Decomposing Mortality Changes: Compression or Shifting Mortality?

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Abstract

Shifting and compression of mortality have been studied through two components of mortality: modal age at death and variability of the age at death. These two components inform us about the timing and age patterns of mortality respectively. The aim of this study is to decompose changes in life expectancy into effects due to changes in the modal age at death and in the variability of the age at death. We introduce a new decomposition method, using recent expression of the Gompertz, and study the changes in its components. Our approach allows differentiating between the two underlying processes in mortality and their relevance to understand the dynamics of mortality. The results suggest that the increase in life expectancy, since the 1950's, is largely driven by a shift in the modal age at death.

1 Background

In the first half of the twentieth century, a compression of the distribution of deaths in a more narrow age interval has been observed in many low-mortality countries. (Cheung et al., 2009; Fries, 1980; Kannisto, 2000, 2001; Wilmoth and Horiuchi, 1999). This compression of mortality occurred as people tend to die more and more around a same age. Those changes in the variability of the age at death were also noticeable in the survival distribution by a rectangularization of the survival curve, and in the hazard distribution by a steeper slope. The slope of the hazard distribution through age has become steeper due to more pronounce decrease in mortality at young ages (Wilmoth, 1997). It has been shown that the reduction of infant and child mortality had led to an important compression of mortality between the late 1870's to the early 1950's (Wilmoth and Horiuchi, 1999).

In the second half of the twentieth century, the compression of mortality has been replaced gradually by a shift of the density curve toward older ages (Cheung et al., 2005; Yashin et al., 2001). The shifting mortality hypothesis suggests a delay in the mortality schedule, but with a shape remaining nearly constant (Bongaarts and Feeney, 2002, 2003; Canudas-Romo, 2008). The study of the modal age at death has been useful to study the shifting mortality hypothesis. By shifting the modal age at death, the deaths around this age move towards older ages (Canudas-Romo, 2008). Changes in mortality leading to a shift of the modal age at death have been linked more strongly to adult mortality reductions (Bongaarts, 2005; Canudas-Romo, 2010; Wilmoth and Horiuchi, 1999).

Changes in the variability of the age at death and modal age at death inform about different processes: changes in the age pattern and timing of mortality respectively. They are also supported by different hypothesis regarding the future of mortality; the compression of mortality being often associated with an upper limit to human longevity. To determine how these two components of mortality have changed over time, authors have been studying the mortality compression and the shifting mortality, by using different measures of dispersion/variability and the modal age at death (Cheung et al., 2009; Kannisto, 2001; Wilmoth and Horiuchi, 1999). As example, Kannisto (2001) suggests using the modal age at death, a measure of longevity, in parallel with the standard deviation of individual life durations above the mode as a measure of variability/compression.

The present study aims to quantify the gains in life expectancy respectively due to a change in the modal age at death and a change in the variability of the age at death. We take advantage of recent expression of the Gompertz, and study the changes of their components. We introduce a new methodology to decompose the change in life expectancy between two distributions by a shift in mortality schedule and change in variability.

2 Methodology

In order to explain the dynamic behind changes in mortality, demographers have developed several techniques to decompose changes in life expectancy by different components of mortality. Some methods focus on discrete differences between two life expectancies (Arriaga, 1984; Pollard, 1982; Pressat, 1985) while other consider continuous changes (Beltrán-Sánchez et al., 2008; Keyfitz, 1977; Vaupel and Canudas-Romo, 2003; Vaupel, 1986). We propose a continuous decomposition using recent expression of the Gompertz model.

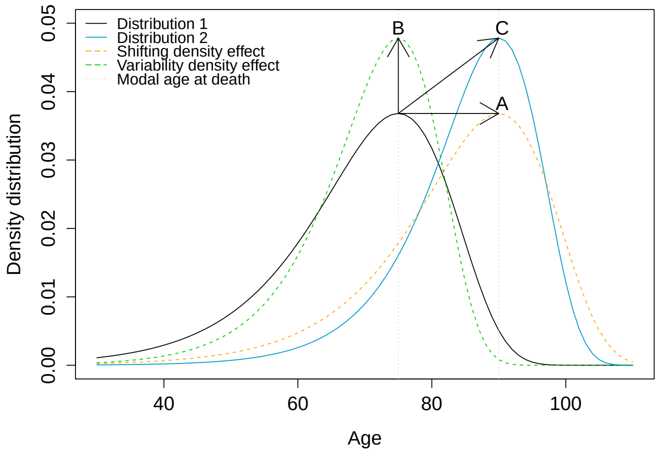
It has been shown by Horiuchi et al. (2013) and Missov et al. (2014) that the hazard rate as expressed by the Gompertz model can be rewritten by using the modal age at death instead of the timing parameter α:

$$\mu_x = \alpha e^{\beta x} = \beta e^{\beta(x-M)}, \tag{1}$$

where β is the slope of the Gompertz hazard function μ_x and M is the modal age at death. This parametrization gives a starting point to decompose changes in life expectancy due to respective changes in the slope and modal age at death (Horiuchi et al., 2013; Missov et al., 2014).

