How many old people have ever lived on earth?

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Abstract

Recently, Cohen (2014) revisited the popular question some demographers have asked: How many people have ever been born? In particular he studies the fraction of those ever born up to a calendar year T, who are alive at time T, for age $y = 0$. The present paper extends this methodology to the proportion of people, who have ever reached a certain age y, say 65 years, and are alive today (hereinafter $\pi(y, T)$).

In this paper, we first analyze this fraction $\pi(y, T)$ by using demographic data based on UN estimates. Opposite to the claim made by Fred Pearce (The Economist, 2014): "it is possible that half of all the humans who have ever been over 65 are alive today", we estimate that the proportion $\pi(65, 2010)$ is much smaller, ranging between 5.5 and 9.5%. Moreover, we show the main properties of $\pi(y, T)$ by age and over time. We complete our analysis by using stable population theory.

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1 Introduction

Global population ageing, caused by fertility decline and increasing survival at older ages, has become a challenging issue of our times. The shift of the age structure of the population will profoundly reshape the social structure of our world as well as its economy.

There are around 600m people aged 65 or older alive today. While their share is now about 8% of the total population, it will increase to some 13% in the next twenty years. According to the UNs population projections the world had 16 people aged 65 and over for every 100 adults between the ages of 25 and 64, but this dependency ratio will rise to 26 by 2035.

A recent article in The Economist (2014) describes how those age invaders are about to change the global economy. Beside of the old-age dependency ratio in this publication another indicator of aging is mentioned: the ratio 65 or older alive today related to all the humans who have ever reached the age of 65 and above. According to The Economist, Fred Pearce claimed that it is possible that half of all people who have ever been over 65 are alive today. Since this is a daring statement depending crucially on historic demographic processes, we decided to check its validity. In the present paper, by using formal demography and some empirical data on population processes, we will try to get a more sound answer to Fred Pearces interesting assertion.

Clearly, it is closely related to a question, which has been posed by several prominent demographers, namely "How many people have ever lived on earth?" In his seminal book on *Applied Mathematical Demography* Keyfitz (1977) gives a brief introduction into the problem. Among the demographers who have dealt with this problem are Petty (1682), Winkler (1959), Deevey (1960), Desmond (1962), and Keyfitz (1966). More recent references are Tattersall (1996), Johnson (1999), Haub (2011) and Cohen (2014).

Cohen (2014) shows a table with various estimates of the number of people ever born by year t starting with Petty (1682) until Haub (2011). It illustrates the wide range of the various estimates. The most reliable seems to be the last one authorized by Haub (2011). This semi-scientific approach yields an estimate of 108 billion births since the dawn of the human race assumed as 50.000 B.C. With a world population in mid-2011 this gives a percentage of those ever born who are living in 2011 of 6.5.

Asking the question whether this fraction rises or falls, Cohen (2014) comes to the robust conclusion that at present it is increasing. On the other hand, if world population would reach stationarity or declines, the fraction would fall. The significance of Cohens analysis lies in the fact that he uses mathematical demography to obtain his results. The present paper follows his reasoning. By extending his approach we study the fraction of people ever surpassing a certain age limit y, say 65 years, who are now alive.

The paper is organized as follows. In Section 2 we introduce an analytic expression of the ratio of the number of people at ages above y in year T to the number of those that ever reached the age y and present a first rough estimate of this number based on given historical population estimates. The empirical assessment is refined in Section 3 by applying a stable and alternatively a non-stable population model. In particular a non-stable population model that takes into account changes of fertility and mortality in history yields a more realistic estimate of our expression of elderly at a specific age currently alive among elderly that have ever reached this age. Section 4 is devoted to an analytic and numerical investigation of the dynamic change in this expression with respect to the age threshold y and the time T. The final section concludes and highlights how far off estimations of our expression could be by using wrong models on historical populations.

2 Formal model and first empirical assessment

In this section we first present the general formula to calculate the fraction of people over age y ever lived who are currently alive in year T, which we denote by $\pi(y, T)$. Second, we calculate using data from several authors the ratio of people at age 65 alive in year 2010 to the number of those ever reached age 65.

2.1 Analytical framework

Let $N(a, t)$ be the population size at age a in year t; $B(c)$ be the number of births in year c; and $\ell(a, c)$ be the survival probability to age a for the birth cohort c. The number of people that ever reached old age y since the original cohort $c = 0$ is:

$$\int_0^{T-y} N(y, c+y)dc = \int_0^{T-y} B(c)\ell(y, c)dc, \tag{1}$$

while the number of people currently alive at ages y and older is (assuming $T > \omega$, where ω is the maximum age):

$$\int_y^{\omega} N(a, T)da = \int_{T-\omega}^{T-y} B(c)\ell(T-c, c)dc. \tag{2}$$

The proportion of interest is the ratio of the number of people currently at ages $y+$ to the number of those ever reached the age y:

$$\pi(y,T) = \frac{\int_y^{\omega} N(a,T)da}{\int_0^{T-y} N(y,c+y)dc} = \frac{\int_{T-\omega}^{T-y} B(c)\ell(T-c,c)dc}{\int_0^{T-y} B(c)\ell(y,c)dc}. \tag{3}$$

The numerator of Eq. (3) accounts for the living population older than age y in year T, which is represented by the vertical solid line in Figure 1, while the denominator of Eq. (3) is the population ever lived to age y until year T, or the solid horizontal line in Figure 1.

