent that parental characteristics affect children's status, the potential impact of the mating pattern on intergenerational mobility is also an important aspect.

Studies on inter-generational mobility usually consider correlation between parents' and children's social status, education level and class as indicators of inequality between generations. Previous studies of stratification have engaged mothers' and fathers' education levels, occupations, class and social status jointly as predictors of intergenerational human capital transmission. However, these are solely individual characteristics, and other characteristics formed mutually by the parents yet unobserved in individual-level data may be omitted. For example, educational homogamy is related with consistency in parenting attitudes and agreement in human capital investments that contribute to their children's performance in school (Beck & Gonzalez-Sancho, 2009). These unobserved mutual characteristics can lead to inconsistent estimation of the partial effects of parents' characteristics on the child's education. Specifically, individuals with higher education or social status may have unobserved matching qualities which are correlated to, for instance, greater consistency in communication and parenting, and other unobserved mutual characteristics that affect children's outcomes.

Taking assortative mating into consideration, researchers need to be careful in examining how parental characteristics affect children's education. If marriage sorting is formed on unobserved matching qualities such as care for children, and housework abilities in addition to observed education and social status, and if the unobserved matching qualities are correlated with unobserved characteristics in the error term of children's education equation, then parents' education level and social status become endogenous. On the other hand, if the unknown qualities underlying matching, such as attractiveness of appearance, are independent to the unobserved characteristics affecting children's education attainment, estimation remains unbiased. Hence, it's important to examine whether a certain matching observed in data causes endogeneity. If it does, then the endogenous assortative matching should be filtered out in order to get consistent estimation.

The classical solution to the endogeneity problem is to estimate Two-stage-least-square models using instrumental variables. A perfect instrument must be independent of the outcome but related to the endogenous variable. However, it's difficult to find such an instrument in this case. The matching of a given man and woman with a certain level of education and other observed characteristics is determined by their mutual marriage decisions. As children's education is always a big concern in marriage, there is hardly an observed variable satisfying the conditions for being a perfect instrument. For example, in a model regressing children's education on parental characteristics, grandparents' education has been always used as instrumental variables to deal with the endogeneity of parental characterisstics, but recent research such as Sharkey and Elwert (2011) shows that grandparents' education level also has a significant impact on grandchildren's education. The same argument applies to other characteristics as well.

To overcome the problem of missing instruments, I build a structural model to address assortative matching explicitly. The nature of using instruments is to take advantage of its exogenous variation. As long as similar variation can be obtained, the consistent estimation still yields. So if we examine the mechanism of assortative matching closely, we can find before a decision is made, each individual strategically iterates with all the agents on the other side because everyone in the marriage market is ranked by certain qualities and each individual would like to choose a partner that is not only ranked highest in his preference profile, but also is available for him. These iterations with individuals to whom they do not marry provide sufficiently large exogenous variation for consistent estimation.

The model consists of two parts. The first part is the main equation estimating the partial effects of father's and mother's characteristics on the child's education level. The second part is an empirical one-to-one matching model. Each individual gets a matching value or utility through matching. The strategic iterations give the model a nontrivial game theoretic foundation. Specifically, if we assume all the matches observed are at an equilibrium level, i.e., stable, then the intervals, in which the latent matching value of each one lies, can be computed. With the matching values representing individuals' preference profiles, the matching process comes to the current results as observed in data. Then, using these values as dependent variables in the matching model, we can regress them on the observed characteristics in the main equation.

Although the equation system is estimated simultaneously, the idea is a two-step procedure. First, the results provide a convenient way to check for endogeneity. This presents evidence to determine the pattern of the assortative matching. More specifically, when only the father's characteristics are endogenous, then men are ranked by women based on unobserved qualities which also affect children's education. When only the mother's characteristics are endogenous, then women are ranked on such qualities. When characteristics of both are endogenous, then they are both ranked on these qualities. Second, when the endogeneity is verified, we can compare the results in the main equation of the system with the results of estimating it alone by OLS. Then we can find how much the assortative matching pattern underlying the endogeneity affect the estimation.

To implement this model, I use household survey data from 2006 China General Social Survey (China GSS) organized by Renmin University of China, which is the first general social survey conducted annually and nationally since 2003 throughout 10,000 households in China. China's post 1978 market economy reform and other large-scale social transformation enable both changes in family formation and intergenerational educational mobility. The new economy provides incentives for people to increase their social status by obtaining more education and accumulating human capital (Bian, 2002). At the same time, Chinese couples had increasingly similar education since the1980s (Han, 2010). The rising education homogamy makes it interesting to look at whether and how patterns of assortative mating and intergenerational education mobility are related.