Figure 1 shows the life table distribution of death for Gompertz parameters under two scenarios. Assuming a change of mortality between the two distributions (in Figure 1 as C), we define the "shifting effect" as the hypothetical change obtained if only the modal age at death would have changed between those two distributions (in Figure 1 as A). The "variability effect" refers to the hypothetical change produced if only the slope of the hazard function would have changed from one distribution to another (in Figure 1 as B). Changing the slope of the hazard distribution also changes the shape of the density and survival distributions and thus their variability (Wilmoth, 1997).

Figure 1: *Illustration of the shifting and variability effects in the density function of the distribution of deaths for simulated data from a Gompertz model with slopes* $\beta_1 = 0.10$ *and* $\beta_2 = 0.13$ *and modal ages at death* $M^1 = 75$ *and* $M^2 = 90$*, respectively*



2.1 Gompertz decomposition

Given a Gompertz hazard function at age x and time t, $\mu_{x,t}$, changes in the force of mortality over time ($\dot{\mu}_{x,t}$) can be decompose into a component of change for the slope ($\dot{\beta}_t$) and a component of change for the mode ($\dot{M}_t$):

$$\dot{\mu}_{x,t} = \dot{\beta}_t \, [\frac{\mu_{x,t}}{\beta_t} + (x - M_t)\mu_{x,t}] \; - \dot{M}_t \, [\beta_t \mu_{x,t}], \tag{2}$$

where the derivative with respect to t is denoted by a dot on the top of the variable. The change in life expectancy at birth through time ($\dot{e}_{0,t}$) can be expressed by:

$$\dot{e}_{0,t} = \int_0^{\omega} \dot{l}_{a,t} \, da = -\int_0^{\omega} l_{a,t} \int_0^{a} \dot{\mu}_{x,t} \, dx \, da, \tag{3}$$

where $\dot{l}_{a,t}$ is the time derivative of the survival function $l_{a,t}$. By substituting equation (2) in equation (3), we can estimate the change in life expectancy at birth due to changes in the modal age at death and changes in the slope such as:

$$\dot{e}_{0,t} = \underbrace{-\int_0^{\omega} l_{a,t} \int_0^{a} \dot{\beta}_t \, [\frac{\mu_{x,t}}{\beta_t} + (x - M_t)\mu_{x,t}] \, dx \, da}_{\Delta\beta} + \underbrace{\int_0^{\omega} l_{a,t} \int_0^{a} \dot{M}_t \, [\beta_t \mu_{x,t}] \, dx \, da}_{\Delta M} . \tag{4}$$

The first term in equation (4) represents the gain in life expectancy resulting from a change in the β_t parameter ($\Delta\beta$) while the second term is the gain in life expectancy produced by a shift in the modal age at death (ΔM).

The Gompertz model does not fit well the young-age mortality and tend to fit better the mortality between 30 and 80 years old. Hence, the application of the Gompertz decomposition, $e_{0,t}$ will represent the life expectancy at age 30.

The application of equations (4) to discrete data is done by adapting a Kitagawa (1955) procedure to our equations.

2.2 Decomposing young age mortality

With the previous methodology, only senescent mortality can be decomposed. The decomposition is then limited to adult and old age mortality and might bring only limited understanding of the mortality changes over time. However, modeling overall ages mortality needs more complex models. The Siler model extends the Gompertz model by including two additional terms, capturing respectively the decrease over ages of infant mortality and the mortality at maturity, often interpreted as the "background" mortality risk or as adult mortality:

$$\mu_{x,t} = \alpha_{1,t} e^{-b_t x} + c_t + \alpha_{2,t} e^{\beta_t x}, \tag{5}$$

where α_1, c and α_2 are the intercepts of the three terms and the parameters b and β are the constant rate of mortality change over age for infant and senescent mortality respectively.