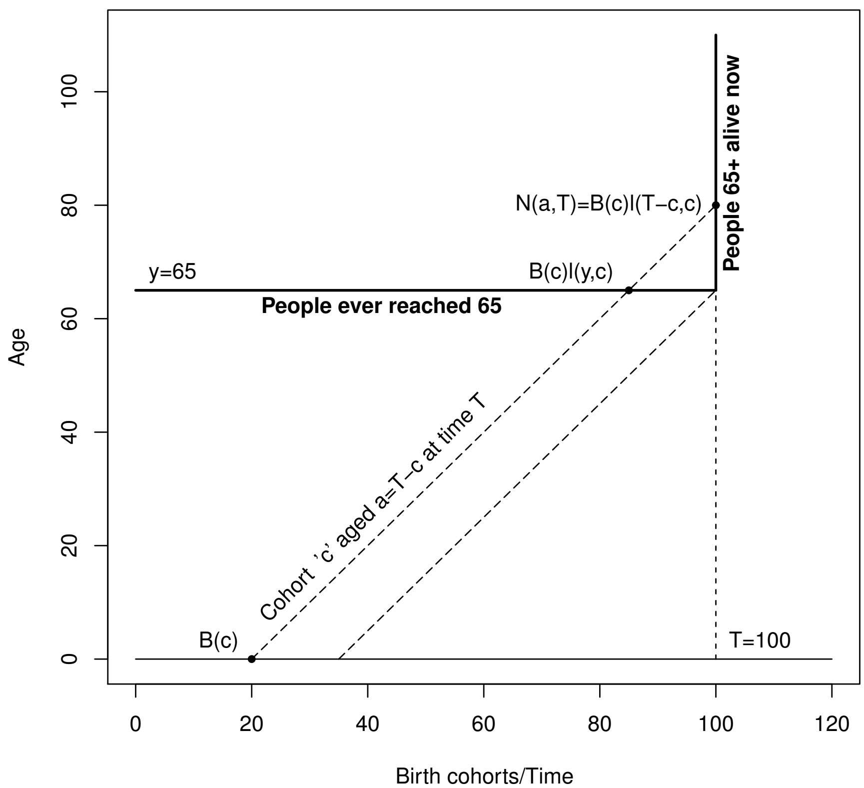


Figure 1: Lexis diagram illustrating the calculations of $\pi(y,T)$

2.2 First empirical assessment

For a first estimate of $\pi(65, 2010)$, we took data on total population and births born before 1945 from Deevey (1960), Keyfitz (1966), Westing (1981), and Haub (2011). These four authors cover plausible minimum (5.5%) and maximum (13.9%) values of the people ever lived on earth in the literature. In all papers, the births born are calculated dividing the human history in several time intervals, in which the population is assumed to grow at a constant rate. Differences in the number of people ever lived on earth among all authors stems mainly from the number of intervals used and the assumed life expectancy at birth in the first periods.[1] For instance, the number of time intervals up to 1945 used by Deevey (1960) is 11, 8 intervals by Haub (2011), 6 intervals by Westing (1981), and 4 intervals by Keyfitz (1966). In the first time intervals, the life expectancy at birth ranges between age 13 (Haub, 2011) and 25 (Deevey, 1960; Keyfitz, 1966), with a middle value of 20 assumed by Westing (1981). To compute the number of people that ever lived to age 65, shown in Table 1, we multiply the total population born by the corresponding survival probability to age 65 in each period. The values of the survival probability to age 65 by different life expectancy are drawn from the UN General Model Life Table. See Table 3 in the Appendix 6.5 for the calculations performed for each author.

Table 1: Fraction of people ever lived to age 65 who are alive in year 2010

	Deevey (1960)	Westing (1981)	Keyfitz (1966)	Haub (2011)
Persons ever born until 1945 (millions)†	83,719	45,951	67,138	99,803
Persons age 65 ever lived (millions)‡	9,575	7,991	6,640	3,762
Persons age 65+ in 2010 (millions)♭	524	524	524	524
$\pi(65, 2010)$	0.055	0.066	0.079	0.139

Source: † Data collected from Johnson (1999). ‡ Author's calculations based on UN Model Life Tables by life expectancy and people ever lived collected by Johnson (1999). ♭ Data taken from UN, Population Division (2013).

These assessments led to estimate that the number of people who have survived to age 65 until 2010 ranges between 3.762 and 9.575 million people. The lowest value obtained by Haub (2011) crucially depends on a low life expectancy even for the most recent decades, while the highest value obtained by Deevey (1960) is due to the combination of a long time span (i.e.

[1]Recall that in a stable population, for a given population growth rate, there exists a one-to-one relationship between the life expectancy at birth and the crude birth rate .

more than one million years) together with a high initial population size (i.e. 125,000 people). Provided that the UN estimates a total number of people age 65+ in year 2010 close to 524 million, we calculate that between 5.5% and 13.9% of the total population ever-reached age 65 were alive in year 2010. It is clear that these values fall below the claim that half of people who have ever been over age 65 are alive today. Due to this high discrepancy, in the next subsections we study whether higher values of $\pi(y, T)$ can be obtained using different demographic models.

3 Empirical assessments

In this section we analyze the behavior of $\pi(y, T)$ in the context of two alternative demographic models: the stable and the non stable population model.

3.1 Stable population

In the simplest case of a stable population, that is life tables are constant across cohorts (i.e. $\ell(a, c) = \ell(a)$) and births grow exponentially at a constant rate r (i.e. $B(c) = B(0)e^{rc}$), $\pi(y, T)$ becomes

$$\begin{cases} \frac{r}{1-e^{-r(T-y)}} \int_y^\omega e^{-r(a-y)} \frac{\ell(a)}{\ell(y)} da & \text{if } r \neq 0, \\ \frac{1}{T-y} \int_y^\omega \frac{\ell(a)}{\ell(y)} da & \text{if } r = 0. \end{cases} \tag{4}$$

The integral in Eq. (4) is the stable population at ages $y+$ divided by the stable population of exact age y, while the fraction in front of the integral is the ratio between the total births born in year $T - y$ and the person-years lived between 0 and $T - y$.

Assuming positive population growth and $T \gg y$, the ratio converges to the limit value:

$$\pi(y, T) = r \int_y^\omega e^{-r(a-y)} \frac{\ell(a)}{\ell(y)} da. \tag{5}$$

Hence, under a stable population, the value of the integral is given by the inverse of the proportion of people aged 65 divided between those age 65+, which according to the UN estimates is about 7.50% at age $y = 65$ in 2010. On the other side, the geometric mean of the long-run population growth rate from the origin of our race (50.000 BC) is 0.035%. Consequently, if we use the existing data to a stable population model, the value of $\pi(65, 2010)$ will be $\frac{0.00035}{0.075} \simeq 0.47\%$, which according to Figure 2 is above the range 0.20-0.35% that is obtained if a stable population with a life expectancy at birth

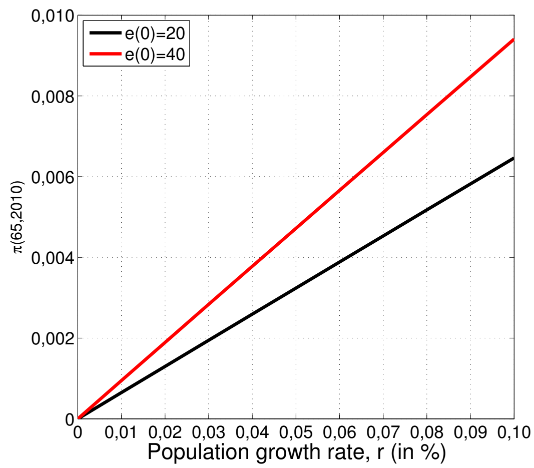


Figure 2: Ratio of people age 65+ who are alive in year 2010 to people ever lived to age 65, by life expectancy at birth and growth rate of births.

Note: Survival probabilities by life expectancy taken from the UN General Model Life Tables.

between age 20 and 40, respectively, were assumed. This result simply implies that current $\pi(65, 2010)$ values cannot be obtained using the same stable population growth rate throughout the whole time period. This is because $\pi(y, T)$ is very sensitive to the number of intervals when the population growth rate accelerates rapidly (Keyfitz, 1966). For this reason, we next analyze the value of $\pi(65, 2010)$ under a non-stable population.

3.2 Non stable population

Unlike the stable population model, we now assume, yet realistically, that the rapid acceleration of the population growth rate is driven by changes in fertility and mortality. To account for these changes, we consider that the survival probability to age a of an individual born in year c and the fertility rate at age a of an individual born in year c are, respectively, given by

$$\ell(a, c) = e^{-M(a,c)}, \tag{6}$$

$$f(a, c) = \begin{cases} f \cdot \exp\{\phi(c)\} & \text{if } a = A, \\ 0 & \text{otherwise,} \end{cases} \quad \text{with } \phi(0) = 0, \tag{7}$$

where $M(a,c) = \int_0^a \mu(x, c+x)dx$ is the cumulative mortality hazard rate at age a for an individual born in year c and $\mu(x, c+x)$ is the mortality hazard rate at age x in year $c+x$. In Eq. (7) it is assumed that fertility is concentrated at the mean age at childbearing, where f is the average number of children of the birth cohort 0, $\phi(c)$ is the cohort-specific change from the initial cohort in the number of children, and A is the unique age of childbearing.

