Peer Effects and Friendship Network Diversity: Evidence from U.S. Adolescents

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March 16, 2015

Abstract

It is controversial whether diversity in an individual's friendship network improves one's performances. Using a unique data set with friendship nomination information from 1350 American high school adolescents, we investigate whether the adolescents are under more peer effects from friends of their same immigration status or ethnicity than friends of different types. Empirical results show that adolescents' same type of friends, as compared to their different types of counterparts, exert differential influences on adolescents' academic grades, frequency of psychological service visits, and participation in extracurricular activities. Counterfactual analyses by restricting adolescents in homogeneous friendship networks show that networks only consisting of their same type of friends reduce some education outcomes. The findings provide supporting evidence to justify actions that enhance friendship network diversity in American high schools. (JEL C31, D10, I24, J15)

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1 Introduction and Literature Review

It is hard to determine whether diversity in an individual's network is a blessing or a curse for the individual's outcomes. When students with different immigration and ethnic backgrounds are flocking to the education system in the U.S., the demographic composition of the schools are changing. High school adolescents have more chances to make friends with peers from diverse backgrounds. Friends with various backgrounds in high school students' networks might exert heterogeneous peer effects on adolescents' education outcomes. This paper investigates whether diversity in a high school student's friendship network is beneficial to the student's education outcomes.

Quantifying peer effects in social network is always challenging because of the endogeneity problem. Literature identifies peer effects by solving a *reflection* problem, which is to investigate whether the average behavior of all individuals in one's network affects his or her behavior. Manski (1993) distinguishes endogenous peer effects, exogenous peer effects, and correlated peer effects under the reflection framework. How an adolescent's education outcome tends to vary with the average education outcome of one's peers in his or her friendship network is defined as *endogenous peer effect*. How an adolescent's education outcome tends to vary with the socioeconomic composition of his or her friendship network is *exogenous peer effect*. How adolescents in the same environment, for example, in the same classroom, tend to behave similarly is *correlated peer effect*. We use the three definitions of peer effects defined in Manski (1993) to distinguish the different aspects of peer effects in our paper.

There is evidence that the influence from peers affect education outcomes in academic performances, psychological health, and extracurricular activities. Students' academic performances are strongly influenced by their peers on campus (Calvó-Armengol et al., 2009) and in neighborhood (Bervoets et al., 2012). Students' psychological health is affected by their peers. For example, peer effects determine how close students choose to be with their friends, especially among immigrants in a Canadian public school (Teja and Schonert-Reichl, 2013). Peer effects cause *social distress*, which is a psychological suffering resulted from bad social interactions. Horenczyk and Tatar (1998) find that social distress exists among young girls in Israel. Students' extracurricular activity participation is also affected by their peers (Bramoullé et al., 2009). Peer effects are not only important factor in education, but also important in other social networks. For example, peers affect outcomes in other fields such as financial behaviors (Cohen-Cole et al., 2010; Greenberg and Mollick, 2014), romantic and sexual relations (Bearman et al., 2004), job market participation(Zenou, 2013; Ioannides and Loury, 2004; Mortensen and Pissarides, 1994), etc.

One of the underlying reasons for adolescents to have similar type of friends is homophily, the tendency of individuals to tie himself with similar peers. "Birds of a feather, flock together" is an old proverb introducing homophily in the 16th century. Homophily is a common phenomenon observed in friendship network in current U.S. education system. Connections that make people feel close include age, gender, religious belief, cultural background, education, geographic proximity, etc. The opposite of homophily is heterophily, the reason for adolescents to have different types of friends. We follow the definitions in Currarini et al. (2009) : Individuals' friends who are same with themselves in immigration status or ethnicity are their same type of friends. Individuals' friends who are different to themselves in immigration status or ethnicity are their different type of friends. An individual gains type-dependent benefits from having same type or different types of friends. Evidence shows that homophily and heterophily are important in friendship formation. Friends with homophily last longer than friends with heterophily (McPherson et al., 2001). Failure to identify similar characterisitcs leads to dissolution of friendships (Watts, 2007; Noel and Nyhan, 2011).

Instead of treating effects from all peers as identical in the previous literature, we consider the heterogeneity of peer effects from various friends with different immigration status and ethnicity. Immigration status and ethnicity are two important factors that affect friendship networks, and therefore affect education outcomes. Students with same immigration status or ethnicity share similar backgrounds (Williams, 1975; Blau, 1977; Fong and Isajiw, 2000; Page, 2008; Moody, 2001; Cutler et al., 2008). Aslund et al. (2011) find that academic performance is increasing in adults who have friends with their same ethnicity. Peer effects are associated with immigration and racial segregation between Russian Jewish and ethnic German Diaspora migrant (Titzmann et al., 2012) and among Dutch, Moroccan, Turkish, Surinamese and others in a high school in the Netherlands (Baerveldt et al., 2004). Arcidiacono et al. (2013) explore interracial friends segregation in the transition period from high schools to universities, finding that individuals with stronger academic backgrounds are more likely to make friends with Asian and white students, but less likely to interact with black students. Cortes (2006) refers the first generation as "immigrant children" and the second generation as "US-born children." He compares the reading and math test scores of the first and the second generation immigrants. He also compares the test scores of the first generation immigrants who attend an immigrant receiving school to those that attend a non-immigrant receiving school. Empirical results show that the test score gap between the first generation and the second generation immigrants narrows with the increasing resident years of the first generation immigrants in the U.S. Results also show that the first and second generation immigrants from immigrant-receiving schools do not have largely different performances in academic grades than those from non-immigrant receiving schools.

We find that in both the immigration and ethnicity cases, having the same type of friends results in a greater frequency of psychological service visits, when compared to those with different types of counterparts. In the ethnicity case, the same type of friends are more influential in academic grades and extracurricular activity participation, as compared to different types of friends. In the immigration case, different types of friends exert more impacts in academic grades and extracurricular activity participation than the same type of friends. Based on the empirical findings, we study the counterfactual outcomes where each adolescent is restricted with a strictly homogeneous friendship network. We find that strictly homogeneous networks lower some of the education outcomes.

This paper contributes to the existing literature in two ways. First, previous literature considers peer effects as homogeneous from all friends in the network. However, in this study we assume the heterogeneity of peer effects from various groups. Based on this assumption, our findings shed light on endogenous and exogenous peer effects from various groups. Second, we use counterfactual analysis by assuming that each student has a strictly homogeneous friendship network. The counterfactual outcomes address the importance of the diversity in students' friendship networks.

2 Theoretical Model and Identification Strategy

2.1 Peer Effects Identification: Linear-in-mean Model

We apply generalized two stage least square (2SLS) methodology as in the literature to identify the endogenous peer effects. Kelejian and Prucha (1998) develop a linear spatial autoregressive model estimated by generalized two-stage least squares, yielding a consistent and asymptotically normal estimator that is applicable to large sample. An endogeneity problem happens because an individual's outcome is affected by the average outcome of one's network. But the individual is also included in his or her own network (Moffitt et al., 2001). Bramoullé et al. (2009) apply generalized 2SLS in addressing the endogenous and exogenous peer effects on recreational activities among adolescents in the United States, using the average characteristics of friends' friends as instruments. Our paper expands the instruments as the average characteristics of the same type of friends' same of type friends and the average characteristics of different type of friends' different type of friends.

Literature uses a row-normalized spatial weighting matrix to indicate a friendship network. If an individual nominates the other as a friend, the corresponding entry of the weighting matrix is the inverse of the person's number of friends. If one does not nominate the other as a friend, the entry is zero (Lin, 2010; Boucher et al., 2010; Bramoullé et al., 2009). We use linear-in-mean model to disentangle endogenous and exogenous peer effects in this paper. Manski (1993) sets up a linear model under the reflection framework. Assuming each member of a population be characterized by a value for (y, x, z, u), the linear-in-mean model is

$$y = \alpha + \beta E(y|x) + E(z|x)'\delta + z'\eta + u, \qquad E(u|x, z) = x'\gamma. \tag{1}$$

y represents the scalar outcome and x is the characteristics of the reference group. The interpretation of β, the coefficient for endogenous peer effects, is the impact from the average education outcome of the reference group on an individual's education outcome. z is the socioeconomic composition of the reference group. u is the unobserved ability that directly affects y. δ is the coefficient for exogenous peer effects, quantifying the impact from the average background of the reference group on an individual's outcome. η is an indicator for correlated effects, which describes similar behaviors of individuals due to their similar background. Taking expectation on both sides of Equation (1), Manski (1993) has

$$E(y|x, z) = \alpha + \beta E(y|x) + E(z|x)'\delta + x'\gamma + z'\eta.$$

Combining Manski (1993) with Moffitt et al. (2001), Bramoullé et al. (2009) apply the linear-in-mean model to identify endogenous and exogenous peer effects. A student's education outcome is affected by the mean outcome of the student's reference group, by the student's own characteristics, and by the mean characteristics of the student's reference group. Let y_i be the education outcomes of adolescent i. Let x_i be a $1 \times K$ vector of characteristics of i. P_i is the reference group for adolescent i, with himself excluded. The size of an individual i's reference group is n_i. Thus, the model is given by:

$$y_i = \alpha + \beta \frac{\sum_{j \in P_i} y_j}{n_i} + \gamma x_i + \delta \frac{\sum_{j \in P_i} x_j}{n_i} + \epsilon_i, \qquad E[\epsilon_i|\mathbf{x}] = 0 \tag{2}$$

where β captures the endogenous peer effects and δ captures the exogenous peer effects. γ is the coefficient for the impact of personal attributes. In this paper, *correlated effects* exist because an adolescent's outcome and the average outcome of the adolescent's peers are correlated with some unobserved common characteristics. For example, students from the same classroom share the same effort from their teacher in mathematics class. A student's math score is affected by both the teacher's effort and the peers' average performance. However, the average performance of the peers is also affected by the teacher's effort. Omitted variable bias occurs if we do not include the teacher's effort in the regression. Lee (2007) introduces fixed group effects model to address the estimation bias in group interaction caused by unobservables. Bramoullé et al. (2009) treat correlated unobservables as either being absent or as network fixed effects to solve the difficulty in separating correlated effects out of the total peer effects. We include class dummies in the model for network fixed effects because the formation of adolescents' networks is highly correlated with the classes and the unobservables might be affecting the whole classroom.