Traditional Chinese family formation tends to be based on negative assortative mating as mothers generally specialize in household work including child bearing and education, while fathers work outside the home. But whether and how traits of assortative mating differ in shaping heterogamy or homogamy, especially as Chinese society has experienced major transformations, remains unknown. Previous research in China has focused on the persistency of occupational transition from parents to children, as well as intergenerational mobility of education, occupations and social status. However, there has been no attempt to explore whether and how assortative mating affects intergenerational educational mobility of modern China. To this end, I show that in this marriage market, the assortative matching pattern is that men are ranked based on the observed and unobserved qualities affecting the child's education level, but women are not ranked by these. Also, I find that this one-sided endogeneity pattern leads to moderate bias. So a reasonable conjecture is that if the major concerns when choosing a wife or husband are not correlated to children's education, assortative matching has no effect on the estimation of intergenerational education mobility. On the contrary, the bias could be large and needs to be addressed.

The remainder of the paper is organized as follows. In Section 2, I briefly discuss related literature. In Section 3, a two-sided one-to-one matching model is built. Section 4 mainly discusses the empirical model. Section 5 describes data and variables. Main findings are presented in Section 6 and Section 7 concludes the study.

2. Literature Review

The topic of how assortative mating impact intergenerational education mobility is related to two branches of literature.

The first branch studies assortative mating and its impact on social mobility. Becker's marriage model (1974, 1981) provides an important paradigm for analysis of assortative mating. When traits are complementary, people tend to marry who are similar to them. Previous studies regard traits like lifestyle, culture, religion and race as determinants shaping this positive assortative mating pattern, which leads to homogamy. At the same time, negative assortative mating may exist and occur as for characteristics that are substitutes, when there are premiums of specialization, which constitutes basis for heterogamy. However, the literature on the relationship between assortative mating and intergenerational inequality is limited (Schwartz, 2013). While most research suggest that both mothers' and fathers' social status, education, and class are significant in describing intergenerational transmission of human capital, other studies find that there is some evidence of assortative mating effects (Beller, 2009). Some researchers go further to explain the mechanism of how assortative mating affects children's education. BGS shows that children from educational homogamy families perform better because educational homogamy is correlated with consistency in parenting attitudes, agreement in human capital investments, and as a result, should have a positive effect on children's education, regardless of parental education level. However, there is no agreement on whether or not education homogamy as a result of assortative mating weakens inequality. Beck & Gonzάlez-Sancho (2009) find that education homogamy does not deepen intergenerational education inequality because the effects are positive regardless of the education levels or distributions, 2009); while Martin et al. (2007) disagrees, finding that children with both less educated parents have the lowest cognitive outcomes at age five, worse than children with only one less educated parent. When both mother' and father's characteristics affect results of children, the ways in which they matter may be different. Besides, there are not much literature on whether there is variance in assortative mating patterns of females and males, and how that differences matter in shaping children's educational outcomes.

The second branch focuses on empirical implementation of assortative matching models. In their seminal paper, Gale and Shapley (1962) develop a marriage model to illustrate matching in two-side markets. This model assumes that the payoff or utility obtained through a match is *ex ante,* i.e., known to the individual before she makes a decision. So everyone in the market tries to maximize their utility through matching. This framework is called *utility non-transferable model.* In the game theoretic setup, the solution concept is *stable matching,* meaning that no Pareto improvement can be achieved by divorce and match a new one. Roth and Sotomayor (1990) generalize the basic model with relaxed assumptions. Based on the theoretical models, Sørensen (2007) and Fox (2010) developed two estimators to estimate the assortative matching model. Based on Bayesian inference, Sørensen estimated an equation system consisted of a main equation of interest and a matching equation. Being a one-to-one matching model, my empirical method is an extension of his one-to-many matching model. The extension is not trivial because the dependent variable here is a continuous while in Sørensen, it's a binary variable. Fox develop another method which takes advantage of a maximum score estimator. However, his estimator is used to estimate a utility transferable matching model, while in this paper, the matching is utility nontransferable.

3. The Matching Model

In this section, I present a special one-to-one matching model of Gale and Shapley (1962), Shapley and Shubik (1972), and Sørensen (2007). In a game theoretical view, the equilibrium concept adopted here is stable matching. In Irving and Leather (1986), it's shown that for a one-to-one two sided matching of size n, there exist at least 2^{n-1} stable matching solutions. Therefore, existence of solutions is already satisfied. However, to carry out empirical analysis, more assumptions are needed so that the uniqueness is also guaranteed. To this end, I show that with the assumption of assortative matching, the equilibrium of the one-to-one matching exists and is unique.

3.1 Preliminaries

Before the formal model is presented, the main assumptions are discussed here.

First, assume that the preference of an agent is assortative. An agent's preference over the partners is assortative if the partners are ranked by this agent in a descending order and the rankings of any two possible partners are irrelevant with other alternatives. In the marriage model, if we assume that information is perfect[2], then information of the quality of every man or woman could be obtained by its potential partner without bias. Hence, observing the performance of each potential partner in the past, an order forms on each side of the market for every person. This assumption leads to uniqueness by the concept of strong substitutability.