Like the Lee and Carter (1992) model, we assume that $\log \mu(x, c+x) = \alpha(x) + k(c+x)\beta(x)$, where $\alpha(x)$ and $\beta(x)$ represent the fixed age effects and the rate of change in mortality at age x in response to a change in k, and $k(c+x)$ is the level of mortality at time $c+x$. Particular functional forms of Eq. (7) have been previously studied in the context of population growth theory. In particular, Coale and Zelnik (1963), Feichtinger and Vogelsang (1978), and Feichtinger (1979) showed that when $\phi(t) = \phi \cdot t$ the birth trajectory is given by $B(t) = B(0)\exp\left\{\frac{\phi}{2}t + \frac{\phi}{2A}t^2\right\}$, where ϕ is the rate of change in the level of fertility. Here, however, we assume that total births depend on both fertility and mortality.

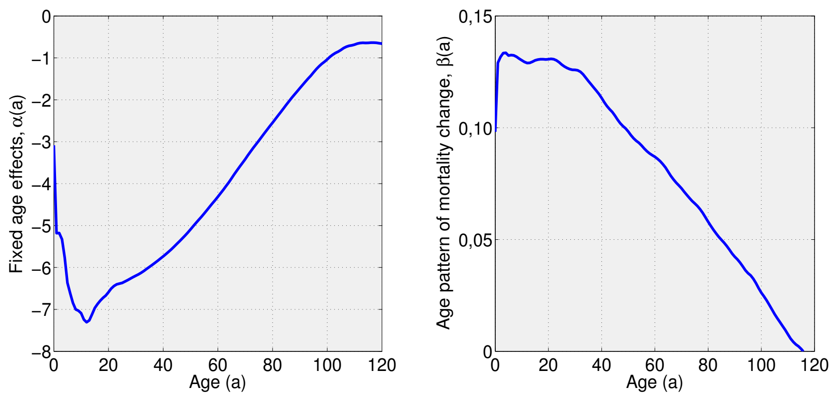
Combining (6)-(7) the total number of births born in year c becomes

$$B(c) = B(0)\exp\left\{\sum_{i=0}^{c/A-1} \phi(iA) - M(A, iA)\right\}. \tag{8}$$

See the proof in Appendix 6.3. Eq. (8) shows to what extent former changes in fertility and in mortality affect on the growth rate of births. Substituting (6) and (8) in (3) we get

$$\pi(y,T) = \frac{\int_{T-\omega}^{T-y} B(c)e^{-M(T-c,c)}dc}{\int_0^{T-y} B(c)e^{-M(y,c)}dc}. \tag{9}$$

Thus, provided $\alpha(x)$ and $\beta(x)$, Eq. (9) implies that $\pi(y,T)$ is a function not only of y and T but also on the history of $\phi(\cdot)$ and $k(\cdot)$. In order to replicate the historical population data shown in Table 2 (column 2), we calculated $\phi(\cdot)$ and $k(\cdot)$ over time using the Generalized Inverse Projection method (GIP) (Lee, 1985; Oeppen, 1993) (see Appendix 6.4 for further details). Our age-specific mortality rates as well as the relative rate of change in mortality across age groups are taken from the model life table by level of life expectancy provided by UN, Population Division (2013). Figure 3 shows the age components of the underlying survival probabilities. These values are derived taking the first principal component from the mortality data by life expectancy reported by UN, Population Division (2013). We set the mean age of childbearing (A) at 27, similar to that of Hutterites. Since the results presented by several scholars mainly differ because of the assumed initial life expectancy at birth and the number of periods, in Table 2 we present the results that would be obtained if we replicate the populations that result from combining the data of Deevey (1960) and Haub (2011) up to year 1900 and the population estimates from 1950 until 2100, reported by UN, Population Division (2013).



(a) Fixed age effects, $\alpha(a)$

(b) Age pattern of mortality change, $\beta(a)$

Figure 3: Underlying mortality model

Replicating Deevey (1960) and Haub (2011) gives us the likely minimum and maximum values of $\pi(65, 2010)$. Table 2 reports the population of age 65 (columns 3 and 6) and persons age 65 ever lived (columns 4 and 7) for each author from year 50000 B.C to 2010 A.C estimated with the GIP method. Notice that the population age 65 ever lived in year 2010 differ from those reported in Table 1 because in this case the population data from 1950 until 2010 is based on UN, Population Division (2013). The first important feature to be highlighted in Table 2 is that the number of people age 65 increased almost by a factor of four during the last century, while the total population increased less than three times in both population scenarios. A similar increase in the population age 65 before the twentieth century took almost 250 years (from 1650 to 1900) according to Haub (2011), which is one-third of the time following Deevey (1960)'s assumptions. The population age 65 ever lived, however, only increased by 40% during the twentieth century according to Haub (2011) and around 20% according to Deevey (1960). The

Table 2: Number of people age 65 ever lived on earth (millions)

	Haub (2011)			Deevey (1960)		
Year t	Pop.	Pop. age 65	Pop. age 65 ever lived	Pop.	Pop. age 65	Pop. age 65 ever lived
-50000	0	0	0	3	0	1,921
-8000	5	0	36	6	0	2,404
1	309	1	1,547	139	1	4,699
1200	432	1	2,350	369	1	5,882
1650	516	1	2,823	544	2	6,743
1750	800	3	3,003	732	3	7,006
1850	1,277	5	3,342	1,199	4	7,389
1900	1,681	7	3,620	1,637	7	7,678
1950	2,587	13	4,118	2,577	13	8,126
1970	3,758	18	4,422	3,760	19	8,427
1990	5,354	27	4,861	5,361	27	8,869
2000	6,177	33	5,159	6,184	33	9,168
2005	6,573	35	5,330	6,579	35	9,340
2010	6,896	39	5,514	6,896	39	9,524

[1] Source: Haub (2011), Deevey (1960), are used until 1900 and UN, Population Division (2013) from 1950 to 2010.

fact that the population age 65 is increasing faster than the population age 65 ever lived suggests that the proportion of people who have ever reached 65 and are alive today is nowadays higher than in the past. In particular, if the population older than 65 in year 2010 was 524 million people, $\pi(65, 2010)$ ranges between 5.5% and 9.5%. Clearly, these $\pi(65, 2010)$ numbers are still far from the 50% claimed in The Economist (2014).

Given this huge discrepancy in the next section we study whether $\pi(y, T)$ can reach the value of 50% at any other age threshold (y) or at any point in time in the recent past or the future (T).

4 Dynamic features of $\pi(y, T)$

Recently, Cohen (2014) has shown that $\pi(0, T)$ (i.e. fraction of people ever born up to time T who are alive at time T) decreases over time for a stable population model, but it can increase or decrease with a super-exponential or with a doomsday model. Therefore, in a population with an increasing population growth rate, it cannot be rejected the possibility that $\pi(y, T)$ might take a value close to 50% at any other point in time or, maybe, at any other age threshold y.

In this section we extend Cohen (2014) analysis by studying the dynamic features of the new indicator $\pi(y,T)$, i.e. the proportion alive of those ever reached age y. To explore the alternative values that $\pi(y,T)$ might take over time and over age, we first differentiate the $\log \pi(y,T)$ with respect to the threshold age y and, second, with respect to time T.