2.2 Setup of the Weighting Matrix

To present the friendship network information in matrix notation, we set up a weighting matrix for the relationship between any two of the individuals in the sample. The weighting matrix $\mathbf{G}$ is an $N \times N$ matrix, where N is the number of observations in the sample. If individual i nominates individual j as a friend, $\mathbf{G}_{ij}$, the (i, j) entry in the weighting matrix $\mathbf{G}$, is $1/n_i$ where n_i is individual i's reference group size. Otherwise $\mathbf{G}_{ij} = 0$, meaning individual i does not nominate individual j as a friend. An individual cannot nominate oneself as one's own friend, so the weighting matrix is a zero-diagonal matrix. The weighting matrix is not symmetric because a one-way nomination does not guarantee a mutually agreed friendship. Following Bramoullé et al. (2009), we plug the weighting matrix into Equation (2) and obtain

$$y = \beta \mathbf{G} y + \gamma x + \delta \mathbf{G} x + \epsilon, \quad E[\epsilon|x] = 0. \tag{3}$$

The reduced form of Equation (3) is

$$y = (I - \beta \mathbf{G})^{-1}(\gamma I + \delta \mathbf{G})x + (I - \beta \mathbf{G})^{-1}\epsilon. \tag{4}$$

2.3 Instrument for Endogenous Peer effects

In Equation (3), the outcome y is affected by the endogenous peer effect $\beta \mathbf{G} y$, which is the average outcome of the network to which an adolescent belongs. $\beta \mathbf{G} y$ is correlated with y. Intuitively, the individual is included in his own network, so the impact from the average outcome of the network contains the impact of the outcome from the individual himself. Therefore, we cannot obtain β by a linear estimation. We apply generalized 2SLS strategy from the setup of Kelejian and Prucha (1998) and the revision from Lee (2007) to remove the endogeneity. Since

$(I - \beta\mathbf{G})^{-1} = \sum_{k=0}^{\infty} \beta^k \mathbf{G}^k$, we plug it in Equation (4) and obtain

$$y = \gamma x + (\gamma\beta + \delta) \sum_{k=0}^{\infty} \beta^k \mathbf{G}^{k+1} x + \sum_{k=0}^{\infty} \beta^k \mathbf{G}^k \epsilon. \tag{5}$$

Multiplying $\mathbf{G}$ on both sides of Equation (5) and taking the expectation, we have

$$E(\mathbf{G}y|x) = \gamma\mathbf{G}x + (\gamma\beta + \delta) \sum_{k=0}^{\infty} \beta^k \mathbf{G}^{k+2} x. \tag{6}$$

Moffitt et al. (2001) assume that the friend groups have same size s including the individual. They denote Γ_s as the interaction matrix within a group. One has $\Gamma_{s,ij} = \frac{1}{s-1}$ if $i \neq j$ and 0 otherwise. The matrix $\mathbf{G}$ is a block-diagonal matrix with diagonal blocks given by Γ_s. It is easy to see that $\mathbf{G}^2 = \frac{1}{s-1}\mathbf{I} + \frac{s-2}{s-1}\mathbf{G}$ if $s \geq 2$. So $\mathbf{G}^2$ can be described as a combination of $\mathbf{I}$ and $\mathbf{G}$. Lee (2007) develops the model into interactions with different sizes. The two group sizes s_1 and s_2 satisfy $s_1, s_2 \geq 2$. The interaction matrix $\mathbf{G}$ is thus

$$\begin{pmatrix} \mathbf{\Gamma}_{s_1} & 0 \\ 0 & \mathbf{\Gamma}_{s_2} \end{pmatrix}.$$

The assumption in Lee (2007) gives that $\mathbf{G}^2 = \lambda_0\mathbf{I} + \lambda_1\mathbf{G}$. The diagonal elements give $\lambda_0 = \frac{1}{s_1-1} = \frac{1}{s_2-1}$, so $s_1 = s_2$. Therefore, with $s_1 \neq s_2$, the matrices $\mathbf{I}$, $\mathbf{G}$ and $\mathbf{G}^2$ are linearly independent. A proposition in Bramoullé et al. (2009) states that under $\gamma\beta + \delta \neq 0$, if the matrices $\mathbf{I}$, $\mathbf{G}$, and $\mathbf{G^2}$ are linearly independent, social effects are identified. In our study, individuals have different sizes of friendship networks, so $s_1 \neq s_2$. Therefore, $\mathbf{I}$, $\mathbf{G}x$, and $\mathbf{G^2}x$ can be used as instruments.[1] The instrument $\mathbf{G}^2 x$ is interpreted in a socioeconomic context as an $N \times K$ vector of weighted averages of the characteristics of the friends' friends of each individual in the network.

3 Data

The National Longitudinal Study of Adolescent to Adult Health data (Add Health) is a four-wave household survey starting in 1994. It records adolescents' backgrounds, including school

[1] A detailed proof of instrumentation is in Section 1 of the online appendix, which can be downloaded from: sites.google.com/site/shanshan333wang/secondyearpaper

performance, friendship network, neighborhood information, personal characteristics, household members, parents' job market participation, etc. The data covers 142 high schools. We take a random draw of 5 high schools containing 1350 adolescents in wave I (1994 -1995) for analysis. One unique feature of Add Health is a friendship nomination dataset. Table (1) is the summary statistics. Table (2) demonstrates the friendship network distribution. Table (3) is the immigration and ethnicity composition of our sample. It justifies the existence of homophily in our sample.

3.1 Immigration Status and Ethnicity

We define an individual's immigration status by checking the individual's birthplace as well as his or her parents' birthplaces. *Native* is an individual who was born with American citizenship[2] and his or her parents were both born in the United States. *First generation immigrant*, or *imm1* for short, is a foreign-born individual with at least one foreign-born parent. *Second generation immigrant*, or *imm2*, is an American-born individual with at least one foreign-born parent. *Other immigrant*, or *other imm*, includes individuals who cannot name their own birthplaces, or individuals who know neither of their parents' birthplaces.

We construct the ethnicity variable according to individuals' self reports. Ethnicity has five subgroups: *white*, *black*, *Hispanic*, *Asian* and *other ethnicity*. In further analysis, all the ethnic groups are denoted by their full names except for the *other ethnicity*, which we call *other eth* for short.

The *other imm* and *other eth*, are individuals who could not clearly identify their profiles. These two groups are equally important as all the other identified groups. They represent the adolescents who have less sense of belonging. They have difficulty in identifying to which group they belong. It is likely due to their early parental separation, frequent move from countries to countries, or other reasons.

The immigration and ethnicity composition is shown in the top panel in Table (1). The sample includes about 80% native, 10% imm1, 6% imm2 and 4% other imm. The sample is

[2]If an individual is the child of an American citizen, this person can be American citizen even if he or she was born in another country.

composed of approximately 60% white, 16% black, 13% Hispanic, 6% Asian and 6% other eth.

3.2 Friendship Network

We use *In-School Friendship Nominations Data* to obtain the friendship network information. An individual nominates up to five closest same-sex and five closest opposite-sex friends. The ten friends were also interviewed within the in-school survey and were in the same school with the individual. With the nominations, we set up a friendship network. It refers to the *friends* or *non-friends* relationship between any two individuals. Table (2) is the distribution of number of friends for all adolescents. On average everyone has 1.6 friends. Half of the adolescents do not have any friend. We keep them in our study because they can be nominated by other adolescents as a friend. Around 1% of the adolescents have 10 friends. Furthermore, we are able to identify whether two individuals are *non-friends*, *friends with same type*, or *friends with different types*, by combing the immigration status or ethnicity information.

3.3 Evidence of Homophily

To show that homophily exists in our sample, we calculate the friendship network composition for each subgroup. For example, we sum up natives' friend number and we calculate the number of native friends in all natives' network. The ratio of native's native friends equals

$$\frac{\text{Number of native friends in native subgroup friend network}}{\text{Number of all natives' friend}}.$$

This is the definition of *homophily index* (Currarini et al., 2009). The underlined numbers in Table (2) on diagonal represent the proportion of same type of friends for a subgroup. The last row is the proportion of a subgroup in the whole population. We compare the percentage on diagonal with the corresponding percentage in the last row. For example, among native's friends, 89.3% are natives. But only 79.4% of the whole population are natives[3]. This indicates that the percentage of the native friends in natives' network are significantly higher than the percentage of natives in the whole population. We find out that all the numbers on diagonal are greater than the numbers in the last row. This verifies the existence of homophily in our sample for all subgroups.