DEFINITION 1. An agent a's preference relation $P(a)$ satisfies substitutability if, for any sets S and S' of partners of a with $S \subset S$,

$$b \in Ch(S' \cap b, P(a)) \text{ implies } b \in Ch(S \cap b, P(a))$$

where $Ch(S \cap b, P(a))$ is the optimal subset of S for agent a under preference relation $P(a)$.

DEFINITION 2. An agent a's preference relation $P(a)$ satisfies strong substitutability if, for any sets S and S', with $S' P(a) S$,

$$b \in Ch(S' \cap b, P(a)) \text{ implies } b \in Ch(S \cap b, P(a))$$

Now, I introduce the first proposition. The proof is in the Appendix.

LEMMA 1. If an agent a's preference relation P(a) is assortative, then it satisfies strong substitutability.

PROPOSITION 1. If a matching is assortative, then the stable pair set is unique.

Proof of proposition 1 can be found in the Appendix and such a unique equilibrium can be found out by an algorithm which is also described in Appendix.

Second, the matching value is non-transferable. As a match is formed, both sides of the match yield a value. This is called matching value or utility. As the assumption of assortative preference is required, we need to assume that the matching value cannot be transferred from one side to another side. If not, then a high-quality woman may match a low-quality man as long as the total utility is sufficiently high and the woman is guaranteed a large share. If such a mechanism exists, then the rankings are disordered.

3.2 Equilibrium Matching

[2] In the period the dataset cover, new couples in China met before marriage in blind dates with prior information gathered by their parents. Due to the small size of the communities, it's conventional to report the truth. So perfect information is not as restrictive as in many other markets.

In this subsection, I characterize the unique equilibrium with a set of inequalities.

Let M and W denote the sets of men and women and they are disjoint and finite. The set of all possible matches is given by $\Omega = M \times W$. A match ω, a subset of Ω, is a set of matches such that $(i,j) \in \omega$ if and only if man i and woman j match. Hence, the equilibrium concept here is pair-wise stability (Roth and Sotomayor, 1990; Echenique and Oviedo, 2006).

Let $\omega(i)$ denote the woman that matches man i, and let $\omega(j)$ be the man matching woman j. Then in the context of pair-wise matching, we have the following equivalent relations

$$m_{ij} = 1 \Leftrightarrow (i,j) \in \omega \Leftrightarrow i = \omega(j) \Leftrightarrow j = \omega(i),$$

As pointed out in assumptions, matching value is non-transferable. Thus we define separately the matching value yielded in a matching of the two sides. The value obtained by i if it matches j is Q_j^W, and the value received by j if it matches i is Q_i^M. If a match is pair-wise stable, then it's equivalent to say there's no blocking pair. A pair (i',j') is a blocking pair if there exists i and j such that $Q_{j'}^W > Q_j^W$ and $Q_{i'}^M > Q_i^M$. Thus, when there's no blocking pair, if i ranks higher than i' and no woman who matches i' ranks higher than the worst woman matches i. So if we list the men and women in two columns by their ranking, and connect the matched pairs by lines, this suggests there is no crossing. This provides a constructive algorithm to find out the unique equilibrium.

Such a non-blocking pair condition provides a set of inequalities to characterize and identify the equilibrium. If we find the lower and upper bound of quality index for either side, i.e. $\underline{Q}_i^M$, $\overline{Q}_i^M$, $\underline{Q}_j^W$ and $\overline{Q}_j^W$, we can define a pair-wise equilibrium as follows.

$$\omega = \omega^{\varepsilon} \Leftrightarrow Q_j^W \in (\underline{Q}_j^W, \overline{Q}_j^W) \text{ and } Q_i^M \in \left(\underline{Q}_i^M, \overline{Q}_i^M\right), \forall (i,j) \in \Omega$$

where ω^{ε} is the stable matching. The bounds are derived in the Appendix and estimation relies on this characterization of equilibrium

4. Empirical Model

The empirical model consists of two parts. The first part is the main equation determining how mothers' and fathers' characteristics affect children's education.

$$educ_{ij} = \alpha_0 + \alpha_1 M + \alpha_2 W + \alpha_3 N + \varepsilon_{ij} \quad (1)$$

where $educ_{ij}$ is the education of the person whose parents are i and j. M and W are characteristics vectors of i and j. N that contain other information of the observant. Because of sorting, M and W are positively correlated thus separate identification fails unless we deal with the assortative matching explicitly. The matching is characterized as follows.

$$Q_i^M = \beta M' + \eta_i \quad (2)$$

$$Q_j^W = \gamma W' + \delta_j \quad (3)$$

These are separate matching value equations. The value received by woman j is determined by the quality of man i, and the value received by man i is determined by the quality of woman j. Note that the prime on M and W suggests that here, it's not necessary to include the same variable of the man and the woman as in the main equation.