Changing age threshold y. In the first case, taking logarithms at both sides of Eq. (3) and differentiating it with respect to y gives

$$\frac{\pi_y(y,T)}{\pi(y,T)} = \frac{N(y,T) + \int_0^{T-y} N(y,c+y)\mu(y,c+y)dc}{\int_0^{T-y} N(y,c+y)dc} - \frac{N(y,T)}{\int_y^{\omega} N(a,T)da}. \quad (10)$$

Eq. (10) is the difference between the fractional change over age in the number of people ever reached age y and the fractional change over age in the number alive above age y. The first term, which is always positive, is the sum of the ratio between the number of people at y in year T and the number of people ever reached age y plus the average mortality rate at y, weighted by the population ever reached age y. The second term, which is always negative, is the proportion of people age y exactly among all age $y+$ in year T. A priori, the sign of Eq. (10) is ambiguous. Higher ages imply a greater contribution of mortality on $\pi(y,T)$ due to the positive correlation between age and mortality. But, higher ages also imply a greater proportion of people age y among all age $y+$ in the same year.

The sign of (10) is, nonetheless, known for some special cases. For example, in a stable population, Proposition 1 shows that $\pi(y,T)$ is monotonically decreasing with respect to the age threshold y (see proof in Appendix 6.1)

Proposition 1 *In a stable population with $r > 0$, $\pi(y,T)$ is monotonically decreasing with respect to the age threshold y if the death rate from age y onwards is non-decreasing.*

Proposition 1 implies that, in a stable population with $r > 0$, the reduction in the number of people alive at age y and older is, in relative terms, smaller than the reduction in the number of people ever reached age y if, and only if, the death rate from age y onwards is non-decreasing. Thereby, in a growing stable population, $\pi(y,T)$ is increasing early in life, due to the fact that infant death rates are historically higher than the proportion of people at age y (y belonging to infant ages) among all $y+$; it reaches a maximum and it monotonically decreases until very old ages (see Figure 3 in Johnson (1999) for an illustration with a constant population growth rate).

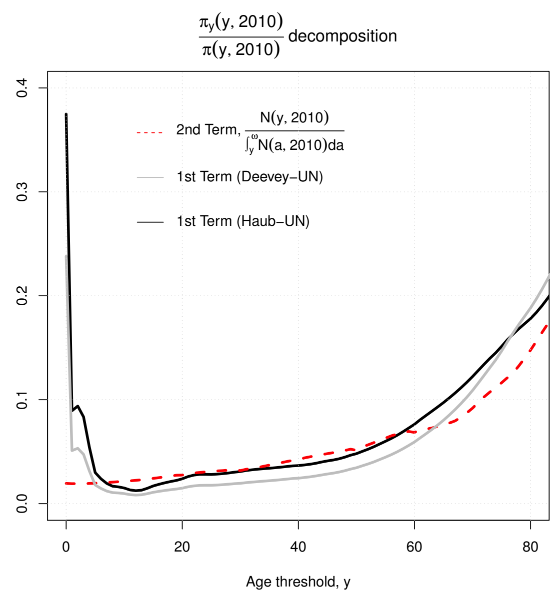


Figure 4: Decomposition of the fractional change over age in the ratio between the number of people above age y ever lived who are alive in year 2010.

In reality, however, the population growth rate is not constant over time. Like in Eq. (8), the population growth rate is driven by gains in life expectancy and by decreases in the fertility rate. Under this setting, Proposition 1 does not necessarily hold and, instead, it is necessary to perform an empirical analysis. Figure 4 shows, for the two extreme cases modeled with the GIP method, the decomposition of the fractional change over age in the fraction of people above age y ever lived who are alive in year 2010. The solid lines (black for Haub-UN and gray for Deevey-UN) represent the fractional change over age in the number of people ever reached age y (or first term in Eq. (10)), while the dashed red line is the fractional change over age in the number of people alive in year 2010 above age y (or second term in Eq. (10)). Note that the second term is the same in both cases since it is based on current population data. In contrast, the black solid line and the gray solid line differ because they are based on historical data. Consequently, since historically the age-specific mortality rates are higher in Haub (2011) than in Deevey (1960), the black solid line is higher than the gray solid line. Recall that Haub (2011) starts with a life expectancy at birth of age 13, while Deevey (1960) assumes, similar to Keyfitz (1966), a life expectancy of 25 at the onset of the homo sapiens. The crossing point between the gray and black solid lines at old age is due to the higher weight of historical data in Deevey (1960) than in Haub (2011), since the former assumed that more people reached old age. From Figure 4, we know that Eq. (10) is positive at young and old ages, i.e. when the solid lines are above the dashed line, and it is negative from age 7 to the end of prime working age (around age 60). Thereby, according to Figure 4, the fraction $\pi(y, 2010)$ should have a local maximum early in life and a local minimum late in life.

Figure 5 shows the fraction of people above different ages y ever lived who are alive in year 2010. The black solid line depicts $\pi(y, 2010)$ under the case of Haub (2011)-UN, Population Division (2013), while the gray solid line corresponds to that of Deevey (1960)-UN, Population Division (2013). As Figure 4 suggests, in both cases we find that $\pi(y, 2010)$ increases early in life, reaching a maximum value between 11 and 13% at age 5 (gray line) and at age 7 (black line), afterwards it declines until age 65 (gray line) and age 60 (black line), and finally it rises, reaching a value of 8% (gray line) and almost 15% (black line) at age 80. Initially, $\pi(y, 2010)$ rises because the historical average mortality rate at age 0 –i.e. the first term in (10)– until 2010 is close to 23 percent (in Deevey-UN) and 35 percent (in Haub-UN), while the proportion of recently born among the total population in year 2010 is close to 2 percent. Second, the faster decrease over age in the gray solid line from age 8 to age 60 compared to the black solid line is explained by the lower mortality rate in the former case relative to the proportion of

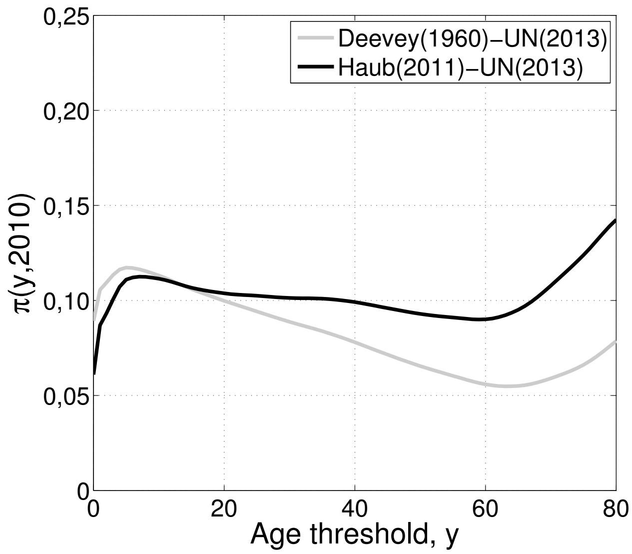


Figure 5: Fraction of people above age y ever lived who are alive in year 2010.

people at age y among all age $y+$ in year 2010 (cf. Figures 4 and 5). As a consequence, $\pi(65, 2010)$ is three percetange points greater in the black solid line (9%) than in the gray solid line (6%). Therefore, according to Figure 5 we cannot expect –based on realistic scenarios– $\pi(65, 2010)$ values close to 50% for any age threshold $y < 80$.[2]