3.4 Dependent Variable

Dependent outcomes are *academic grade*, *psychological service* and *extracurricular activity*. The descriptive data is in the middle panel in Table (1).

Academic grade is the grade summation of four subjects, mathematics, English, history and science. A student's grade in each subject falls into one of the four categories, A, B, C or D or below. We assign 4 points to A, 3 to B, 2 to C and 1 to D or below. The maximum of the academic grade is 16 and the mean is 11.2.

Psychological service refers to the index of one's frequency of psychological service visits. The index is measured by the question "When was your last counseling, psychological testing, or any mental health or therapy service?" We assign value 1 for "within the last 12 months", 2 for "1 to 2 years ago", 3 for "more than 2 years ago", and 4 for "never had". The average frequency of psychological service visit is 2.8.

Extracurricular activity is the total number of clubs[4] in which a student participates. On average an adolescent joins 2.3 clubs.

[3]We conduct a hypothesis test to determine whether the difference between the two proportions is significant. The null hypothesis of the two-proportion z-test is that the proportion of natives in the whole population equals the proportion of natives in natives' friends. The alternative is that the proportion of natives in the whole population is greater than the proportion of natives in natives' friends. The low p-value, 0.00, indicates that the test rejects the null hypothesis. So the proportion of natives in the whole population is greater than the proportion of natives in natives' friends. We also conduct the two-proportion z-test for all the other subgroups. Their p-values are 0.00 for imm1, 0.00 for imm2, 0.36 for other imm, 0.00 for white, 0.00 for black, 0.00 for hispanic, 0.00 for Asian, and 0.36 for other eth. So the two proportions are significantly different for most of the subgroups.

[4]The clubs include French club, German club, Latin club, Spanish club, book club, computer club, debate team, drama club, future farmers of America, history club, math club, science club, band, cheerleading/dance team, chorus or choir, orchestra, other club or organization, baseball/softball, basketball, field hockey, football, ice hockey, soccer, swimming, tennis, track, volleyball, wrestling, other sport, newspaper, honor society, student council and yearbook.

3.5 Independent Variable

The last panel of Table (1) is about personal characteristics. Around 40% of the adolescents are female. The adolescents' age range is from 10 to 19 with an average of 15 years old. 87% of the mothers live with kids. 78% of the adolescents have at least one of their parents involved in job market. Mother's education dummies include 10% not finishing high school ("no high school"), 24% finishing high school but not entering college ("finish high school"), 11% entering college but not finishing ("no college"), 23% finishing college ("finish college"), and the rest unknown education level ("do not know"). Physical health index is between 1 and 5 with the average of 3.9. Value 5 represents an "excellent" health condition, 4 for "very good," 3 for "good," 2 for "fair", and 1 for "poor." Time spent on watching television or video cassettes on a weekday is 2.4 hours on average. Hard work index is set up with the self evaluation of how hard an adolescent tries on his or her school work. Value 3 is for "very hard," 2 for "hard enough but not as hard as one could," 1 for "don't try very hard", and 0 for "never try." The average hard work index is 1.97.

4 Empirical Framework

4.1 Homophily Matrix and Heterophily Matrix Setup

A friendship network can be divided into two networks, one from an adolescent's same type of peers and the other from different types of peers. We assign each of them a weighting matrix. $\mathbf{G}_1$ is a 1350×1350 weighting matrix for one's same type of friends and $\mathbf{G}_2$ is of the same size but for different types. Suppose n_i is the size of the reference group of individual i. The (i, j) entry of $\mathbf{G}_1$ is $1/n_i$ if i nominates j as a friend and they are of the same type. It is 0 if i and j are friends of different types, or if they are not friends. The (i, j) entry of $\mathbf{G}_2$ is $1/n_i$ if i nominates j as a friend and they are of different types. It is 0 if i and j are friends of the same type, or if they are not friends. $\mathbf{G}$ is the traditional weighting matrix that considers every friend identical in terms of types. It satisfies that $\mathbf{G} = \mathbf{G}_1 + \mathbf{G}_2$. We call $\mathbf{G}_1$ *homophily weighting matrix* because it activates the entries of friends in $\mathbf{G}$ that are of the same type and deactivates the entries that are of different types. The homophily weighting matrix describes a friendship network with the same type of friends. According to the notation in Currarini et al. (2009), we call $\mathbf{G}_2$ *heterophily weighting matrix*, which highlights how a person is interacting with different types of friends.

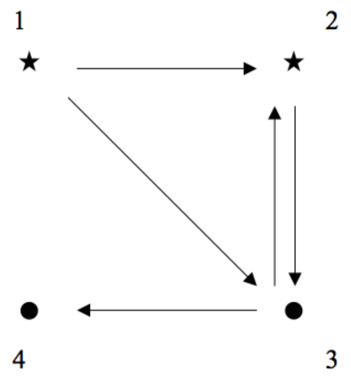

Figure 1: A Friend Network with Different Types

The following is an example of how to split a weighting matrix into two matrices. In Figure (1), a social network consists of 4 individuals. The direction of an arrow from individual i to individual j indicates that i nominates j as a friend. We are only interested in direct friends. The number of the outgoing arrows represents the number of friends an individual has. For example, individual 3 has two friends. He nominates individual 4 as his friend but individual 4 does not choose him as a friend. Individual 2 and individual 3 nominate each other as friends. A traditional weighting matrix without distinguishing types is illustrated as follows:

$$\mathbf{G} = \begin{array}{cc} & \begin{array}{cccc} \star & \star & \bullet & \bullet \end{array} \\ \begin{array}{c} \star \\ \star \\ \bullet \\ \bullet \end{array} & \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{1} & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{array}.$$

To distinguish types in this network, we assign each node a shape. Nodes with the same shape are the same type of friends. Therefore, individual 1 with 2, and 3 with 4 are of the same type. We identify *same type of friends*, *different type of friends*, and *non-friends* relationships by combining the direction of an arrow and the shapes of its connecting nodes. For example, individual 1 points at individual 2 and they are of the same shape, so they are *same type of friends*. Individual 1 points at individual 3 but they are of different shapes, so they are *different* *types of friends*. Individual 1 and 4 do not have any connection, so they are *non-friends* even though they are of the same shape.

The homophily weighting matrix is as follows:

$$\mathbf{G}_1 = \begin{array}{cc} & \begin{array}{cccc} \star & \star & \bullet & \bullet \end{array} \\ \begin{array}{c} \star \\ \star \\ \bullet \\ \bullet \end{array} & \begin{pmatrix} 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{array}.$$

The heterophily weighting matrix is as follows:

$$\mathbf{G}_2 = \begin{array}{cc} & \begin{array}{cccc} \star & \star & \bullet & \bullet \end{array} \\ \begin{array}{c} \star \\ \star \\ \bullet \\ \bullet \end{array} & \begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{1} & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{array}.$$

It is easy to check in this example that $\mathbf{G} = \mathbf{G}_1 + \mathbf{G}_2$, which verifies the assumption that a friendship network can be divided into two networks, one from the same type of friends, and the other from different types of friends.

4.2 Empirical Model and Instruments

The empirical model is derived from Equation (3) and the splitting rule of the weighting matrix. Note that even though mathematically the equation $\mathbf{G} = \mathbf{G}_1 + \mathbf{G}_2$ satisfies, we cannot plug this equation directly into Equation (3) because the assumptions on the two sides of the equation are different. $\mathbf{G}$ is set up under the assumption that all individuals are of one type. However, $\mathbf{G}_1$ and $\mathbf{G}_2$ are split under the hypothesis that one's same type of friends and different types of friends generate different impacts. Because $\mathbf{G}_1$ and $\mathbf{G}_2$ influence the outcome differently, we assign different coefficients for these two weighting matrices. We set up a model with the following specification,

$$y = \beta_1 \mathbf{G}_1 y + \beta_2 \mathbf{G}_2 y + \gamma x + \delta_1 \mathbf{G}_1 x + \delta_2 \mathbf{G}_2 x + \epsilon, \quad E[\epsilon|x] = 0. \tag{7}$$

y is the outcome variable. $\mathbf{G}_1$ is the weighting matrix representing the same type of friends' network and $\mathbf{G}_2$ is for different types. β_1 and β_2 are the coefficients for endogenous peer effects from friends with same and different types. γ is the coefficient for the effects from an individual's own characteristics. δ_1 and δ_2 are the coefficients for exogenous peer effects from friends with same and different types. The regression is applied to immigration and ethnicity cases, respectively. The reduced form is

$$y = (I - \beta_1 \mathbf{G}_1 - \beta_2 \mathbf{G}_2)^{-1}(\gamma I + \delta_1 \mathbf{G}_1 + \delta_2 \mathbf{G}_2)x + (I - \beta_1 \mathbf{G}_1 - \beta_2 \mathbf{G}_2)^{-1}\epsilon$$

To solve the endogeneity problem, we apply instruments in the split weighting matrix context. With the validity of instruments proved in Section 2.3, Bramoullé et al. (2009) adopt $S = [x \quad \mathbf{G}x \quad \mathbf{G}^2 x]$ as instrument. We expand them into five instruments,[5]

$$S = [x \quad \mathbf{G}_1 x \quad \mathbf{G}_1^2 x \quad \mathbf{G}_2 x \quad \mathbf{G}_2^2 x].$$

The socioeconomic interpretation of the instrument $\mathbf{G}_1^2 x$ is the average characteristics of the same type of friends' same type of friends. The instrument $\mathbf{G}_2^2 x$ is the average characteristics of the different type of friends' different type of friends. With the five instruments, the model is overidentified and we obtain the results by the generalized 2SLS methodology. It is important to test the validity of the instruments to verify that 2SLS model is a better fit than OLS (Kelley Pace and LeSage, 2008). We apply Wu-Hausman test and find that the instruments we construct are valid.