By substituting the senescent term of equation (5) by equation (1), we can express the senescent mortality in terms of the modal age at death M,

$$\mu_{x,t} = \alpha_t e^{-b_t x} + c_t + \beta_t e^{\beta_t (x - M_t)}. \tag{6}$$

The implication of forcing this change in the hazard equation is unexplored at this point of the study. This might however help understanding the role of young age mortality on shifting or compressing of mortality.

Decomposing changes in the force of mortality with the Siler model is then expressed by changes in 5 different parameters:

$$\dot{\mu}_{x,t} = \dot{\alpha}_t [e^{-b_t x}] - \dot{b}_t [\alpha_t e^{-b_t x} x] + \dot{c}_t + \dot{\beta}_t \, [e^{\beta_t (x-M_t)} (1 + \beta_t (x - M_t))] - \dot{M}_t \, [\beta_t^2 e^{\beta_t (x-M_t)}], \tag{7}$$

where $\dot{\alpha}_t$ is the change respect to t in the initial level of mortality (age 0), $\dot{b}_t$ is the change in the infant rate of mortality decrease over age, $\dot{c}_t$ is the change in the background mortality level, $\dot{\beta}_t$ is the change in the rate of mortality increase over age for senescent mortality and $\dot{M}_t$ is the change in the modal age at death.

Canudas-Romo (2010) analytically demonstrated that when reduction of mortality occurs at younger ages than the modal age, the mode will be maintain. The first four terms of the above equation would then have an impact on variability reduction and should not influence the modal age at death. The changes in variability could then be divided into four distinct effects ($\dot{\alpha}_t$, $\dot{b}_t$, $\dot{c}_t$ and $\dot{\beta}_t$).

As for the Gompertz, we can estimate the change in life expectancy at birth due to changes in the different parameters by substituting equation (7) in equation (3). The gain in life expectancy at birth due to a change in the different parameters is denoted by a Δ in front of the respective parameters.

$$\begin{aligned} \dot{e}_{0,t} = & -\underbrace{\int_0^\omega l_{a,t} \int_0^a \dot{\alpha}_t [e^{-b_t x}] \, dx \, da}_{\Delta\alpha} + \underbrace{\int_0^\omega l_{a,t} \int_0^a \dot{b}_t [\alpha_t e^{-b_t x} x] \, dx \, da}_{\Delta b} - \underbrace{\int_0^\omega l_{a,t} \, \dot{c}_t \, a \, da}_{\Delta c} \\ & - \underbrace{\int_0^\omega l_{a,t} \int_0^a \dot{\beta}_t \, [e^{\beta_t (x-M_t)} (1 + \beta_t (x - M_t))] \, dx \, da}_{\Delta\beta} + \underbrace{\int_0^\omega l_{a,t} \int_0^a \dot{M}_t \, [\beta_t^2 e^{\beta_t (x-M_t)}] \, dx \, da}_{\Delta M} . \end{aligned} \tag{8}$$

3 Illustration

3.1 Gompertz

A Gompertz model has been fitted to the mortality of Swedish females at age 30 and older by using a maximum likelihood procedure. Table 2 presents the decomposition in life expectancy at age 30 by M and β for Swedish females at the beginning, middle and end of the twentieth century. For the three periods selected, the change in the modal age at death is the main factor of change in life expectancy.

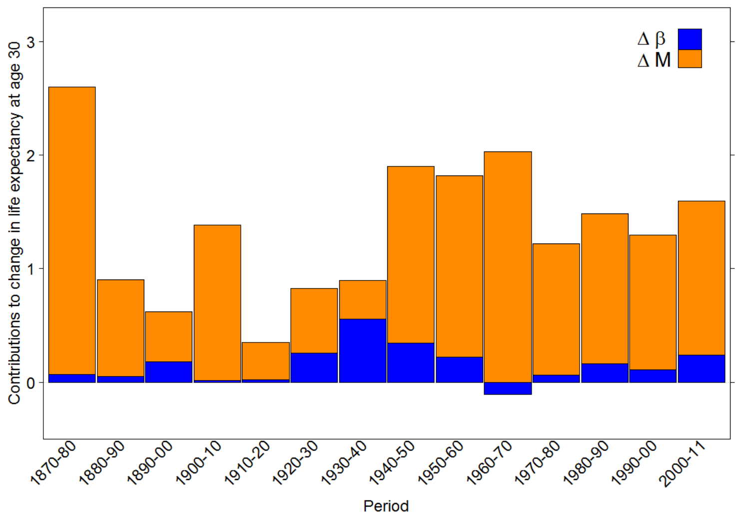
Table 1: *Female life expectancy at age 30, $e_{30,t}$, and its decomposition due to change in the modal age at death, ΔM, and slope, $\Delta\beta$, Sweden, 1900, 1950 and 2000*

	1900	1950	2000
$e_{30,t}$	39.60	44.94	52.67
$e_{30,t+10}$	40.98	46.75	54.27
$\dot{e}_{30,t}$	1.38	1.82	1.60
$\Delta\beta$	0.01	0.22	0.24
ΔM	1.37	1.60	1.36
$\Delta\beta + \Delta M$	1.38	1.82	1.60

Source: HMD (2014) and author's own calculation.