I need to estimate the three equations simultaneously. If the assortative matching issue does exist, then at least one of the estimates of the parameters in the following equation should be significant.

$$\varepsilon_{ij} = \kappa\eta_i + \lambda\delta_j + \nu_{ij} \tag{4}$$

4.1 Identification: The Structure of Error Terms

Assume that ε_{ij}, η_i, δ_j and ν_{ij} follow normal distribution. Specifically, $\varepsilon_{ij} \sim N(0,\ \sigma_\varepsilon)$, $\eta_i \sim N\left(0,\ \sigma_\eta\right)$, $\delta_j \sim N(0,\ \sigma_\delta)$ and $\nu_{ij} \sim N(0,\ \sigma_\nu)$. To fix the scale, let $\sigma_\eta = \sigma_\delta = 1$. Then from the equation of error terms, we have

$$\begin{pmatrix} \varepsilon_{ij} \\ \eta_i \\ \delta_j \end{pmatrix} \sim N(0, \quad \begin{bmatrix} 1+\kappa^2 & \kappa & \eta \\ \kappa & 1 & 0 \\ \eta & 0 & 1 \end{bmatrix})$$

The signs in the second part model are identified by λ. Opposite sign of λ gives opposite estimates of (β, γ, κ). As a woman with higher quality will more likely to have a child with longer education, I assume λ is non-negative and follows a normal distribution truncated on the left at 0.

As for other parameters in the model, we assume that the prior distribution of $(\alpha,\ \beta,\ \gamma)$ is multivariate normal and the prior distribution of κ is normal. The prior means for all the parameters are all 0 and their variance-covariance matrix are $10I$, where I is an identity matrix. The prior distribution of $\frac{1}{\sigma_\nu^2}$ follows $G(2,1)$ where $G(\cdot)$ is gamma distribution.

4.2 Iteration and Estimation

In this model, matching decisions interact by the algorithm in Appendix. For estimation, iteration means that we cannot analyze the matching decisions in isolation. So we need to run over all other agents' choices when analyzing one agent's choice. Hence the dimension of this integral is often the thousands thus makes the traditional estimating methods like MLE get lost into a computational nightmare. Hence, following Sørensen (2007), we adopt the Bayesian estimation using Markov Chain Monte Carlo (MCMC) to deal with this integration problem.

Estimation uses the augmented data with the latent variables Q^M and Q^W. From the prior distribution, each parameter is estimated by data. Then the prior distribution is changed. Together with data, the new distribution is used in the second iteration to get a further posterior distribution. Repeat this process to a sufficiently large number, and we can see the posterior means and variance converge.

Estimation results are reported in Section 5. They are based on 500 draws and the first 10 percent are discarded for burn-in.

5. Data

5.1 A Description of the Sample

The source of the data is China General Social Survey (China GSS). This database is an annual or biannual survey of China's urban and rural households designed to gather repeated cross-sectional data on social trends and the changing relationship between social structure and quality of life in China, which includes information of households' education, employment, marriage and family, economic, social identification, social capital, physical & metal health, political participation, rural construction, civil rights and so no. Because people tend to complete their education by mid-twenties, so cross sectional setting is appropriate for research in education (Black & Devereux, 2010). Concerning education, measurement error needs to be noticed in data from self-report survey, especially as with self-reported years of education. The majority of the individuals in the survey are from 40 to 60 years old, so the assortative mating pattern of their parents marks the traditional family formation culture and how that affects intergenerational education inequality of Chinese born from 1950s to 1970s.

5.2 Description of Variables

In the empirical model, variables are divided into four categories: education attainment of an individual, mother's characteristics, father's characteristics and other covariates of the individual.

Education attainment of the individual. *eduyear* is constructed to indicate the education attainment. The variable ranges from 1 to 21. So the largest possible number of total years of education from primary school to a doctor's degree can be 21 years, and the support of the variable in the sample fits it well.

Mother's characteristics. I include three variable indicating mother's characteristics, i.e., *mleader, medu,* and *mhukou. mleader* goes from 0 to 9 as status improves. *medu* ranges from 0 to 3, indicating the education level is none, primary school, middle school, high school and college or higher respectively. *mhukou* indicates the mother's hukou status. Hukou is a typical household registration system of China. It connest different social welfare program such as housing, medical treatment, etc. This variable ranges from 1 to 6, where the numbers respectively indicates rural, township, county, city, provincial capital, and municipality directly under the Central Government. Since the hukou status and education level are ususally time-invariant, they are included in the main equation and the matching equation. But the leadership status does not necessarily remain unchanged since marriage, so it's only included in the main equation.

Father's characteristics. *fleader, fedu* and *fhukou* are included. The notations share the same meaning with mother's characteristics.

Other individual covariates. Variables in this category are those affecting one's education attainment yet not completely captured by parents' characteristics. *sex* and *gender* are exogenous and used as control variables. *sex* is a dummy variable with 1 indicating female and 0 indicating male. *age* shows self-reported ages of individuals in 2006.

Details of categorical variables are shown in Table 1, and descriptive statistics of all variables are presented in Table 2.