5 Estimation Results

Estimation results are presented from Table (4) to Table (7). Table (4) and Table (5) are the coefficients for endogenous and exogenous peer effects for the same and different types of friends. β_is are the coefficients for endogenous peer effects and δ_is are the coefficients for exogenous peer effects, where $i = 1, 2$. Standard errors are indicated under every coefficient. We apply Wald test to compare the corresponding β_1 with β_2, and δ_1 with δ_2. A low p-value

[5]Proof of the instruments is in Appendix A

indicates that β_1 and β_2, or δ_1 and δ_2, are significantly different. Table (6) and Table (7) are γs, estimates for impacts from personal attributes.

5.1 Endogenous Peer Effects Pattern

In Table (4), the Wald test between β_1 and β_2 shows that β_1 and β_2 in the immigration case are statistically different for each of the three outcomes. For the academic grades, a one-point increase in the average grade of an adolescent's same type of friends causes a 0.16 point increase in one's academic grade. A one-point increase from different types of friends causes a 0.3 point decrease in one's academic grade. A one-unit increase in the frequency of psychological service visits of one's same type of friends increases one's psychological service visit frequency by 0.67 significantly. A one-unit increase in the frequency of psychological service visits of one's different types of friends increases one's psychological service visit frequency by 0.2. Regarding extracurricular activity participation, having the same type of friends exerts positive endogenous effects on adolescents' participation. When the participation of different types of friends increases, an adolescent participates in fewer clubs.

In Table (5), β_1s and β_2s in the ethnicity case display the same signs with the corresponding β_is in the immigration case. Adolescents obtain positive endogenous impacts from having the same type of friends and negative impacts from having different types of friends in academic grades. Both the same type of friends and different types of friends exert positive endogenous impacts on adolescents in the frequency of psychological service visits. The more the same type of friends participate in extracurricular activities, the more an adolescent will tend to join the clubs. Different types of friends' participation in clubs discourages an adolescent to join the clubs.

We compare the absolute values of β_1s and β_2s. If $|\beta_1| > |\beta_2|$, the endogenous peer effects from having the same type of friends are greater than the endogenous peer effects from having different types of friends. If $|\beta_1| < |\beta_2|$, the endogenous peer effects from having the same type of friends are less than the endogenous peer effects from having different types of friends.

Regarding the frequency of psychological service visits in either the immigration case or the ethnicity case, β_1 and β_2 are both positive. This indicates that an adolescent increases his or her psychological service visits if either the same type of friends or different types of friends increase their frequency of psychological service visits. Also $|\beta_1| > |\beta_2|$ indicates that having the same type of friends is more influential than having different types of friends in the frequency of psychological service visits.

Regarding the academic grades and extracurricular activity participation in the ethnicity case, $\beta_1 > 0$ and $\beta_2 < 0$. Also β_1 and β_2 satisfy $|\beta_1| > |\beta_2|$. This indicates that for academic grades and extracurricular activity participation in the ethnicity case, having the same type of friends has positive endogenous peer effects on the adolescents and having different types of friends has negative endogenous peer effects on the adolescents. Having the same type of friends is more influential on each of the two outcomes than having different types of friends.

Regarding the academic grades and extracurricular activity participation in the immigration case, $\beta_1 > 0$ and $\beta_2 < 0$. Also β_1 and β_2 satisfy $|\beta_1| < |\beta_2|$. This indicates that for the outcomes of academic grades and extracurricular activity participation in the immigration case, having the same type of friends exerts positive endogenous peer effects on the adolescents and having different types of friends exerts negative endogenous peer effects on the adolescents. Having the same type of friends exerts less influence than having different types of friends on these two outcomes in the immigration case.

For the academic grades and extracurricular activity participation in the immigration case, having the same type of friends exerts less endogenous peer effects than having different types of friends. For the rest outcomes, having the same type of friends is more influential than having different types of friends. This is empirical evidence to support the theory of *social effects* as introduced in Montgomery and Casterline (1996). Social effects include two aspects, *social learning* and *social influence*.

Social learning happens in one's problem solving process. In solving a problem, individual i refers to an information set $\mathbb{I}$, which helps the individual to make a specific decision. The information set might include a listing and description of other individuals $N = \{N_j\}$ whose actions, communications, or perceived traits might help person i to resolve uncertainties in the problem. Person i may contact j as information made available by these individuals may already be embedded in person i's expectations. Social learning takes place interpersonally when the other individuals $N = \{N_j\}$ provide information that shapes person i's subjective beliefs about the keys in solving the problem. In this study, high school adolescents want to solve three problems including how to improve academic grades, how to deal with psychological problems, and how many clubs to join. These adolescents refer to their friendship networks to find the information $\mathbb{I}$ for the three topics. In an individual's network, having the same type of friends might obtain more convincing information for the individual to solve the problem. This is likely because friends of the same immigration status or ethnicity may have similar expectations in solving a problem. In contrast, friends with different immigration status or ethnicity are not as influential as friends of the same type because an individual and one's different types of peers may not share similar beliefs about how to solve a problem.

Social influence refers to the fact that an individual has a desire to avoid conflict within social groups. In one's social network, one practices "social conformity," the pressure to be similar to peers. This pressure provides a motivation for change to prevent within group conflict. So the existence of different peers could result in changes in the attitudes and behaviors of the members in a network of adolescents, as they try to reduce the gap between themselves and the different peers. Therefore, this accounts for the facts that some outcomes demonstrate that having different types of friends exerts more impacts on others than having the same type of friends.

In academic grades and extracurricular activity participation in both the immigration and the ethnicity cases, β_1 and β_2 have opposite signs. This indicates that having the same type of friends exerts positive endogenous peer effects and having different types of friends exerts negative endogenous peer effects on the two outcomes. The reason for the opposite signs between the two types of friends has been elusive to those who study peer effects. Further exploration will be a part of our larger research agenda.

5.2 Exogenous Peer Effects

In both of the immigration and ethnicity cases related to the academic grades, the exogenous effects from the age of the same type of friends are significantly negative. This indicates that if the same type of friends of an adolescent are older on average, it is likely to be harmful for one's academic grades. The exogenous effects on academic grades from the age of different types of friends are positive. Comparing the two absolute values of exogenous peer effects from the covariate age, it satisfies $|\delta_1| > |\delta_2|$. Therefore, the same type of friends' average age exerts more exogenous peer effects on adolescents' academic grades than different types of friends' average age. Regarding the covariate age, the same type of friends exert a larger negative impact on an adolescent's psychological service visits frequency, relative to different types of friends.

6 Counterfactual Analysis and Policy Implication

After quantifying endogenous peer effects from the same type of friends and different types of friends, we want to answer the following two questions in friendship network diversity. First, in order to improve students' education outcomes, should a school encourage students to make friends with different types of students or only to cluster with their own type? We answer this by studying an extreme case for friendship networks, strictly homogeneous friendship networks. Second, how do strictly homogeneous friendship networks affect peer effects for each immigration and ethnic subgroup? We achieve this by comparing the subgroup counterfactual outcomes in the extreme case with the corresponding subgroup outcome in the real data.

6.1 Full Sample Counterfactual Analysis

6.1.1 Strictly Homogeneous Networks Model Setup

A strictly homogeneous network contains individuals who only make friends with their own type. We obtain counterfactual outcomes by putting the students with the real characteristics in the above hypothesis. We compare the average counterfactual outcome with the average real outcome. If the average counterfactual outcome is higher, it indicates that the network in the hypothesis is enhancing the outcome. Otherwise, the network assumption will not increase the education outcome.

The following paragraph contains the steps of counterfactual analyses for strictly homogenous networks. In the previous estimation, $\mathbf{G}_1$ and $\mathbf{G}_2$ are constructed through the real nomination data. We denote the average outcome from the real data y_0. The original model estimation yields $\hat{\beta}_1$, $\hat{\beta}_2$, $\hat{\delta}_1$, $\hat{\delta}_2$, and $\hat{\gamma}$ in estimation results from Table (4) to Table (7). If we restrict $\mathbf{G}_2$ to a zero matrix and re-row-normalize $\mathbf{G}_1$, the outcome y_c will be the counterfactual outcome under the strictly homogeneous networks. A strictly homogeneous network can be obtained by restricting $\mathbf{G}_2 = \mathbf{0}$. We count each adolescent's number of the same type of friends. The adolescent's number of the same type of friends is also the number of all friends since in this case each adolescent doesn't have different types of friends. We row-normalize the new $\mathbf{G}_1$ with the number of one's same type of friends. The re-row-normalization of $\mathbf{G}_1$ ensures that entries of each row sum up to 1. We call the re-row-normalized matrix $\mathbf{G}_1'$.