Figure 2 presents the decomposition from 1870 until 2011 for 10-years period. No matter the period, the gain in life expectancy at age 30 was in great part the result of a shift in the modal age at death. From the 1920's until the 1950's, the variability contribution to changes in life expectancy have been more important than in the previous and following periods. However, even during those years, the change in life expectancy was mainly driven by a change in the mode.

Figure 2: *Shifting (ΔM) and variability ($\Delta\beta$) contributions to the decomposition of the female life expectancy at age 30, Sweden, 1870-2011*



Source: HMD (2014) and author's own calculation.

When looking exclusively at senescent mortality, the changes in life expectancy seems to be mainly the result of shifting mortality.

3.2 Siler

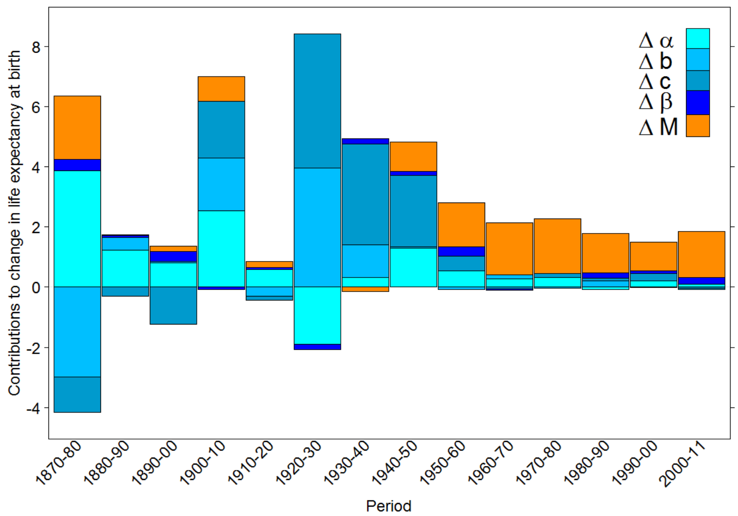
A Siler model has been fitted to the mortality of Swedish females at age 0 and older, using also a maximum likelihood procedure. Table 2 and Figure 3 present the results of the decomposition. These results suggest that the changes in the life expectancy at birth before 1950 were mainly the result of variability reduction. Different parameters influencing the variability were however involved. Until 1920, changes in life expectancy were mainly driven by lower level of mortality during immaturity (parameters α and b). After this period and until the 1950s, there was an important decrease in the background mortality (parameter c). After 1950, the modal age at death is the key parameter of the changes in life expectancy.

Table 2: *Female life expectancy at age 0, $e_{0,t}$, and its decomposition due to change in the Siler parameters, Sweden, 1900, 1950 and 2000*

	1900	1950	2000
$e_{0,t}$	51.84	72.12	81.91
$e_{0,t+10}$	58.74	74.84	83.68
$\dot{e}_{0,t}$	6.90	2.72	1.77
$\Delta\alpha$	2.54	0.53	0.10
Δb	1.74	-0.08	-0.03
Δc	1.89	0.51	-0.05
$\Delta\beta$	-0.08	0.31	0.22
ΔM	0.82	1.45	1.53
$\Delta\alpha+\Delta b+\Delta c+\Delta\beta+\Delta M$	6.91	2.72	1.77

Source: HMD (2014) and author's own calculation

Figure 3: *Siler parameters contributions to the decomposition of the female life expectancy at age 30, Sweden, 1870-2011*



Source: HMD (2014) and author's own calculation.

4 Conclusion

Our results suggest that mortality compression was the main driver of change in life expectancy at birth before 1950, by a decrease in infant and background mortality. After this period, changes in life expectancy come from a shift in the modal age at death, which is associated to a decrease in senescent mortality. These results are consistent with the findings of other studies looking at changes in the modal age at death and at different variability measures (Robine, 2001; Wilmoth and Horiuchi, 1999; Yashin et al., 2001). Our methodology allows however to quantify the gain in life expectancy resulting from the compression and shifting of mortality over time.

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