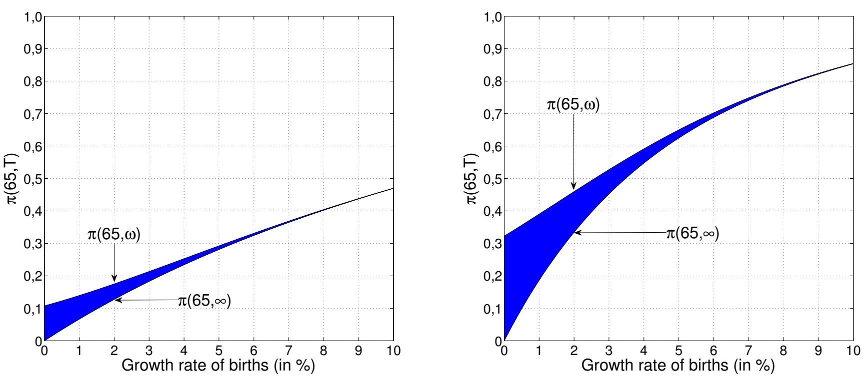
Changing time T. To analyze whether $\pi(y, T)$ might reach values close to 50% in the near future, we differentiate the $\log \pi(y, T)$ with respect to time T. After rearranging terms, we obtain[3]

$$\frac{\pi_T(y,T)}{\pi(y,T)} = \frac{N(y,T) - \int_y^\omega N(a,T)\mu(a,T)da}{\int_y^\omega N(a,T)da} - \frac{N(y,T)}{\int_0^{T-y} N(y,c+y)dc}. \tag{11}$$

[2]Values of $\pi(y, 2010)$ for $y > 80$ are not shown because of lack of data above age 80 for the period 1950-1990. Nevertheless, based on data for living super-centenarians the fraction π for supercentenarians in year 2000 seems to be close to 12% (see `http://www.grg.org/Adams/E.HTM`) .

[3]For an illustration of the derivative of $\pi(y, T)$ with respect to T see Figure 9 in Appendix 6.2

Eq. (11) is the difference between the fractional change over time in the number of people alive above age y and the fractional change over time in the number ever reached age y. Eq. (11) coincides with Eq. (2) in Cohen (2014), page 1562, when $y = 0$. The fractional change over time in the number of people alive above age y in year T can be either positive or negative. Indeed, the first term is the crude growth rate in year T of the population older than age y. In contrast, the second term in Eq. (11) is always negative. As a result, our fraction $\pi(y, T)$ can either increase or decrease over time. Another important difference is that the first term in Eq. (11) only depends on current information, whereas the second term depends on the historical population.



(a) Life expectancy (at birth)=20 (b) Life expectancy (at birth)=80

Figure 6: Ratio of people age 65+ who are alive in year T to people ever lived to age 65 until year T, by life expectancy at birth and growth rate of births.

Note: Survival probabilities by life expectancy taken from the UN General Model Life Tables.

Assuming a stable population, we know from Eq. (4) that $\pi(y, T)$ is a decreasing function with respect to time T, which converges in the limit to Eq. (5). Figure 6 illustrates all possible values of $\pi(y = 65, T)$ between year $T = \omega$ and $T \uparrow \infty$ by different life expectancies at birth and growth rates of births. Note that all feasible values are contained in the blue area. Since $\pi(y, T)$ decreases over time, the highest value of $\pi(65, T)$ for a given population growth rate occurs when $T = \omega$, while the lowest value occurs when $T \uparrow \infty$. Any other intermediate value of $\pi(65, T)$, or ξ, should satisfy the following equation:[4]

$$\tilde{T} = y - \frac{1}{r}\log\left(1 - \frac{r}{\xi}\int_y^{\omega} e^{-r(a-y)}\frac{\ell(a)}{\ell(y)}da\right). \tag{13}$$

where $\tilde{T}$ is the exact time at which $\pi(y, \tilde{T})$ equals ξ. For example, in a stable population, a value of $\xi = 50\%$ is unattainable unless that the (annual) growth rate of births would be higher than 10% for a population with a life expectancy at birth of 20 years or higher than 3% for a population with a life expectancy at birth of 80 years.

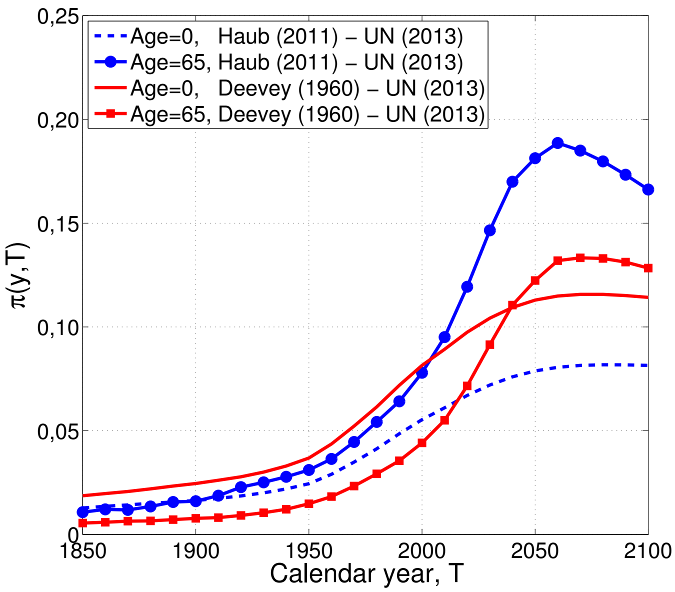


Figure 7: Fraction of people above alternative threshold ages ever lived who are alive in year T

Similar to the previous analysis with the age threshold y, the sign of Eq. (11) is ambiguous and we need to perform an empirical analysis when the

[4]Notice that Eq. (13) exists if, and only if,

$$r\int_y^{\omega} e^{-r(a-y)}\frac{\ell(a)}{\ell(y)}du < \xi. \tag{12}$$

The region that satisfies the above condition is depicted in blue color in Figure 6.

population is non-stable. Nevertheless, the first term in Eq. (11) will typically be higher than the second one when the growth rate of births increases, because the population reaching age y increases faster than the deaths above that age (Cohen, 2014). For this reason, as it is shown in Figure 7, $\pi(y, T)$ has continuously increased during the twentieth century at all ages analyzed. In the twenty first century, however, the proportion $\pi(y, T)$ may eventually decline at different ages after reaching a maximum due to the expected slowdown in the growth rate of births. For instance, $\pi(0, T)$ is expected to reach a maximum value between 8-12% during the second half of the twenty first century, $\pi(65, T)$ will peak between 13-19% in the 2060s.

5 Conclusion and discussion

The question on how many people have ever lived on earth has been discussed extensively in the demographic literature. The first mathematically correct solution has been derived by Keyfitz (1966). By dividing the interval from -1,000,000 to 1960 into 4 subintervals and fitting successive geometric growth for each interval, Keyfitz obtained that 69 billion people have ever lived as compared to 3 billion people alive in 1960. Put differently, the population in 1960 amounted to about 4% of all people that have ever lived. Compared to Keyfitz, Winkler (1959) obtained much higher values above 3,000 billion people since he has not been aware of the different subperiods of population growth in human history.

In a recent study Cohen (2014) followed this earlier research and studied the change over time in the fraction of people ever born who are currently alive. In this paper, we extend the analysis by Cohen and investigate the fraction of people above a specific age threshold y alive at time T to the population that ever was alive and reached this age threshold, which we denote it by $\pi(y, T)$. Such a measure may yield a new view on the pace of population ageing over time. More specifically, through our analysis we can show that the claim of Fred Pearce (The Economist, 2014), that half of all people who have ever reached the age of 65 is alive today, is not true. Indeed, such a number would be never attainable, neither theoretically (in a stable population), nor empirically according to existing data. Hence, though such investigations are important to understand the dynamics of ageing populations, we need to be careful in applying correct calibrations.