We plug $\mathbf{G}_2 = \mathbf{0}$, $\mathbf{G}_1'$, the real characteristics x, and the real estimation coefficients $\hat{\beta}_1$, $\hat{\beta}_2$, $\hat{\delta}_1$, $\hat{\delta}_2$, and $\hat{\gamma}$ into Equation (7). We obtain

$$y_c = \hat{\beta}_1 \mathbf{G}_1' y_c + \hat{\gamma} x + \hat{\delta}_1 \mathbf{G}_1' x$$

or

$$y_c = (\mathbf{I} - \hat{\beta}_1 \mathbf{G}_1')^{-1} (\hat{\gamma} \mathbf{I} + \hat{\delta}_1 \mathbf{G}_1') x. \tag{8}$$

y_c is the counterfactual outcome under $\mathbf{G}_2 = 0$, where all the adolescents only have their same type of friends.

We use a t-test to check if the counterfactual outcome y_c equals the real outcome y_0.[6] The null hypothesis is $y_c = y_0$. Table (8) presents the difference between y_c and y_0. A low p-

[6]The estimation equation with real friendship networks is $y_0 = \beta_1 \mathbf{G}_1 y_0 + \beta_2 \mathbf{G}_2 y_0 + \gamma x + \delta_1 \mathbf{G}_1 x + \delta_2 \mathbf{G}_2 x + \epsilon$. The equation with counterfactual friendship networks is $y_c = \beta_1 \mathbf{G}_1' y_c + \gamma x + \delta_1 \mathbf{G}_1' x$. To compare the counterfactual and real equation by parts, in section 3 of the online appendix we list the average value for each part of the two equations, y_0, $\hat{\beta}_1 \mathbf{G}_1 y_0$, $\hat{\beta}_2 \mathbf{G}_2 y_0$, $\hat{\gamma} x$, $\hat{\delta}_1 \mathbf{G}_1 x$, $\hat{\delta}_2 \mathbf{G}_2 x$, y_c, $\hat{\beta}_1 \mathbf{G}_1' y_c$, $\hat{\gamma} x$, and $\hat{\delta}_1 \mathbf{G}_1' x$. The online appendix can be downloaded from
sites.google.com/site/shanshan333wang/secondyearpaper

value indicates that the two outcomes are significantly different. A positive difference indicates that the strictly homogeneous network hypothesis will increase the education outcome. A negative difference indicates that the strictly homogeneous network hypothesis will decrease the education outcome.

6.1.2 Full Sample Counterfactual Results

The full sample counterfactual results are in Table (8). For each of the three outcomes in the immigration and ethnicity cases, the difference between the counterfactual outcome and the real outcome under $\mathbf{G}_2 = 0$ is significant. This indicates that as compared to the peer effects from the friendship network in the real data, the peer effects in the friendship network with strictly homogeneous immigration status or ethnicity exert a significantly different influence on all outcomes.

Each of the differences between counterfactual outcomes under $\mathbf{G}_2 = 0$ in the immigration and ethnicity cases is negative. This indicates that individuals in strictly homogeneous immigration or ethnic friendship networks will get lower grade, visit psychological services less frequently and participate in fewer extracurricular activities than those individuals in the friendship networks constructed by the Add Health data.

6.2 Subgroup Counterfactual Analysis

6.2.1 Subgroup Model Setup

We want to evaluate the impact of strictly homogeneous networks[7] on different subgroups. First we divide the full sample into subgroups. In each subgroup, the individuals are from same immigration status or ethnicity. We then estimate

$$y = \beta_1^s \mathbf{G}_1^s y + \beta_2^s \mathbf{G}_2^s y + \gamma^s x + \delta_1^s \mathbf{G}_1^s x + \delta_2^s \mathbf{G}_2^s x + \epsilon,$$

[7] We are also interested in the impact of strictly heterogeneous networks. But the subgroup counterfactual weighting matrix is not calculable. The detailed reason and example are in Section 2 of the online appendix. The online appendix can be downloaded from sites.google.com/site/shanshan333wang/secondyearpaper

where s = *native*, *imm1*, *imm2* or *other imm* in the immigration case and s = *white*, *black*, *Hispanic*, *Asian* or *other eth* in the ethnicity case. $\mathbf{G}_1^s$ and $\mathbf{G}_2^s$ are the real weighting matrices for each subgroup. We obtain $\hat{\beta}_1^s$, $\hat{\beta}_2^s$, $\hat{\delta}_1^s$, $\hat{\delta}_2^s$ and $\hat{\gamma}^s$ for each subgroup. y_0^s is the subgroup outcome in the real data. The analyses are similar to Equation (8), but they are applied to each subgroup. We set $\mathbf{G}_2^s = 0$. We re-row-normalize $\mathbf{G}_1^s$ and we get $\mathbf{G}_1'^s$. Then we have

$$y_c^s = (\mathbf{I} - \hat{\beta}_1^s \mathbf{G}_1'^s)^{-1}(\hat{\gamma}^s \mathbf{I} + \hat{\delta}_1^s \mathbf{G}_1'^s)x.$$

y_c^s is the counterfactual outcome under $\mathbf{G}_2^s = 0$. $y_c^s - y_0^s$ is the difference between subgroup counterfactual outcome and subgroup real outcome.

6.2.2 Subgroup Counterfactual Results

The subgroup counterfactual results are in Table (9). In the immigration case, the average grade of natives is 0.06 points lower in strictly homogeneous friendship networks, as compared to the grade in the real network. Natives participate in fewer extracurricular activities if their networks only have native friends, as compare to the networks with various types of friends.

In the ethnic case, the average grade of whites is 0.5 points lower in strictly homogeneous friendship networks, as compared to the grade in the real network. Whites decrease their participation in extracurricular activities by 2 clubs if they are in the networks which only have white friends, as compare to the networks with various types of friends.[8].

6.3 Policy Implication

The results indicate that the strictly homogeneous friendship networks in both immigration and ethnicity cases could reduce some outcomes. Thus, diversified friendship networks will probably improve some education outcomes. The best network for an adolescent should be a combination of one's same type of friends and different types of friends. High school should

[8]In Table (9), there are four differences with very large absolute value. This is likely because the observations of the subgroups are with small sample size. A coefficient obtained from a larger sample tends to produce a more accurate estimate of the coefficient. The insufficiency of the observations causes a relative inaccurate estimate for βs in the subgroups, and it will further cause the inaccuracy in the subgroup counterfactual outcomes. This problem can be solved by enlarging the sample size in the future work.

prevent adolescents from making friends only with those who are from their same immigration status or ethnicity. It is highly encouraged to increase the communication across groups of different immigration status and ethnicities. School policy to increase diversity in friendship networks requires school diversity. Schools may arrange students to sit near other types of students. Schools may suggest students have greetings in hallways with all types of friends and join different clubs with other types of students. Students are also encouraged to communicate with various types of peers outside of school. For example, they can attend a party or activity outside of school with other types of students, or they can invite other types of students to their homes. Teachers can also help to set up study groups of students from different immigration and ethnic backgrounds. The introduction of interracial communication courses may help students to understand more about other ethnic groups and make friends with students from other immigration and ethnicity groups.

7 Conclusion

Social networks play an important role in adolescents' education in the United States, including academic grades, frequency of psychological service visits and participation in extracurricular activity. Previous studies do not consider the heterogeneity of peer effects from heterogeneous friends. We consider the heterogeneity in effects from various types of peers. Using Add Health data with a unique friendship nomination dataset, we quantify endogenous and exogenous peer effects among American adolescents. We create homophily matrix and heterophily matrix to disentangle the impacts from two types. The generalized 2SLS is applied with additional instruments from the average characteristics of the same type of friends' same type of friends and the average characteristics of the different type of friends' different type of friends. Empirical results show that adolescents' friends of their same type are more influential than friends of their different types in some outcome such as the frequency of psychological service visits in both immigration and ethnicity cases. Adolescents' friends of their different types have more endogenous peer effects than friends of their same type on other outcomes such as academic grades and extracurricular activity participation in the immigration case. In the ethnicity case, adolescents' friends of their same type have more endogenous peer effects than friends of their different types on academic grades and extracurricular activity participation. The counterfactual study assumes that each student is interacting in a strictly homogeneous friendship network. The strictly homogeneous network setting implies some lower education outcomes. School should encourage students to communicate with the same type of friends as well as different types of friends.

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Table 1: Summary Statistics

Variable	**Mean**	**Standard Deviation**
Immigration and Ethnic Group		
Immigration Status		
Native	0.794	0.484
Imm1	0.104	0.349
Imm2	0.062	0.267
Other Imm	0.040	0.360
Ethnicity		
White	0.585	0.494
Black	0.158	0.406
Hispanic	0.131	0.425
Asian	0.060	0.227
Other Eth	0.066	0.276
Dependent Variable		
Academic Grade	11.189	3.263
Frequency of Psychological Service Visits	2.766	0.935
Extracurricular Activity Participation	2.293	3.463
Control Variable		
Female	0.426	0.495
Age	14.996	1.741
Live with mother	0.870	0.336
Parents' Participation in Job Market	0.779	0.415
Mother's Education		
No High School	0.101	0.301
Finish High School	0.240	0.427
No College	0.112	0.316
Finish College	0.230	0.421
Do not know	0.317	0.466
Physical Health Index	3.893	0.985
TV Watching Index	2.434	1.137
Hard Work Index	1.969	0.976
Observations	1350	

Table 2: Friends Number Distribution

Number of friends	Freq.	Percent	Cum.
0	288	55.71	55.71
1	51	9.86	65.57
2	46	8.9	74.47
3	26	5.03	79.5
4	30	5.8	85.3
5	34	6.58	91.88
6	7	1.35	93.23
7	18	3.48	96.71
8	9	1.74	98.45
9	3	0.58	99.03
10	5	0.97	100
Total	517	100	

Note: Half of the adolescents do not have friends. They are kept because they can be nominated as other people's friends.