TABLE 1 Categorical Variables

mhukou/ fhukou		*mleader/ fleader*		*medu/ fedu*	
1	Rural	0	No leadership	0	No education
2	Township	1	Production Team leader	1	Primary school
3	County	2	Village leader	2	middle school
4	Prefecture-level city	3	Village head	3	high school
5	Provincial capital	4	Township Commune leader	4	College or higher
6	Municipality directly under the central government	5	Township Commune head		
		6	Group leader		
		7	Leader in lower branch		
		8	Middle-level manager		
		9	Main leader		

TABLE 2 Descriptive Statistics

Variable	Obs	Mean	SD	Min	Max
edu year	5118	8.2804	3.2829	1	21
sex	5118	.5639	.4960	0 (43.61%)*	1 (56.39%)
age	5118	52.4639	11.7497	25	77
mleader	5118	.1292	.9649	0	9
fleader	5118	.5492	1.9546	0	9
fhukou	5118	2.1884	1.7720	1	6
fedu	5118	1.8490	.8591	1	4
medu	5118	1.5039	.7133	1	4
mhukou	5118	2.1102	1.7573	1	6

TABLE 3 Correlation Coefficients of Parental Characteristics

Correlation	*fleader*	*mleader*	*fhukou*	*mhukou*	*fedu*	*medu*
fleader	1.0000					
mleader	0.7800	1.0000				
fhukou	0.0600	0.0633	1.0000			
mhukou	0.0658	0.0784	0.9203	1.0000		

fedu	0.0268	0.0135	0.0547	0.0582	1.0000	
medu	0.0332	0.0319	0.0716	0.0830	0.7790	1.0000

6. Empirical Findings

In this section, I present the empirical findings from the estimates of the structural model. The findings are consisted of three parts. First, I examine the matching pattern revealed from the sample. Different patterns of assortative matching have different impact on the estimates of the main equation. Then, I analyze the main equation after the endogenous matching is filtered out. At last, I compare the results with those obtained by regress the main equation alone using OLS.

6.1 The Pattern of the Assortative Matching

To examine the pattern of the assortative matching, we first notice in that κ in table 4 is significant. Recall equation (4). Hence the error term of the main equation and the error of the mother's matching value equation are correlated. This implies that when a woman chooses a man to marry, she considers qualities known to her but unobserved in the data (η). Moreover, these qualities are correlated to the unobserved mutual characteristics in the main equation (ε) that affetcs the child's education attainment.

TABLE 4 Estimates of matching Equations

Mother's Matching Value Equation	Coefficients (SE)
Fedu	0.0477
	(0.0156**)
Fhukou	0.0277
	(0.0073**)
Father's matching value	
Mother education	-0.0002
	(0.0164)
Mother hukou	0.0002
	(0.0063)
κ	-0.1379
	(0.0828*)
λ	-0.2667
	(0.2387)

*, **, and *** indicate significance at the 10%, 5% and 1% level.

To see that the correlation does cause endogeneity, notice that the coefficients on *feduc* and *fhukou* are both significant. This shows that these characteristics together with the unobserved qualities are all considered by a woman. Hence for each child observed, her father's education level and hukou status are correlated to the unobserved characteristics epsilon, then endogeneity arises.

On the contrary, lambda is not significant. So following the logic above, the mother's observed characteristics have no endogeneity, in the context of assortative matching. In the equation of father's matching value, the observed characteristics are also insignificant. This indicates that men consider some other qualities that are not captured by education level or hukou status. This is still assortative matching, but since those unknown qualities are not correlated to epsilon, this matching pattern has no effect on the child's education outcome.

Hence, the assortative matching pattern in the market characterized by the sample is asymmetric; the qualities considered by men and women are distinct, and only the men's qualities bring endogeneity to the main equation when estimating their partial effects on the child's education.

6.2 Net Effects of Intergenerational Education Mobility

From the estimates of the coefficients in the main equation of the structural model (see table 5), we can analyze the inter-generational education inequality. As is significant in the matching equation, it shows that there is endogeneity of father's education and *hukou* in reduce form regression. By controlling for the effects of education and hukou assortative mating, net effects of parental characteristics on children's education are obtained. From the first column of Table 3, it can be learnt that father's characteristics including education level, leadership, *hukou* status, mother's characteristics of education level, and *hukou* status positively affect children's education. Mother's leadership does not have a significant effect on children's education. A male on average has 0.9151 year more education than a female. For the sample of individuals in CGSS 2006, the younger an individual is, the more years of education he has on average. Mother's leadership status does not have a significant effect on children's total years of education, while the leadership position of father has a slightly positive effect on children's education.