We have applied simple mathematical demography to analytically express $\pi(y, T)$ and use the framework of the Lexis diagram to illustrate this fraction. Assuming a stable population model, we were able to derive analytical expressions of $\pi(y, T)$. For the specific case of a stationary population this fraction converges to 0 for T going to infinity. Assuming, however, a stable population with positive growth rate $r > 0$ we could analytically derive an expression of the fraction $\pi(y, T)$ which amounts to a weighted integral of the further life expectancy at age y with the weights being an exponential discount with the stable population growth rate.

Since the stable population model is quite a restrictive approximation over such a long time period, we extended our analysis to a nonstable population model where we indirectly estimated the time series of fertility and mortality over time allowing for differences across various subperiods. Our estimates for the fraction $\pi(65, 2010)$ ranges from 5.5% to about 9.5% which is clearly well below the estimates cited in Pearce (The Economist, 2014).

In the rest of the paper we studied the sensitivity of the fraction $\pi(y, T)$ with respect to the age y and the time T. For a given contemporaneous time T, we demonstrate that the shape of the fraction is non monotonic. It first increases with the age threshold at younger ages, then starts to decline before it increases again for older ages. This property can be explained by two opposite forces. The first one is positive and depends on the average historical mortality rate at age y. The second is negative and it is the proportion of people at age y among all age $y+$ in year T, which depends on contemporaneous data. The non decreasing property of $\pi(y, T)$ over the age threshold at young and old ages is explained by the fact that the high mortality rates at these ages in the past dominate over the existing mortality rates at these two life periods. Nevertheless, and despite $\pi(y, T)$ increasing at old ages, our results clearly indicate for all age thresholds the value of the faction $\pi(y, T)$ in year $T = 2010$ is far below 50% and ranges from 0.05 to at most about 0.15.

The behavior of the fraction $\pi(y, T)$ with respect to T may also exhibit a non monotonic path as we have demonstrated in our numerical calculations for values of $\pi(y, T)$ for T between 1850 and 2100 in case of $y = 65$. In this case, $\pi(y, T)$ first increases with T, while it decreases afterwards starting at time periods around $T = 2050$. The behavior of $\pi(y, T)$ over time is also explained by two terms. The first one is the crude growth rate in year T of the population older than age y, which can be either positive or negative. The second one is the fractional change over time in the number of people ever reached age y. During the twentieth century and first half of the twenty first century the first term will typically be higher than the second one when the growth rate of births increases, because the population reaching age y increases faster than the deaths above that age. Though, the values obtained for various time periods and different age thresholds are again well below 50% and could be as low as 1% for early time periods $T = 1850$ and go up to about 20% in 2050.

For illustrations we also provided the range of values for $\pi(65, T)$ for extreme values of T (i.e. T being the maximum age 130 versus T going to ∞) given a stable population under various growth rates of births and for alternative values of the life expectancy at birth. Only in case of a very high growth rate of births we could obtain numbers of the fraction $\pi(65, T)$ similar to Pearce (The Economist, 2014) or even exceeding these values.

Summing up our analytical and numerical derivations we may conclude that for realistic time series of historic and future fertility and mortality patterns the fraction of people alive today at ages above 65 among all those ever lived to age 65 is much lower as the one given in Pearce. Nevertheless our results indicate that the fraction has increased over time supporting the argument that the pace of ageing has increased.

6 Appendix

6.1 Proof of Proposition 1

Assuming time-constant death rate at age y (μ_y), let us define

$$\pi = \frac{A}{B}, \qquad \pi' = \frac{A'}{B'}, \tag{14}$$

where

$$B' = (B - \Delta_B)(1 - \mu_y \Delta) = B - \Delta_B - B\mu_y\Delta + \mu_y \Delta \Delta_B, \tag{15}$$
$$A' = A - \Delta_A = A - \Delta_B + 0.5\mu_y \Delta \Delta_B \tag{16}$$

(By contradiction:) if $\pi' > \pi$ it should be satisfied that

$$\frac{A - \Delta_B + 0.5\mu_y\Delta\Delta_B}{B - \Delta_B - B\mu_y\Delta + \mu_y\Delta\Delta_B} > \frac{A}{B}. \tag{17}$$

Rearranging terms and multiplying by -1 gives

$$\Rightarrow \frac{\Delta_B}{A} - 0.5\mu_y\Delta\frac{\Delta_B}{A} < \frac{\Delta_B}{B} + \mu_y\Delta - \mu_y\Delta\frac{\Delta_B}{B}. \tag{18}$$

Defining $\Delta_B = b\Delta$ and simplifying

$$\Rightarrow \frac{b}{A} - 0.5\mu\Delta\frac{b}{A} < \frac{b}{B} + \mu - \mu\Delta\frac{b}{B}. \tag{19}$$

Notice that b is the total number of births per year who have survived to age y, whereas Δ is an infinitesimal number. Rearranging terms and using the definition of π gives

$$\Rightarrow \frac{b}{A}(1-\pi) < \mu_y \left[1 + \Delta\frac{b}{A}(0.5-\pi)\right]. \tag{20}$$

Provided that for any stable population $\lim_{T\to\infty} \pi = 0$, we obtain

$$\Rightarrow \mu_y > \frac{b}{A}\frac{1}{1+0.5\frac{b}{A}\Delta}. \tag{21}$$

Under a stationary population $b/A = 1/e_y$. Hence,

$$\Rightarrow \mu_y > \frac{1}{e_y + 0.5\Delta}. \tag{22}$$

If the death rate from age y is non decreasing and $\Delta \to 0$, $1/e_y \geq \mu_y$, which contradicts the above inequality. Therefore, we have shown that $\pi' < \pi$ when the population is stationary.

It is important to realize that $\pi' < \pi$ also applies to a stable population with a fixed mortality schedule across cohorts. If the death rate at age y is constant, it can be shown for $\Delta \to 0$

$$\begin{aligned}\Delta_B(T) &= \int_{T-\Delta}^{T} N(y,t)dt = \int_{T-\Delta}^{T} B(t-y)\ell(y)dt \\ &= B(T-y)\ell(y)\Delta = N(y,T)\Delta = b(T)\Delta.\end{aligned} \tag{23}$$

Using Eq. (30), we have

$$\begin{aligned}A(T) &= \int_{y}^{\omega} N(a,T)da = N(y,T)\int_{y}^{\omega}\frac{N(a,T)}{N(y,T)}da \\ &= N(y,T)\int_{y}^{\omega} e^{-r(a-y)}\frac{\ell(a)}{\ell(y)}da = b(T)\int_{y}^{\omega} e^{-r(a-y)}\frac{\ell(a)}{\ell(y)}da.\end{aligned} \tag{24}$$

Therefore, if $r > 0$

$$\frac{b(T)}{A(T)} = \frac{1}{\int_y^{\omega} e^{-r(a-y)}\frac{\ell(a)}{\ell(y)}da} > \frac{1}{e_y} \geq \mu_y, \tag{25}$$

which also proves by contradiction that $\pi' < \pi$.