Table 3: Friendship Network Composition for Subgroups

(a) Friendship Network Composition for Immigration Subgroups

	Native	Imm1	Imm2	Other Imm
Native's friend	<u>89.3%</u>	6.2 %	1.6 %	2.9 %
Imm1's friend	61%	<u>25.1%</u>	10.9%	3.0 %
Imm2's friend	38.9%	25.1%	<u>31.3%</u>	4.7 %
Other Imm's friend	80.2%	8.0%	6.2 %	<u>5.5%</u>
Sample Mean	79.4%	10.4%	6.2%	4.1%

(b) Friendship Network Composition for Ethnicity Subgroups

	White	Black	Hispanic	Asian	Other Eth
White's friend	<u>83.8%</u>	2.2 %	5.0 %	2.6%	6.3%
Black's friend	9.6 %	<u>78.6%</u>	7.4 %	1.6%	2.8%
Hispanic's friend	35.5%	11.5%	<u>41.6%</u>	5.6 %	5.8%
Asian's friend	34.6%	4.9%	9.4 %	<u>46%</u>	5.1%
Other Eth's friend	69.5%	7.0%	9.4 %	4.4 %	<u>9.7%</u>
Sample Mean	58.4%	15.9%	13.2%	6.0%	6.6%

Note: Sample mean is the percentage of a certain group in the whole sample.

Table 4: Endogenous and Exogenous Estimation Result - Immigration

Outcome	**Academic Grade**			**Psychological Service**			**Extracurricular Activity**		
	Same Type *Coeff* *(se)*	Different Type *Coeff* *(se)*	Wald	Same Type *Coeff* *(se)*	Different Type *Coeff* *(se)*	Wald	Same Type *Coeff* *(se)*	Different Type *Coeff* *(se)*	Wald
β Endogenous Effects	β_1	β_2	p-value	β_1	β_2	p-value	β_1	β_2	p-value
Endogenous Effects	0.163	−0.298	0.024	0.672*	0.198	0.032	0.233	−0.476	0.055
	(0.336)	(0.256)		(0.371)	(0.279)		(0.534)	(0.358)	
δ Exogenous Effects	δ_1	δ_2	p-value	δ_1	δ_2	p-value	δ_1	δ_2	p-value
Age	−0.183**	0.074	0.000	−0.103***	−0.019	0.000	−0.096	−0.036	0.355
	(0.091)	(0.057)		(0.030)	(0.023)		(0.085)	(0.053)	
Female	0.104	0.139	0.911	−0.060	−0.004	0.575	0.080	0.047	0.923
	(0.436)	(0.303)		(0.133)	(0.090)		(0.489)	(0.307)	
Mother presence	−0.259	0.440	0.472	−0.394	−0.191	0.446	0.273	0.088	0.803
	(1.353)	(0.932)		(0.370)	(0.247)		(0.986)	(0.686)	
Mother no HS	−0.677	−0.305	0.609	0.462	0.409*	0.805	−0.449	−0.077	0.536
	(1.095)	(0.790)		(0.320)	(0.231)		(0.830)	(0.618)	
Mother finish HS	0.205	0.137	0.900	0.139	0.033	0.516	−0.117	0.085	0.673
	(0.749)	(0.533)		(0.224)	(0.162)		(0.730)	(0.534)	
Mother no college	0.680	0.637	0.942	0.094	0.063	0.862	0.142	0.501	0.517
	(0.838)	(0.593)		(0.250)	(0.182)		(0.814)	(0.569)	
Mother finish college	0.443	0.324	0.853	0.040	0.028	0.948	0.128	0.588	0.433
	(0.906)	(0.645)		(0.246)	(0.175)		(0.879)	(0.594)	
Parents work	−0.436	0.380	0.218	0.073	−0.005	0.727	0.442	0.689	0.684
	(0.912)	(0.690)		(0.317)	(0.223)		(0.829)	(0.570)	
Physical health	0.264	0.242	0.921	−0.004	−0.020	0.759	0.246	0.261	0.925
	(0.304)	(0.258)		(0.075)	(0.061)		(0.236)	(0.161)	
Watch TV	0.020	0.073	0.730	0.045	0.038	0.896	−0.350*	−0.235	0.472
	(0.211)	(0.138)		(0.080)	(0.052)		(0.193)	(0.143)	
Hard work	0.092	0.207	0.741	−0.042	−0.041	0.990	0.163	0.197	0.859
	(0.555)	(0.394)		(0.139)	(0.097)		(0.282)	(0.196)	
Classroom fixed effects considered, estimates omitted									

Note: The sample size is 1350 . Standard errors are in parentheses. * Significant at the 10% level. ** Significant at the 5% level. *** Significant at the 1% level.

Table 5: Endogenous and Exogenous Estimation Result - Ethnicity

Outcome	**Academic Grade**			**Psychological Service**			**Extracurricular Activity**		
	Same Type *Coeff* *(se)*	Different Type *Coeff* *(se)*	Wald	Same Type *Coeff* *(se)*	Different Type *Coeff* *(se)*	Wald	Same Type *Coeff* *(se)*	Different Type *Coeff* *(se)*	Wald
β Endogenous Effects	β_1	β_2	p-value	β_1	β_2	p-value	β_1	β_2	p-value
Endogenous Effects	0.548	-0.003	0.011	0.232	0.172	0.790	0.326	-0.308	0.080
	(0.354)	(0.279)		(0.357)	(0.293)		(0.611)	(0.380)	
δ Exogenous Effects	δ_1	δ_2	p-value	δ_1	δ_2	p-value	δ_1	δ_2	p-value
Age	-0.251^{***}	0.022	0.000	-0.068^{**}	-0.021	0.061	-0.063	-0.0609	0.961
	(0.092)	(0.059)		(0.034)	(0.023)		(0.078)	(0.053)	
Female	-0.170	0.094	0.436	-0.124	0.019	0.126	0.117	0.151	0.915
	(0.444)	(0.307)		(0.129)	(0.087)		(0.431)	(0.284)	
Mother presence	-0.496	-0.067	0.647	-0.444	-0.237	0.425	-0.433	0.00731	0.532
	(1.387)	(0.947)		(0.367)	(0.239)		(0.957)	(0.652)	
Mother no HS	0.516	0.087	0.599	0.062	0.390^{*}	0.191	0.157	0.148	0.988
	(1.122)	(0.789)		(0.363)	(0.225)		(0.807)	(0.566)	
Mother finish HS	0.622	0.323	0.615	-0.018	0.122	0.425	0.243	0.171	0.906
	(0.791)	(0.536)		(0.223)	(0.156)		(0.833)	(0.503)	
Mother no college	0.457	0.510	0.934	-0.064	0.140	0.331	0.362	0.473	0.873
	(0.886)	(0.599)		(0.268)	(0.176)		(0.939)	(0.553)	
Mother finish college	0.173	0.167	0.993	-0.046	0.078	0.512	0.699	0.571	0.859
	(0.949)	(0.662)		(0.246)	(0.169)		(1.004)	(0.588)	
Parents work	-0.883	0.005	0.199	0.223	0.006	0.271	0.178	0.605	0.444
	(0.957)	(0.702)		(0.278)	(0.220)		(0.763)	(0.544)	
Physical health	0.035	0.054	0.932	0.093	-0.017	0.038	0.280	0.264^{*}	0.933
	(0.318)	(0.273)		(0.077)	(0.061)		(0.260)	(0.157)	
Watch TV	0.122	0.046	0.619	0.132^{*}	0.044	0.107	-0.168	-0.139	0.875
	(0.206)	(0.139)		(0.078)	(0.052)		(0.232)	(0.137)	
Hard work	-0.617	-0.226	0.271	0.051	-0.014	0.454	-0.208^{*}	0.0426	0.195
	(0.555)	(0.420)		(0.123)	(0.097)		(0.263)	(0.178)	
Classroom fixed effects considered, estimates omitted									

Note: The sample size is 1350. Standard errors are in parentheses. * Significant at the 10% level. ** Significant at the 5% level. *** Significant at the 1% level.