TABLE 5 Estimates of the Main Equation, Reduce Form, and Differences

	Structural Model		Reduce Form		Difference in coef.
	coefficient	SE	coefficients	SE	
constant	9.5848	0.2067***	9.6284	0.2115***	0.0436
mleader	0.0378	0.0374	0.0377	0.0375	-0.0001
mhukou	0.2372	0.0686***	0.2332	0.0711***	-0.0040
medu	0.1984	0.0589***	0.2012	0.0592***	0.0028
fleader	0.0764	0.0189***	0.0762	0.01985***	-0.0002
fhukou	0.4138	0.0711***	0.4201	0.0716***	0.0063
fedu	0.5478	0.0617***	0.5430	0.0583***	-0.0048
sex	-0.9151	0.0802***	-0.9194	0.0782***	-0.0043
age	-0.0536	0.0034***	-0.0545	0.0035***	-0.0009

*, **, and *** indicate significance at the 10%, 5% and 1% level.

Inter-generational education correlation is strong for Chinese people as for 2006.

After controlling for the effect of assortative mating, mother's education and father' education both have significantly positive effects on children's education. Indeed, when mother' education increases by one level, children's years of education on average increases 0.1984 year, holding other factors unchanged; when a level of father's education upward, children's education increases 0.5478 year on average, holding other factors stable.

There are many ways how education of parents matter for children's outcomes. Gary Becker and Nigel Tomes (1994) explained the role of human capital transmission by the model of home investment in children. Parents with higher education and social status tend to have higher incomes, and parental incomes along with education, social status, and other characteristics could impact on both the quantity and quality of investment into children, which determines the final education results of children (Haveman & Wolfe, 1995). Empirical research of intergenerational education mobility also gives coherent evidences.

After controlling for education assortative mating, the coefficients of mother's education and father's education can be more securely compared. The coefficient of father's education 0.5478 is greater than mother's education, which is 0.1984, which supports some previous research in the area. A recent study uses the Chinese Household Income Project (CHIP, 2003) dataset, and finds that with both a father's and the mother's education have significantly positive effects on their children's education, the effect of the father's education is larger than that of the mother's (Sato & Li, 2007). The greater father-children education correlation is partly due to the stark gender educational inequality for Chinese people born before the 1970s. Along with the social transformation of China, there is steep increase in education attainment of Chinese. In the process of human capital investment, fathers' education determines the quantity and quality of inputs that children can have access to. The fact that fathers' hukou also has a larger effect than mothers' supports the discussion. Generally, fathers are more likely to be the head of a family, and their social status and education dominantly determines children's results.

6.3 Effects of Assortative Mating

The structural model allows for counterfactual analysis of absolute education and hukou heterogamy, as the assortative mating based on education and hukou has been filtered out by the matching model. Estimating the reduce form of the main equation, however, is inconsistent due to the endogeneity of father's characteristics. Consequently, by comparing the results in the structural model and reduce form, some implications of education assortative mating can be obtained.

Although education and hukou assortative mating does exist, the effects of are small, as the differences between structural results and reduce form are of small scales. Connected with the results in matching models, the reason underlying the small variance is very likely the different sorting patterns of men.

Chinese men sort women according to different patterns other than education and hukou. Although what those specific traits that men seek for are not observable, it could be deduced that they may not be contributable to children's education outcomes.

Looking at the changes in coefficients of education more closely, I find that without women's sorting pattern of education and hukou, mothers' education would have a slightly smaller effect on children's education, and fathers' education have a slightly greater effect on children's education.

There's no evidence that education and hukou assortative mating increase educational correlation across generation, thus add to intergenerational education stratification. However, the strategy engaged in this paper does show that assortative mating pushes up mother-children education correlation, and decrease father-children education correlation, with the decrease a bit greater, although still small.

7. Conclusion

This paper develops a method to analyze education and hukou assortative matching patterns and their effect on estimating intergenerational human capital transmission. It finds in the sample selected, the assortative matching is asymmetric; a woman considers a man's education level, hukou status and other unobserved qualities, while the education level and hukou status of a woman are not a man's concerns. Further, it shows that only the man's unobserved qualities are correlated to the unobserved mutual characteristics (like consistency in parenting style, and agreements in human capital investment) affecting the child's education attainment. After filtering out the endogenous matching, the results show that higher education level and hukou status of parents as well as higher level of leadership of father cause a higher education level of the child. This is not merely a significant positive *correlation;* the endogeneity is addressed so what we obtain is the true causal effects. Therefore, the results add evidence to the literature of social mobility.

The findings suggest that the pattern of assortative matching is crucial to determine whether the parents' characteristics are contaminated by endogenous matching. The results do not show that all kinds of assortative matching have an unobserved effect on the child's education level. On the other hand, when it does, the degree of the bias and inconsistency of estimating the main equation of interest alone depends on whether both parents' characteristics are endogenous or only one of them is. In the market analyzed in this paper, only fathers' characteristics are endogenous. Hence the difference between the structural model and the reduced form is moderate.

It is tempting to apply this approach to other samples where culture and time make the potential assortative pattern differ from the case studied here. Also, by discovering the underlying matching pattern, one can examine other issues where similar endogeneity may exist. For example, when estimating the partial effects of wives' characteristics on husbands' income, these characteristics may also be endogenous because they are likely to be correlated with unobserved qualities of the husband which both affect his income and their marriage decision. A better understanding of these stories hinges on identifying the matching pattern and distinguishing the assortative matching and the net effects of interest.