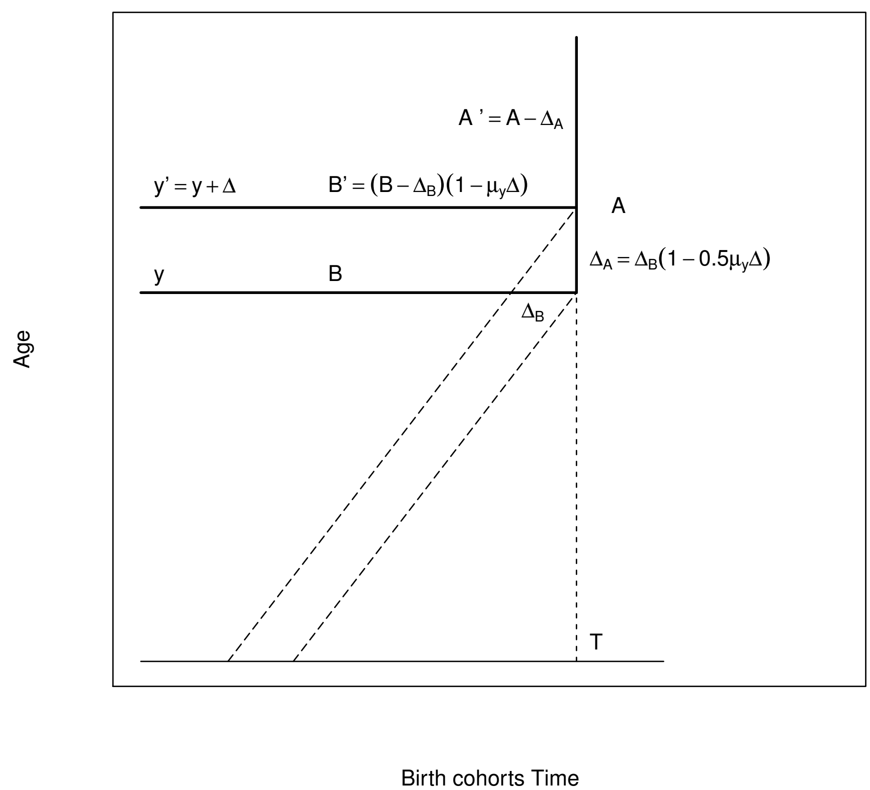


Figure 8: Illustration to change over age of the fraction alive of those ever-survived to old age

6.2 Sensitivity analysis of $\pi(y, T)$ with respect to T

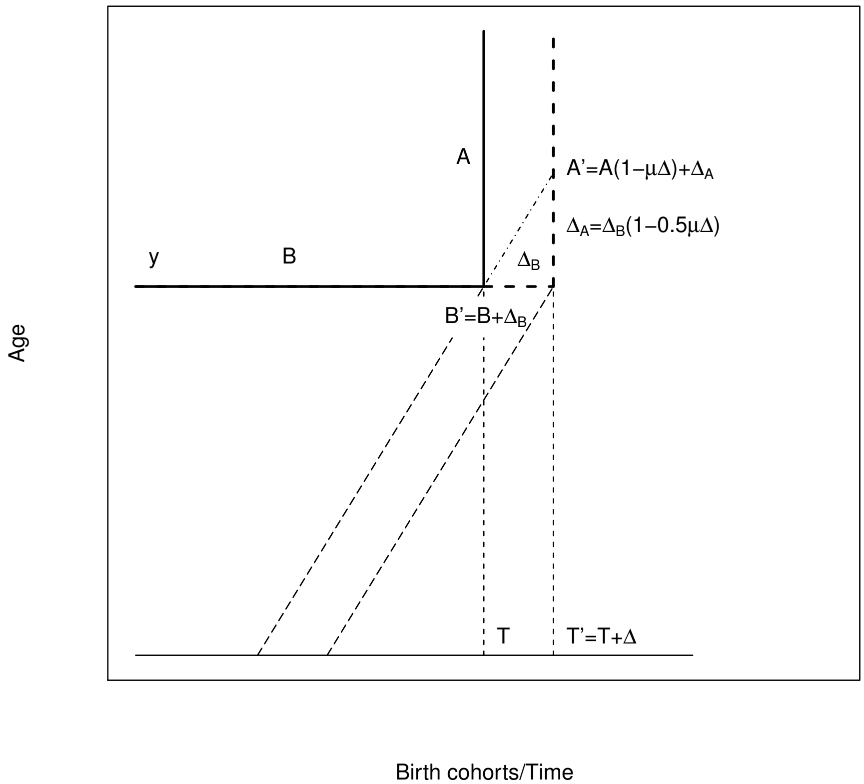


Figure 9: Illustration to change over time of the fraction alive of those ever-survived to old age

6.3 Simultaneous variation in longevity and fertility model

Assuming a unique age of childbearing (A), the renewal equation at time $s + A$ is

$$B(s + A) = B(s)f(A, s)\ell(A, s). \tag{26}$$

From (6)-(7), taking logarithms to both sides of (26) and differentiating with respect to s gives

$$r(s+A) = r(s) + \phi_s(s) - M_s(A, s), \tag{27}$$

where $r(s)$ is the growth rate of births in year s. Iterating (27) recursively until time 0 gives

$$r(s+A) \approx r(0) + \sum_{i=0}^{s/A} \phi_s(s - iA) - M_s(A, s - iA). \tag{28}$$

Integrating (28) with respect to time equals the total contribution of changes in mortality and fertility on the growth rate of births until time t (i.e. $\log\{B(t)/B(0)\}$)

$$\int_0^t r(s)ds \approx r(0)t + \int_0^t \sum_{i=1}^{s/A} \phi_s(s - iA) - M_s(A, s - iA)ds.$$

By changing the order of integration and rearranging terms, we have

$$\int_0^t r(s)ds \approx r(0)t + \sum_{i=1}^{t/A} \int_{iA}^t \phi_s(s - iA) - M_s(A, s - iA)ds.$$

Solving the integral and assuming $r(0)A = \phi(0) - M(A, 0)$ gives

$$\int_0^t r(s)ds \approx \sum_{i=0}^{t/A-1} \phi(iA) - M(A, iA),$$

which is equivalent to Eq. (8).

6.4 Reconstruction of the historical population reported in Table 2

Under a constant population growth rate r, the following demographic relations are satisfied:

$$N(0) = B(-\tau)e^{r\tau} \int_0^\omega e^{-ra}\ell(a)da, \tag{29}$$

$$B[-\tau, 0] = B(-\tau) \int_{-\tau}^0 e^{rt}dt, \tag{30}$$

where $B[-\tau, 0]$ denotes the total number of births born during the period $[-\tau, 0]$. Thus, provided a survival probably profile and the length of the interval τ, combining (29) and (30) the initial population growth rate (r) solves

$$\frac{N(0)}{B[-\tau, 0]} = \frac{r \int_0^\omega e^{-ra} \ell(a) da}{1 - e^{-r\tau}}.$$

Relying on the data most frequently reported in the literature we set 0 at -8000 B.C. and assume $N(0) = 5.000.000$ similar to Deevey (1960), Westing (1981), and Haub (2011). Like Keyfitz (1966) we choose an initial life expectancy at birth (e_0) of 25 years. Notice that this value of e_0 guarantees the survival of the *Homo Sapiens* with a TFR close to 6 even when random negative demographic shocks kill a sizable proportion of the population (see Figure 10).[5] Finally, since we use the same time data points of Haub (2011) we rely on his estimated total number of births born within the period $[-\tau, 0]$.