Table 6: Personal Characteristics and Constant Estimation Result: Immigration

	Academic Grade	**Psychological Service**	**Extracurricular Activity**
	Coeff	*Coeff*	*Coeff*
	(se)	*(se)*	*(se)*
Age	0.306***	0.105***	0.062**
	(0.033)	(0.010)	(0.028)
Female	0.405**	−0.099*	0.147
	(0.182)	(0.053)	(0.159)
Mother presence	0.544	0.165	−0.125
	(0.431)	(0.128)	(0.378)
Mother no HS	−0.872**	−0.085	−0.481
	(0.360)	(0.112)	(0.321)
Mother finish HS	0.155	0.046	−0.186
	(0.300)	(0.094)	(0.280)
Mother no college	0.198	−0.044	−0.066
	(0.326)	(0.101)	(0.300)
Mother finish college	0.854***	0.082	0.434
	(0.329)	(0.097)	(0.321)
Parents work	1.681***	0.237**	0.321
	(0.330)	(0.104)	(0.291)
Physical health	0.395***	0.074***	0.112
	(0.092)	(0.029)	(0.083)
Watch TV	−0.011	0.051**	0.097
	(0.080)	(0.024)	(0.079)
Hard work	1.167***	0.163***	0.073
	(0.120)	(0.036)	(0.107)

Note: The sample size is 1350. Standard errors are in parentheses.
* Significant at the 10% level.
** Significant at the 5% level.
*** Significant at the 1% level.

Table 7: Personal Characteristics and Constant Estimation Result: Ethnicity

	Academic Grade	**Psychological Service**	**Extracurricular Activity**
	Coeff	*Coeff*	*Coeff*
	(se)	*(se)*	*(se)*
Age	0.305***	0.0973***	0.0549**
	(0.033)	(0.010)	(0.026)
Female	0.509***	−0.0819	0.227
	(0.179)	(0.051)	(0.145)
Mother presence	0.275	0.15	0.0366
	(0.432)	(0.122)	(0.371)
Mother no HS	−0.796**	−0.061	−0.527*
	(0.364)	(0.109)	(0.312)
Mother finish HS	0.138	0.0644	−0.247
	(0.301)	(0.090)	(0.277)
Mother no college	0.237	−0.0505	−0.157
	(0.328)	(0.098)	(0.298)
Mother finish college	0.952***	0.0706	0.461
	(0.331)	(0.094)	(0.294)
Parents work	1.421***	0.307***	0.388
	(0.337)	(0.095)	(0.279)
Physical health	0.395***	0.083***	0.109
	(0.091)	(0.028)	(0.080)
Watch TV	−0.0256	0.0488**	0.0545
	(0.081)	(0.024)	(0.070)
Hard work	1.225***	0.179***	0.122
	(0.117)	(0.035)	(0.100)

Note: The sample size is 1350. Standard errors are in parentheses.
* Significant at the 10% level.
** Significant at the 5% level.
*** Significant at the 1% level.

Table 8: Full Sample Counterfactual Analysis: t-test

		y_o	y_c	$y_0 - y_c$
Immigration Status	Academic Grade	11.189	11.128	-0.061^{***}
				(0.000)
	Psychological Service	2.766	2.751	-0.015^{***}
				(0.000)
	Extracurricular Activity	2.293	2.261	-0.031^{**}
				(0.035)
Ethnicity	Academic Grade	11.189	11.112	-0.077^{***}
				(0.000)
	Psychological Service	2.766	2.746	-0.020^{***}
				(0.000)
	Extracurricular Activity	2.293	2.278	-0.014^{***}
				(0.000)

Note:
1. y_c is the counterfactual outcome when $\mathbf{G}_2 = \mathbf{0}$. y_0 is the outcome from the real data. Negative coefficient indicates the counterfactual outcome is less than real data outcome.
2. P-values are in parentheses. * y_c and y_0 are significantly different at the 10% level. ** y_c and y_0 are significantly different at the 5% level. *** y_c and y_0 are significantly different at the 1% level.

Table 9: Subgroup Counterfactual Analysis: t-test

$y_0 - y_c^s$	Native	Imm1	Imm2	Other imm	
Academic Grade	-0.059^{**}	0.059	26.908^{**}	-68.935	-
	(0.033)	(0.582)	(0.016)	(0.323)	-
Psychological Service	-0.012	-0.005	-0.902^{**}	6.359	-
	(0.539)	(0.678)	(0.037)	(0.323)	-
Extracurricular Activity	-2.240^{***}	0.020	13.168^{**}	-36.300	-
	(0.000)	(0.264)	(0.036)	(0.323)	-

$y_0 - y_c^s$	White	Black	Hispanic	Asian	Other eth
Academic Grade	-0.500^{***}	0.082	0.014	-0.077	-0.093
	(0.000)	(0.230)	(0.141)	(0.409)	(0.375)
Psychological Service	-0.007	0.049	-0.017	-0.025	-0.025
	(0.700)	(0.470)	(0.920)	(0.930)	(0.676)
Extracurricular Activity	-2.040^{***}	0.347^{**}	-0.012	0.005	-0.046
	(0.000)	(0.040)	(0.986)	(0.430)	(0.703)

Note:
1. y_c^s is the counterfactual outcome when $\mathbf{G}_2 = \mathbf{0}$. y_0 is the outcome from the real data.
2. In the immigration case, there are four differences in imm2 and other imm with very big absolute value. This is likely because in subgroup counterfactual analysis, the sample size for the subgroup is too small. The insufficiency of the observations causes a relative inaccurate estimate for βs in the subgroups, and it will further cause the inaccuracy in the subgroup counterfactual outcomes. This can be solved in future works by enlarging the sample size.

Appendix

A Proof of instruments in same and different-type linear-in-mean model

The empirical model in our paper[9] is

$$y = \alpha + \beta_1 \mathbf{G}_1 y + \beta_2 \mathbf{G}_2 y + \gamma x + \delta_1 \mathbf{G}_1 x + \delta_2 \mathbf{G}_2 x + \epsilon.$$

The reduced form is

$$\begin{aligned} y &= \alpha(\mathbf{I} - \beta_1 \mathbf{G}_1 - \beta_2 \mathbf{G}_2)^{-1}\iota + \gamma(\mathbf{I} - \beta_1 \mathbf{G}_1 - \beta_2 \mathbf{G}_2)^{-1} x \\ &\quad + (\mathbf{I} - \beta_1 \mathbf{G}_1 - \beta_2 \mathbf{G}_2)^{-1}(\delta_1 \mathbf{G}_1 + \delta_2 \mathbf{G}_2) x + (\mathbf{I} - \beta_1 \mathbf{G}_1 - \beta_2 \mathbf{G}_2)^{-1}\epsilon. \end{aligned} \tag{9}$$

Similar to Bramoullé et al. (2009), we write$(\mathbf{I} - \beta_1 \mathbf{G}_1 - \beta_2 \mathbf{G}_2) = \sum_{k=0}^{\infty} (\beta_1 \mathbf{G}_1 + \beta_2 \mathbf{G}_2)^k$. Plug it in Equation (9), we have

$$\begin{aligned} y &= \alpha \sum_{k=0}^{\infty} (\beta_1 \mathbf{G}_1 + \beta_2 \mathbf{G}_2)^k \iota + \sum_{k=0}^{\infty} (\beta_1 \mathbf{G}_1 + \beta_2 \mathbf{G}_2)^k \gamma x \\ &\quad + \sum_{k=0}^{\infty} (\beta_1 \mathbf{G}_1 + \beta_2 \mathbf{G}_2)^k (\delta_1 \mathbf{G}_1 + \delta_2 \mathbf{G}_2) x + \sum_{k=0}^{\infty} (\beta_1 \mathbf{G}_1 + \beta_2 \mathbf{G}_2)^k \epsilon. \end{aligned}$$

Plug $\sum_{k=0}^{\infty} (\beta_1 \mathbf{G}_1 + \beta_2 \mathbf{G}_2)^k = \sum_{k=0}^{\infty} (\mathbf{I} + \beta_1 \mathbf{G}_1 + \beta_2 \mathbf{G}_2)^{k+1}$ into the above equation and call the constant "cons", we get

$$\begin{aligned} y &= cons + \gamma x + \sum_{k=0}^{\infty} (\beta_1 \mathbf{G}_1 + \beta_2 \mathbf{G}_2)^{k+1} \gamma x + \\ &\quad + \sum_{k=0}^{\infty} (\beta_1 \mathbf{G}_1 + \beta_2 \mathbf{G}_2)^k (\delta_1 \mathbf{G}_1 + \delta_2 \mathbf{G}_2) x + \sum_{k=0}^{\infty} (\beta_1 \mathbf{G}_1 + \beta_2 \mathbf{G}_2)^k \epsilon. \end{aligned} \tag{10}$$

Expand the equation, we have

$$\begin{aligned} \sum_{k=0}^{\infty} (\beta_1 \mathbf{G}_1 + \beta_2 \mathbf{G}_2)^{k+1} &= \sum_{k=0}^{\infty} ((\beta_1 \mathbf{G}_1)^{k+1} (\beta_2 \mathbf{G}_2)^0 + (\beta_1 \mathbf{G}_1)^k (\beta_2 \mathbf{G}_2)^1 \\ &\quad + (\beta_1 \mathbf{G}_1)^{k-1} (\beta_2 \mathbf{G}_2)^2 + ... + (\beta_1 \mathbf{G}_1)^1 (\beta_2 \mathbf{G}_2)^k + (\beta_1 \mathbf{G}_1)^0 (\beta_2 \mathbf{G}_2)^{k+1}). \end{aligned}$$