Appendix A. Uniqueness of Equilibrium Matching

PROOF TO LEMMA 1. For two sets S and S', with $S'P(a)S$, if $b \in Ch(S' \cap b, P(a))$, then b is ranks top in a's profile of $S' \cap b$. Since $S'P(a)S$, if a names an order in $S' \cap S$, there must be at least one element in S which ranks below the corresponding element in S', given that other elements are ranked in the same order. If the element is top in S, then b is preferred than it, so $b \in Ch(S \cap b,\ P(a))$. If it's not, then b is still the top and the case is the same as in S'. Hence, $b \in Ch(S \cap b, P(a))$ still holds. Q.E.D

The proof of uniqueness is more constructive. First, I'll introduce an algorithm to characterize the equilibrium.

ALGORITHM. Assume that the preference over the agents of either side is homogeneous and we index the men and women according to the preference ranking such that $i > i' \Leftrightarrow i \succ_j i'$ and $j > j' \Leftrightarrow j \succ_i j'$. In the first step, man I matches woman J. In step 2, man $I-1$ matches woman $J-1$. Following the procedure and give that any dataset containing children's information have same number of fathers and mothers, all the stable matching can be recovered.

PROOF TO PROPOSITION 1. Suppose not. Then there exists at least two equilibria ω and ω' such that $\omega \neq \omega'$. Then there is at least one match in ω but not in ω'. Now consider the first step of the algorithm that forms such a match, which is denoted by $(i',\ j')$. Thus by the assumption of assortative matching, any j' available for i' ranks below j'. Hence $\omega'(i')$ ranks below j'. The same goes for i'. Thus $(i',\ j')$ is a blocking pair, which contradicts the fact that ω' is an equilibrium. Q.E.D

Appendix B. Characterizing the Unique Equilibrium by Inequalities

Since the equilibrium can be defined by non-blocking-pair condition, it can be characterized by inequalities, i.e. the non-crossing condition discussed in the paper.

Consider a matching ω. Suppose man i and woman j are not matched in ω. (i,j) is a blocking pair iff $Q_j^W > Q_{j'}^W$ and $Q_i^M > Q_{i'}^M$. So if it's not a blocking pair, then $Q_j^W < \overline{Q}_{ji}^W$ where $\overline{Q}_{ji}^W = minQ_{j'}^W$ if $Q_i^M > minQ_{i'}^M$ and is infinity otherwise. $\overline{Q}_{ij}^M$ is defined likewise.

Now suppose that man i and woman j match in $\omega. i$ or j is part of a blocking pair iff $Q_j^W < Q_{j'}^W$ or $Q_i^M < Q_{i'}^M$, where i' are the set of men that do not match j but would prefer to do so, and i' is just an analog of j'. Thus neither i nor j is part of a blocking pair iff $Q_j^W > \underline{Q}_{ji}^W$ and $Q_i^M > \underline{Q}_{ij}^M$, where $\underline{Q}_{ji}^W = maxQ_{j'}^W$ and

$\underline{Q}_{ij}^{M} = maxQ_{i\prime}^{M}$.

Therefore, ω is an equilibrium iff $Q_j^W \in (\underline{Q}_{ji}^W,\ \overline{Q}_{ji}^W)$ and $Q_i^M \in (\underline{Q}_{ij}^M,\ \overline{Q}_{ij}^M)$.

Appendix C. Conditional Posterior Distribution

We obtain the conditional posterior distribution by examining the kernels of the conditional posterior densities.

The conditional posterior distribution of Q_i^M is $N(\hat{Q}_i^M,\ \hat{\sigma}^2_{\hat{Q}_i^M})$ truncated to the interval $(\underline{Q}_{ij}^M,\ \overline{Q}_{ij}^M)$, where

$$\hat{Q}_i^M = M_i'\beta + \frac{\kappa[r_{ij} - U_{i,j}'\alpha - \lambda(Q_j^W - W_j'\gamma)]}{\sigma_v^2 + \kappa^2}, \text{ and}$$

$$\hat{\sigma}^2_{\hat{Q}_i^M} = \frac{\sigma_v^2}{\sigma_v^2 + \kappa^2}$$

The conditional posterior distribution of Q_j^W is $N(\hat{Q}_j^W,\ \hat{\sigma}^2_{\hat{Q}_j^W})$ truncated to the interval $(\underline{Q}_{ji}^W,\ \overline{Q}_{ji}^W)$, where

$$\hat{Q}_j^W = W_j'\gamma + \frac{\lambda[r_{ij} - U_{i,j}'\alpha - \kappa(Q_i^M - M_i'\beta)]}{\sigma_v^2 + \lambda^2}, \text{ and}$$

$$\hat{\sigma}^2_{\hat{Q}_j^W} = \frac{\sigma_v^2}{\sigma_v^2 + \lambda^2}$$

The prior distributions of α, β, γ and κ are $N(\overline{\alpha},\ \overline{\Sigma_\alpha})$, $N(\overline{\beta},\ \overline{\Sigma_\beta})$, $N(\overline{\gamma},\ \overline{\Sigma_\gamma})$, and $N(\overline{\kappa},\ \overline{\Sigma_\kappa})$. The prior distribution of λ is truncated on the left at 0. The prior distribution of $\frac{1}{\sigma_v^2}$ is gamma, $\frac{1}{\sigma_v^2} \sim G\ (a, b), a, b > 0$.