Provided an initial number of births $B(0)$ and a time series of demographic values $\{N(t), e_0(t), \mathrm{tfr}(t)\}_{t=0}^{T}$ and a set of population distributions $\{N(a,t)\}_{a=0,\ldots,\omega}^{t=0,\ldots,T}$, our historical population is consistently calculated over time using the Generalized Inverse Projection method (Lee, 1985; Oeppen, 1993), which consist in solving the following problem

$$\begin{aligned}\min_{\mathbf{k},\boldsymbol{\phi}} F(\mathbf{k}, \boldsymbol{\phi}) = &\sum_{t=0}^{T} \left(\frac{N(t) - \hat{N}(t)}{N(t)} \right)^2 + \sum_{a=0}^{K} \left(\frac{N(a,T) - \hat{N}(a,T)}{N(a,T)} \right)^2 \\ &+ \sum_{t=0}^{T} \left(\frac{\mathrm{tfr}(t) - \hat{\mathrm{tfr}}(t)}{\mathrm{tfr}(t)} \right)^2 + \sum_{t=0}^{T} \left(\frac{e_0(t) - \hat{e}_0(t)}{e_0(t)} \right)^2\end{aligned}$$

[5] For example, if every generation is assumed to suffer a negative demographic shock (e.g. war, famine, pandemic, etc.) that kills 1/3 of the population, the potential population growth rate should be approximately 1% per year.

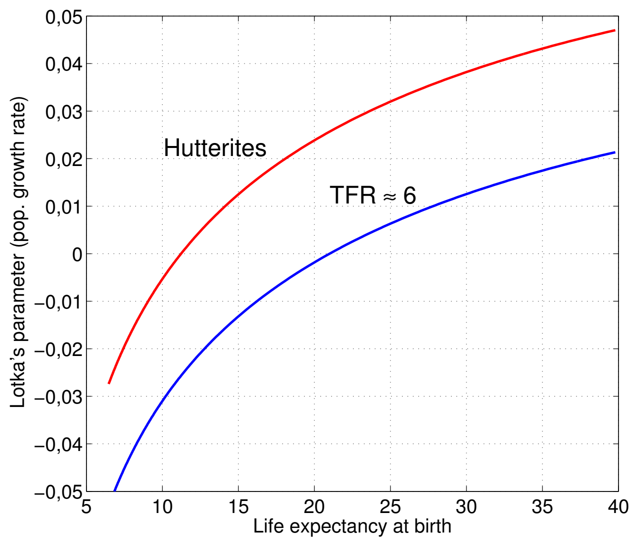


Figure 10: Constant population growth rate by fertility level and life expectancy

subject to

$$\begin{aligned}
\hat{B}(t) &= \hat{B}(t-A)f(A,t-A)\ell(A,t-A), \\
\hat{N}(a,t) &= \hat{B}(t-a)\ell(a,t-a), \\
\hat{N}(t) &= \sum_{a=0}^{\omega} \hat{N}(a,t), \\
\hat{e}_0(t) &= \sum_{a=0}^{\omega-1} 0.5\left[\ell(a,t-a)+\ell(a+1,t-a-1)\right] \\
\hat{\mathrm{tfr}}(t) &= f(A,t-A)(f_{fab})^{-1} \\
\left(I_{2T}\otimes\begin{bmatrix}1\\-1\end{bmatrix}\right)\begin{bmatrix}\mathbf{k}\\ \boldsymbol{\phi}\end{bmatrix} &\leq \begin{bmatrix}\bar{k} & \bar{\phi} & -\underline{k} & -\underline{\phi}\end{bmatrix}' \otimes \mathbf{1}_{T\times 1}
\end{aligned}$$

where $\ell(a,t-a) = e^{\left\{-\sum_{s=0}^{a-1}\exp(\alpha(s)+k(t-a+s)\beta(s))\right\}}$, $f(A,t-A) = f\cdot e^{\phi(t-A)}$, $\mathbf{k} = [k(0),\ldots,k(T)]$, $\boldsymbol{\phi} = [\phi(0),\ldots,\phi(T)]$, and $[\bar{k},\bar{\phi},-\underline{k},-\underline{\phi}]$ are the maximum and minimum values of $\{k(t),\phi(t)\}_{t=0,\ldots,T}$, which are set at $[30, 0.5, 50, 2]$, and $f_{fab} = 0.4886$ is the fraction of female at birth. The main property of the generalized population reconstruction method is that gives a population structure that is consistent by age and over time for non-stable populations.

6.5 Population ever lived data

Table 3: Estimates of people ever born by different authors

Exact date	Population (millions)	Lotka's r (in %)	Crude birth rate	Life expectancy (years)	Births at exact date (millions)	Births in period (millions)	Cumulated births (millions)	Survival prob. age 65	Population ever lived to age 65 (millions)	Cumulated population to age 65 (millions)
Haub (2011)										
-50000	0	0.035	0.080	13.0	0	1140		0.0151		
-8000	5	0.051	0.080	13.0	0	46118	1140	0.0153	17	17
1	300	0.034	0.060	17.0	18	26614	47259	0.0330	704	721
1200	450	0.023	0.060	17.0	27	12813	73872	0.0328	879	1600
1650	500	0.464	0.050	22.0	25	3181	86686	0.0671	420	2020
1750	795	0.464	0.040	28.0	32	4047	89866	0.1172	213	2234
1850	1265	0.539	0.040	29.0	51	2903	93914	0.1226	474	2708
1900	1656	0.837	0.033	38.0	55	2986	96817	0.2337	356	3064
1945	2516		0.031		78		99803		698	3762
Keyfitz (1966)										
-1000000	0	0.001	0.040	25.0	0	13508		0.0889		
-5000	5	0.078	0.040	25.5	0	12525	13508	0.0931	1201	1201
0	250	0.047	0.040	25.3	10	24983	26034	0.0914	1166	2367
1650	545	0.550	0.040	28.7	22	16121	51017	0.1234	2283	4651
1945	3000		0.040		120		67138		1989	6640
Westing (1981)										
-298000	0	0.006	0.050	20.0	0	2725		0.0514		
-40000	3	0.002	0.040	25.0	0	5014	2725	0.0889	140	140
-8000	5	0.046	0.034	30.0	0	14270	7739	0.1364	446	586
0	200	0.056	0.029	35.0	6	15681	22009	0.1929	1946	2532
1650	500	0.347	0.028	40.0	14	3992	37690	0.2576	3025	5557
1850	1000	0.877	0.029	45.0	29	4269	41682	0.3292	1028	6585
1945	2300		0.037	50.0	85		45951		1405	7991
Deevey (1960)										
-998040	0	0.000%	0.040	25.0	0	11782		0.0889		
-298040	1	0.000%	0.040	25.0	0	21344	11782	0.0889	1048	1048
-23040	3	0.003%	0.040	25.0	0	2552	33126	0.0889	1898	2945
-8040	5	0.070%	0.040	25.4	0	4658	35678	0.0927	227	3172
-4040	87	0.011%	0.035	28.7	3	15132	40336	0.1228	432	3604
-40	133	0.083%	0.035	29.2	5	17278	55468	0.1282	1858	5462
1650	545	0.290%	0.035	30.9	19	2212	72746	0.1457	2215	7677
1750	728	0.437%	0.035	32.2	25	1424	74958	0.1597	322	7999
1800	906	0.575%	0.035	33.3	32	4286	76383	0.1729	227	8227
1900	1610	0.798%	0.035	35.5	56	3051	80668	0.1990	741	8968
1945			0.035				83719		607	9575

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