Reorganize the equation to take the first item and the last item together and the rest together,

$$\begin{aligned} \sum_{k=0}^{\infty} (\beta_1 \mathbf{G}_1 + \beta_2 \mathbf{G}_2)^{k+1} &= \sum_{k=0}^{\infty} [((\beta_1 \mathbf{G}_1)^{k+1} (\beta_2 \mathbf{G}_2)^0 + (\beta_1 \mathbf{G}_1)^0 (\beta_2 \mathbf{G}_2)^{k+1} \\ &\quad + ((\beta_1 \mathbf{G}_1)^k (\beta_2 \mathbf{G}_2)^1 + (\beta_1 \mathbf{G}_1)^{k-1} (\beta_2 \mathbf{G}_2)^2 + ... + (\beta_1 \mathbf{G}_1)^1 (\beta_2 \mathbf{G}_2)^k)] \\ &= \sum_{k=0}^{\infty} [((\beta_1 \mathbf{G}_1)^{k+1} (\beta_2 \mathbf{G}_2)^0 + (\beta_1 \mathbf{G}_1)^0 (\beta_2 \mathbf{G}_2)^{k+1})] + N(\cdot) \end{aligned} \tag{11}$$

$$\text{where} \qquad N(\cdot) = \sum_{k=0}^{\infty} [(\beta_1 \mathbf{G}_1)^k (\beta_2 \mathbf{G}_2)^1 + (\beta_1 \mathbf{G}_1)^{k-1} (\beta_2 \mathbf{G}_2)^2 + ... + (\beta_1 \mathbf{G}_1)^1 (\beta_2 \mathbf{G}_2)^k].$$

[9] Detailed proof of instrumentation in a single weighting matrix model is in Section 1 of an online appendix, which can be downloaded from:
sites.google.com/site/shanshan333wang/secondyearpaper

$$\text{Similarly} \quad \sum_{k=0}^{\infty}(\beta_1\mathbf{G}_1+\beta_2\mathbf{G}_2)^k = \sum_{k=0}^{\infty}[((\beta_1\mathbf{G}_1)^k(\beta_2\mathbf{G}_2)^0+(\beta_1\mathbf{G}_1)^0(\beta_2\mathbf{G}_2)^k)]+M(\cdot), \tag{12}$$

$$\text{where} \quad M(\cdot) = \sum_{k=0}^{\infty}[(\beta_1\mathbf{G}_1)^{k-1}(\beta_2\mathbf{G}_2)^1+(\beta_1\mathbf{G}_1)^{k-2}(\beta_2\mathbf{G}_2)^2+...+(\beta_1\mathbf{G}_1)^1(\beta_2\mathbf{G}_2)^{k-1}]. \tag{13}$$

Plug Equation (11) and Equation (12) in Equation (10)

$$\begin{aligned} y &= cons+\gamma x+\gamma[\sum_{k=0}^{\infty}(\beta_1^{k+1}\mathbf{G}_1^{k+1}+\beta_2^{k+1}\mathbf{G}_2^{k+1})+N(\cdot)]x+ \\ &\quad [\sum_{k=0}^{\infty}(\beta_1^{k}\mathbf{G}_1^{k}+\beta_2^{k}\mathbf{G}_2^{k})+M(\cdot)](\delta_1\mathbf{G}_1+\delta_2\mathbf{G}_2)x+\sum_{k=0}^{\infty}(\beta_1\mathbf{G}_1+\beta_2\mathbf{G}_2)^k\epsilon. \end{aligned}$$

Expand and reorganize the equation

$$\begin{aligned} y &= cons+\gamma x+\gamma\sum_{k=0}^{\infty}\beta_1^{k+1}\mathbf{G}_1^{k+1}x+\gamma\sum_{k=0}^{\infty}\beta_2^{k+1}\mathbf{G}_2^{k+1}x+\sum_{k=0}^{\infty}(\beta_1^{k}\mathbf{G}_1^{k})\delta_1\mathbf{G}_1x+\sum_{k=0}^{\infty}(\beta_1^{k}\mathbf{G}_1^{k})\delta_2\mathbf{G}_2x \qquad (14) \\ &\quad +\sum_{k=0}^{\infty}(\beta_2^{k}\mathbf{G}_2^{k})\delta_1\mathbf{G}_1x+\sum_{k=0}^{\infty}(\beta_2^{k}\mathbf{G}_2^{k})\delta_2\mathbf{G}_2x+\gamma N(\cdot)x+M(\cdot)(\delta_1\mathbf{G}_1+\delta_2\mathbf{G}_2)x+\sum_{k=0}^{\infty}(\beta_1\mathbf{G}_1+\beta_2\mathbf{G}_2)^k\epsilon. \end{aligned}$$

Define $C(\cdot)=\sum_{k=0}^{\infty}(\beta_1^k\mathbf{G}_1^k)\delta_2\mathbf{G}_2x+\sum_{k=0}^{\infty}(\beta_2^k\mathbf{G}_2^k)\delta_1\mathbf{G}_1x$, define $T(\cdot)=\gamma N(\cdot)+M(\cdot)(\delta_1\mathbf{G}_1+\delta_2\mathbf{G}_2)x+C(\cdot)$, and define $error=\sum_{k=0}^{\infty}(\beta_1\mathbf{G}_1+\beta_2\mathbf{G}_2)^k\epsilon$. Reorganize Equation (14), we have

$$\begin{aligned} y &= cons+\gamma x+\gamma\sum_{k=0}^{\infty}\beta_1^{k+1}\mathbf{G}_1^{k+1}x+\gamma\sum_{k=0}^{\infty}\beta_2^{k+1}\mathbf{G}_2^{k+1}x+\sum_{k=0}^{\infty}(\beta_1^{k}\mathbf{G}_1^{k})\delta_1\mathbf{G}_1x+\sum_{k=0}^{\infty}(\beta_2^{k}\mathbf{G}_2^{k})\delta_2\mathbf{G}_2x+T(\cdot)+error \\ &= cons+\gamma x+(\gamma\beta_1+\delta_1)\sum_{k=0}^{\infty}\beta_1^{k}\mathbf{G}_1^{k+1}x+(\gamma\beta_2+\delta_2)\sum_{k=0}^{\infty}\beta_2^{k}\mathbf{G}_2^{k+1}x+T(\cdot)+error \end{aligned}$$

To identify instruments, we take expectations of $\mathbf{G}_iy$ on x, respectively.

$$E[\mathbf{G}_1y|x]=cons+\gamma\mathbf{G}_1x+(\gamma\beta_1+\delta_1)\sum_{k=0}^{\infty}\beta_1^{k}\mathbf{G}_1^{k+2}x+(\gamma\beta_2+\delta_2)\sum_{k=0}^{\infty}\beta_2^{k}\mathbf{G}_1\mathbf{G}_2^{k+1}x+E[\mathbf{G}_1T(\cdot)|x]$$

The instruments can be a combination of $\mathbf{I};\mathbf{G}_1;\mathbf{G}_1^2,\mathbf{G}_1^3,...,\mathbf{G}_2^{\infty};\mathbf{G}_1\mathbf{G}_2,\mathbf{G}_1\mathbf{G}_2^2,\mathbf{G}_1\mathbf{G}_2^3,...,\mathbf{G}_1\mathbf{G}_2^{\infty}$.

$$E[\mathbf{G}_2y|x]=cons+\gamma\mathbf{G}_2x+(\gamma\beta_1+\delta_1)\sum_{k=0}^{\infty}\beta_1^{k}\mathbf{G}_2\mathbf{G}_1^{k+1}x+(\gamma\beta_2+\delta_2)\sum_{k=0}^{\infty}\beta_2^{k}\mathbf{G}_2^{k+2}x+E[\mathbf{G}_2T(\cdot)|x]$$

The instruments can be a combination of $\mathbf{I};\mathbf{G}_2;\mathbf{G}_2\mathbf{G}_2,\mathbf{G}_2\mathbf{G}_1^2,\mathbf{G}_2\mathbf{G}_1^3,...\mathbf{G}_2\mathbf{G}_1^{\infty};\mathbf{G}_2^2,\mathbf{G}_2^3,...,\mathbf{G}_2^{\infty}$.
We select $\mathbf{I},\mathbf{G}_1x,\mathbf{G}_1^2x,\mathbf{G}_2x,\mathbf{G}_2^2x$ as instruments. To proof the instruments are linear independent, we analyze the matrix. The interaction matrix is $\mathbf{G}_i=\begin{pmatrix}\mathbf{\Gamma}_{s_i} & 0 \\ 0 & \mathbf{\Gamma}_{s_i}\end{pmatrix}$ where Γ_{s_i} is block diagonal with diagonal blocks $\frac{1}{s_i-1}$ and $i=1,2$. So combining $\mathbf{I}$, $\mathbf{G}_i$, and $\mathbf{G}_i^2$, we find that

$$\mathbf{G}_i^2=\frac{s_i-2}{s_i-1}\mathbf{G}_i+\frac{1}{s_i-1}\mathbf{I} \quad \text{and} \quad \mathbf{G}_1^2+\mathbf{G}_2^2=\frac{s_1-2}{s_1-1}\mathbf{G}_1+\frac{1}{s_1-1}\mathbf{I}+\frac{s_2-2}{s_2-1}\mathbf{G}_2+\frac{1}{s_2-1}\mathbf{I}.$$

The linear independence requirement is satisfied, so $\mathbf{I},\mathbf{G}_1x,\mathbf{G}_1^2x,\mathbf{G}_2x,\mathbf{G}_2^2x$ are instruments to identify various group model.