The conditional posterior distribution of α is $N(\hat{\alpha}, \hat{\Sigma}_\alpha)$, where

$$\hat{\Sigma}_\alpha = \left(\overline{\Sigma}_\alpha^{-1} + \frac{1}{\sigma_v^2} U_{i,j} U_{i,j}'\right)^{-1}, \text{ and}$$

$$\hat{\alpha} = -\hat{\Sigma}_\alpha\{-\overline{\Sigma}_\alpha^{-1} - \frac{1}{\sigma_v^2} U_{i,j}\left[r_{ij} - \kappa(Q_i^I - I_i'\beta) - \lambda\left(Q_j^W - W_j'\gamma\right)\right]\}$$

The conditional posterior distribution of β is $N(\hat{\beta}, \hat{\Sigma}_\beta)$, where

$$\hat{\Sigma}_\beta = (\overline{\Sigma}_\beta^{-1} + \frac{\sigma_v^2 + \kappa^2}{\sigma_v^2} M_i M_i')^{-1}, \text{ and}$$

$$\hat{\beta}=-\hat{\Sigma}_{\beta}\{-\overline{\Sigma}_{\beta}^{-1}\overline{\beta}+\frac{\kappa}{\sigma_v^2}M_i\left(r_{i,j}-U_{i,j}'\alpha-\kappa Q_i^I-\lambda\left(Q_j^W-W_j'\gamma\right)\right)-Q_i^M M_i\}$$

The conditional posterior distribution of γ is $N(\hat{\gamma},\ \widehat{\Sigma_{\gamma}})$, where

$$\hat{\Sigma}_{\gamma}=(\overline{\Sigma}_{\gamma}^{-1}+\frac{\sigma_v^2+\lambda^2}{\sigma_v^2}W_jW_j')^{-1}, \text{ and}$$

$$\hat{\gamma}=-\hat{\Sigma}_{\gamma}\{-\overline{\Sigma}_{\gamma}^{-1}\overline{\gamma}+\frac{\lambda}{\sigma_v^2}W_i\left(r_{i,j}-U_{i,j}'\alpha-\kappa(Q_i^I-I_i'\beta)-\lambda Q_j^W\right)-Q_j^W W_j\}$$

The conditional posterior distribution of κ is $N(\hat{\kappa},\ \widehat{\Sigma_{\kappa}})$, where

$$\hat{\sigma}_{\kappa}^2=[\frac{1}{\overline{\sigma}_{\kappa}^2}+\frac{(Q_i^M-M_i'\beta)^2}{\sigma_v^2}]^{-1}, \text{ and}$$

$$\hat{\kappa}=-\hat{\sigma}_{\kappa}^2\{-\frac{\overline{\kappa}}{\overline{\sigma}_{\kappa}^2}-\frac{1}{\sigma_v^2}\left(r_{i,j}-U_{i,j}'\alpha-\lambda\left(Q_j^W-W_j'\gamma\right)\right)(Q_i^M-M_i'\beta)\}$$

The conditional posterior distribution of λ is $N(\hat{\lambda},\hat{\sigma}_{\lambda}^2)$ truncated on the left at 0, where

$$\hat{\sigma}_{\lambda}^2=[\frac{1}{\overline{\sigma}_{\lambda}^2}+\frac{(Q_j^W-W_j'\gamma)}{\sigma_v^2}]^{-1}, \text{ and}$$

$$\hat{\lambda}=-\hat{\sigma}_{\lambda}^2\{-\frac{\overline{\lambda}}{\overline{\sigma}_{\lambda}^2}-\frac{1}{\sigma_v^2}\left(r_{i,j}-U_{i,j}'\alpha-\kappa(Q_i^M-M_i'\beta)\right)(Q_j^W-W_j'\gamma)\}$$

Let n denote the total number of matching in all the markets. The conditional posterior distribution of $\frac{1}{\sigma_v^2}$ is $G(\hat{a},\hat{b})$, where

$$\hat{a}=\alpha+\frac{n}{2}, \text{ and}$$

$$\hat{b}=[\frac{1}{b}+\frac{1}{2}\left(\gamma_{i,j}-\kappa(Q_i^M-M_i'\beta)-\lambda\left(Q_j^W-W_j'\gamma\right)\right)^2]^{-1}$$

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